Math 3201

9.1 Financial Mathematics: Borrowing Money

Simple vs. Compound Interest

Simple Interest: the amount of interest that you pay on a loan is calculated ONLY based on the amount of money that you borrow. That is, you only pay interest on the money that you borrow.

Compound Interest: interest is paid on two different things:
(i) the amount of money that you borrow and
(ii) the interest that you pay on the money that you borrowed.

When we talk about borrowing money, there are two groups of people involved:

1. The **customer** is the person/group who borrows the money.

2. The **lender** is the the person/institution who loans out the money. It may be a friend, a family member, bank, etc.

Different groups benefit differently from various types of interest. For example, a compound rate involves a greater amount of interest being paid than a simple rate. The lender, or person/group who loans out money would benefit more from this, since they will earn more money in interest. The customer, or person/group who borrows the money is disadvantaged by this however because it means they will pay out more in interest.

There are two main factors that affect the advantages and disadvantages of different types of interest:

1. Whether you are a **customer or lender**: ie. compound interest is an advantage for a lender, but adisadvantage for a customer, as discussed previously.

2. Whether you are **borrowing or investing** money: simple interest would be better for a customer who is borrowing money since it results in the customer having to pay out less money in interest. However, compound interest is better for a customer who is investing money since it means they will gain more money on their investment through interest.

When are simple and compound interest rates typically used?

Simple Interest:
- loans from family members or friends
- loans or investments of a year or less

Compound Interest:
- Most products offered by financial institutions. See table below.

Borrowing	Investing
Loan Credit Card Mortgage Line of Credit Student Loan	Savings Account Chequing Account GIC Canada Savings Bond

***Exception:** GICs and Canada Savings Bonds can have either simple or compound interest.

Calculations Involving Simple and Compound Interest

Simple Interest Formula

$$A = P + Prt$$

where
- A represents the amount present
- P represents the principal amount
- r = interest percentage divided by 100
- t represents the number of years

We can factor the P out of the right hand side of the equation to give:

$$A = P(1 + rt)$$

Compound Interest Formula:

$$A = P(1 + i)^n$$

where
- P is the the principle amount
- i is the interest rate per compounding period
- n is the number of compounding periods

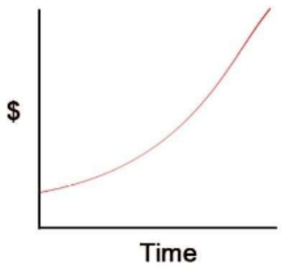
Notice that i is the interest rate **per compounding period**. If interest is compounded x times each year, then the given percentage must be divided by x to come up with i. Compounding periods are usually daily, weekly, semimonthly, monthly, quarterly, semiannually or annually. The table below shows how many times interest is paid, and the interest rate for each of these options.

Compounding Period	Number of Times Interest Is Paid	Interest Rate per Compounding Period, i
daily	365 times per year	$i = \frac{\text{annual rate}}{365}$
weekly	52 times per year	$i = \frac{\text{annual rate}}{52}$
semi-monthly	24 times per year	$i = \frac{\text{annual rate}}{24}$
monthly	12 times per year	$i = \frac{\text{annual rate}}{12}$
quarterly	4 times per year	$i = \frac{\text{annual rate}}{4}$
semi-annually	2 times per year	$i = \frac{\text{annual rate}}{2}$
annually	1 time per year	$i = \frac{\text{annual rate}}{1}$

i = anhnal rate

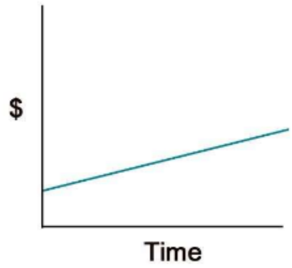
Compound interest increases ~~linearly~~ over time.

exponential

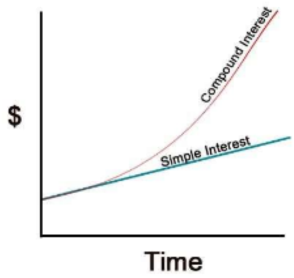


Simple interest increases ~~exponentially~~ over time.

linear



Both types graphed on the same axis.



Example 1:

James intends to go to university in five years. His grandmother decides to invest $2000 in a Guaranteed Investment Certificate (GIC) to help with his first-year expenses.

(A) How much would the GIC be worth in 5 years if she chooses a simple interest GIC at 3% annual interest?

$P = 2000$
$r = 0.03$
$t = 5$

$A = P(1 + rt)$
$= 2000(1 + 0.03(5))$
$= 2000(1.15)$
$= \boxed{2300}$

Interest $2300 - 2000$
$= 300$

(B) How much would it be worth if the interest is compounded monthly?

$P = 2000$
$i = \frac{0.03}{12} = 0.0025$
$n = 12 \times 5$
$= 60$

$A = P(1 + i)^n$
$= 2000(1.0025)^{60}$
$= \boxed{2323.23}$

Interest $2323.23 - 2000$
$= 323.23$

(D) Which is better for James?

Compounded interest. Gets more $.
Simple interest would be better for the bank.

Suppose Peter borrows $1000 from his parents.

(A) How much will he have to pay back in 2 years if they charge 3% simple interest per year?

$P = 1000$
$r = 0.03$
$t = 2$

$A = P(1 + rt)$
$= 1000(1 + 0.03(2))$
$= 1000(1.06)$
$= \boxed{1060.00}$

(B) How much will you have to pay back in 2 years if they charge 3% interest compounded monthly?

$P = 1000$
$i = \frac{0.03}{12} = 0.0025$
$n = 12 \times 2$
$= 24$

$A = 1000(1 + 0.0025)^{24}$
$= 1000(1.0025)^{24}$
$= \boxed{1061.76}$

Example 2:

(C) Explain which is the better option for him.

Simple interest, doesn't pay back as much money!

Predict which of the following investments would yield the greater return:

Option 1: $1000 at 3.5% annual simple interest

Option 2: $1000 at 3% annual compound interest

Verify your prediction by calculating the value of the investments after 5 years.

Option 1

Simple

$A = 1000(1 + 0.035(5))$

$= 1000(1.175)$

$\boxed{= \$1175.00}$

Best Option

Option 2

Compound

$i = \frac{0.03}{1}$

$= 0.03$

$n = 5$

$A = 1000(1 + 0.03)^5$

$= 1000(1.03)^5$

$\boxed{\$1159.27}$

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9.2A Paying Back Loans

A loan can involve **regular loan payments** over the term of the loan or a **single payment** at the end of the term.

We will consider two cases:

1. A loan is paid off using a single payment at the end of the term.
2. A loan is paid off by making regular loan payments only cases in which payment frequency matches the compounding period.

We will start off by looking at loans that are paid off using a single payment at the end of the term.

Examples:

- a farmer making a single lump sum payment on his loan after his crop has been harvested
- a payday loan offered by certain financial service providers

Single Loan Payments

For single loan payments when the loan is paid off in full at the end of the term, we can use the formulas from lesson 1 to determine things such as amount and interest.

Most commonly, there will be a compound interest rate, so we can use the formula:

$$A = P(1+i)^n$$

We could also create a table to show the amount and the interest at the end of each year.

Example 1:

Shannon's employer loaned her $10000 at afixed rate of 6%, compounded annually, to pay for college tuition and textbooks. The loan is to be repaid in full in a single payment on the maturity date, which is at the end of 5 years.

(A) Complete the following table to show the amount owed at the end of each year, along the interest accumulated.

End of Year	Amount ($)	Interest ($)
0		
1		
2		
3		
4		
5		

$P=10\,000$
$i=0.06$
$n=5$

(B) Use the compound interest formula to determine how much Shannon owes her employer at the end of the 5 years.

$A=10000(1+0.06)^5 = \boxed{\$13\,382.26}$

(C) How much interest does she pay on the loan?

$$\begin{array}{r} 13382.26 \\ -10000.00 \\ \hline \boxed{\$3382.26} \end{array}$$

$P=10000$
$i=\frac{0.06}{12}=0.005$
$n=5\times12=60$

(D) Calculate the amount that she would ~~own~~ owe after 5 years if interest was compounded monthly instead.

$A=10000(1.005)^{60}$
$=\boxed{\$13488.50}$

(E) How much interest would she pay in this case?

$$\begin{array}{r} 13\,488.50 \\ -10\,000 \\ \hline \boxed{\$3488.50} \end{array}$$

twice per year

Example 2:
Mary borrows \$1000 at 10% interest, compounded semiannually. Sean borrows \$1000 at 10% interest compounded annually. How much interest will each pay at the end of two years?

Mary

$P = 1000$

$i = \frac{0.10}{2} = 0.05$

$n = 2 \times 2 = 4$

$A = 1000(1.05)^4$

$\boxed{\$1215.51}$

$$\begin{array}{r} 1215.51 \\ -1000.00 \\ \hline \$215.51 \end{array}$$

Sean

$P = 1000$

$i = 0.10$

$n = 2$

$A = 1000(1.10)^2$

$= 1210.00$

$$\begin{array}{r} 1210.00 \\ -1000.00 \\ \hline \$210.00 \end{array}$$

In the previous examples, the principal amount, the interest rate and the term of the loan were kept constant, but the number of compounding periods changed.

What impact does the number of compounding periods have on the total amount of interest paid on a loan?

more compounding periods $\Rightarrow$ more interest

When making financial decisions, it is important for students to understand the **rate of interest charged**, as well as the **compounding**, as these can create large differences over long periods of time.

Example 3:
Which represents the lowest interest that would be paid?

(A) 10% compounded daily → 365

(B) 10% compounded monthly

(C) 10% compounded annually

(A) $i = \frac{0.10}{365} = 0.00027$
$n = 1 \times 365 = 365$

$A = P(1.00027)^{365}$
$= 1.105P$ $\boxed{10.5\%}$

(B) $i = \frac{0.10}{12} = 0.0083$
$n = 1 \times 12 = 12$
$A = P(1.0083)^{12}$
$= 1.104P$ $\boxed{10.4\%}$

(C) $i = 0.10$
$n = 1$
$A = P(1.10)^{1}$
$= 1.10P$ $\boxed{10\%}$

lowest interest

Example 4:
Which represents the lowest interest that would be paid?

(A) 8% compounded daily
(B) 12% compounded monthly

(A) $i = \frac{0.08}{365} = 0.000219$
$n = 365$

$A = P(1.000215)^{365}$
$= 1.083P$ $\boxed{8.3\%}$

Lowest

(B) $i = \frac{0.12}{12} = 0.01$
$n = 12$

$A = P(1.01)^{12}$
$= 1.127P$ $\boxed{12.7\%}$

Determining Interest Rates from A Compound Interest Equation

Example 5:
A loan has interest that is compounded annually. The amount is represented by $A = 1000(1.06)^5$. What is the interest rate?

$A = 1000(1.06)^5$

$1 + i = 1.06$

$i = 1.06 - 1$

$i = 0.06$

compounding period

interest rate $= 0.06 \times 1 = 0.06$ or $\boxed{6\%}$

Calculating the Principal Amount When the Amount Repaid is Known

Example 6:
A student repaid a total of $8456.65 including both the principal and interest. If the interest rate was 3% compounded quarterly for 5 years, what was the principal amount of the loan?

Your Turn:

1. Which situation would result in the greatest interest being paid?
   (A) 8% compounded monthly

   (B) 6% compounded annually

   (C) 6% compounded semiannually

a) $A = P\left(1 + \frac{0.08}{12}\right)^{12} = 1.083P$ $\boxed{8.3\%}$

Greatest interest

b) $A = P\left(1 + \frac{0.06}{1}\right)^{1} = 1.06P$ $\boxed{6.0\%}$

c) $A = P\left(1 + \frac{0.06}{2}\right)^{2} = 1.061P$ $\boxed{6.1\%}$

2. A student repaid a total of $6532.45 including both the principal and interest. If the interest rate was 5% compounded quarterly for 6 years, what was the principal amount of the loan?

$A = 6532.45$

$i = \frac{0.05}{4} = 0.125$

$n = 6 \times 4 = 24$

$6532.45 = P(1.0125)^{24}$

$\frac{6532.45}{1.347} = \frac{P(\cancel{1.347})}{\cancel{1.347}}$

$\boxed{P = \$4849.62}$

3. A loan has interest that is compounded annually. The amount is represented by $A = 6400(1.04)^5$. What is the interest rate?

$1 + i = 1.04$

$i = 1.04 - 1$

$i = 0.04$

$\boxed{4\%}$

$n = 4 \times 2$
$= 8$

4. Jill borrows \$850 at 9% interest, compounded semiannually. Jack borrows \$925 at 9% interest compounded annually. How much interest will each pay at the end of four years?

Jill

$A = 850\left(1 + \frac{0.09}{2}\right)^8$

$= 850(1.045)^8$

\$1208.78

Interest:

$$\begin{array}{r} 1208.78 \\ -\ 850.00 \\ \hline \boxed{358.78} \end{array}$$

Jack

$A = 925(1.09)^4$

$= 1305.71$

Interest:

$$\begin{array}{r} 1305.71 \\ -\ 925.00 \\ \hline \boxed{\$380.71} \end{array}$$

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9.2B Paying Back Loans

Regular/Multiple Loan Payments

For many types of loans, regular loan payments are made at certain time intervals. This is common for mortgages and vehicle loans.

There are three common types of regular payment schedules:

- Biweekly
- Semi Monthly
- Accelerated Biweekly

To see the differences between these, we will consider an example from the Curriculum Guide in a loan payment of $600 per month is made.

The amount paid each year will vary depending on which type of payment plan you choose. The $600 per month payment will work out as follows:

- Biweekly → 600 × 12 ÷ 26 = $276.92
  paid 26 times per year
- Semi Monthly → 600 ÷ 2 = $300.00,
  paid 24 times per year (1st and 15th of each month)
- Accelerated Biweekly → 600 ÷ 2 = $300.00
  paid 26 times per year

For a loan that has a $600 per month payment, determine how much will be paid out at 3 years, using each of the three payment options.

Biweekly $276.92 × 26 × 3 = $21599.76

times per year

years

Semi-Monthly

$300 × 24 × 3 = $21600.00

Accelerated Bi-weekly

$\$300 \times 26 \times 3 \quad \$23\,400.00$

Example 1:

(A) 130 biweekly payments are required to pay off a loan. How many years does this represent?

$\frac{130}{26} = 5$ years

(B) 288 semi monthly payments are required to pay off a loan. How many years does this represent?

$\frac{288}{24} = 12$ years

(C) 390 accelerated biweekly payments are required to pay off a loan. How many years does this represent?

$\frac{390}{26} = 15$ years

When people purchase a vehicle, they often link their loan payment schedule to their payroll schedule. Why is this the case?

When it comes to regular loan payments, we will only examine situations in which the **payment frequency matches the compounding period** For example, if the interest is compounded monthly, then the loan repayment occurs monthly.

The formulas that we learned previously apply **only** to **single loan payments at the end of a term**, and thus **cannot** be used in situations in which there is a regular loan payment. What we will do for these types of questions is refer to a table which shows the payment, interest principal and balance.

Example 2:
Mark is buying an ATV for the summer. The bank offers him a loan of $7499.99 to pay for his ATV with an interest rate of 4.5% compounding monthly. If Mark makes 36 monthly payments of $223.10, calculate the total interest paid at the end of the loan.

(A) Complete the first three rows of the table:

$i = \frac{0.045}{12} = 0.00375$

Month	Payment	Interest	Principal	Balance
				7,499.99
1	223.10	7499.99 × 0.00375 = 28.12	223.10 − 28.12 = 194.98	7499.99 − 194.98 = $7305.01
2	223.10	7305.01 × 0.00375 = 27.39	223.10 − 27.39 = 195.71	7305.01 − 195.71 = 7109.30
3	223.10	7109.30 × 0.00375 = 26.66	223.10 − 26.66 = 196.44	7109.30 − 196.44 = 6912.86

The entire table, as created using software, is shown below:

#/Year	Date	Payment	Interest	Principal	Balance
Loan:	12/01/12				7,499.99
1/01	01/01/13	223.10	28.12	194.98	7,305.01
2/01	02/01/13	223.10	27.39	195.71	7,109.30
3/01	03/01/13	223.10	26.66	196.44	6,912.86
4/01	04/01/13	223.10	25.92	197.18	6,715.68
5/01	05/01/13	223.10	25.18	197.92	6,517.76
6/01	06/01/13	223.10	24.44	198.66	6,319.10
7/01	07/01/13	223.10	23.70	199.40	6,119.70
8/01	08/01/13	223.10	22.95	200.15	5,919.55
9/01	09/01/13	223.10	22.20	200.90	5,718.65
10/01	10/01/13	223.10	21.44	201.66	5,516.99
11/01	11/01/13	223.10	20.69	202.41	5,314.58
12/01	12/01/13	223.10	19.93	203.17	5,111.41
13/02	01/01/14	223.10	19.17	203.93	4,907.48
14/02	02/01/14	223.10	18.40	204.70	4,702.78
15/02	03/01/14	223.10	17.64	205.46	4,497.32
16/02	04/01/14	223.10	16.86	206.24	4,291.08
17/02	05/01/14	223.10	16.09	207.01	4,084.07
18/02	06/01/14	223.10	15.32	207.78	3,876.29
19/02	07/01/14	223.10	14.54	208.56	3,667.73
20/02	08/01/14	223.10	13.75	209.35	3,458.38
21/02	09/01/14	223.10	12.97	210.13	3,248.25
22/02	10/01/14	223.10	12.18	210.92	3,037.33
23/02	11/01/14	223.10	11.39	211.71	2,825.62
24/02	12/01/14	223.10	10.60	212.50	2,613.12
25/03	01/01/15	223.10	9.80	213.30	2,399.82
26/03	02/01/15	223.10	9.00	214.10	2,185.72
27/03	03/01/15	223.10	8.20	214.90	1,970.82
28/03	04/01/15	223.10	7.39	215.71	1,755.11
29/03	05/01/15	223.10	6.58	216.52	1,538.59
30/03	06/01/15	223.10	5.77	217.33	1,321.26
31/03	07/01/15	223.10	4.95	218.15	1,103.11
32/03	08/01/15	223.10	4.14	218.96	884.15
33/03	09/01/15	223.10	3.32	219.78	664.37
34/03	10/01/15	223.10	2.49	220.61	443.76
35/03	11/01/15	223.10	1.66	221.44	222.32
36/03	12/01/15	223.15	0.83	222.32	0.00
	12/31/15	8,031.65	531.66	7,499.99	

Relationships from the Table

To determine the amount of interest paid each month, we need to determine the i-value. Then we multiply the balance from the previous pay period by the i-value.

$$\text{Interest Paid} = \text{i} \times \text{Balance during last payment period}$$

When you make a monthly payment, so much of the payment goes down on the loan amount itself, called the principal, while the rest is paid out as interest.

$$\text{Monthly Payment} = \text{Interest Paid} + \text{Principal Paid}$$

We can rearrange the previous equation to determine how much gets paid on the principal once we figure how much goes on interest.

$$\text{Monthly Payment Interest} - \text{Paid} = \text{Principal Paid}$$

The balance remaining on a loan can be calculated each month by taking the previous balance and subtracting the principal paid during that payment period. Interest paid has no effect on the balance remaining. It is only the principal paid each payment period that will decrease the overall amount left owing on the loan.

$$\text{Current Balance} = \text{Previous Balance Principal} - \text{Paid during Current Payment Period}$$

Your Turn:

1. Lorna is buying a car second hand through a private sale. The bank offers her a loan of $5500 to pay for her car with an interest rate of 4.2% compounding monthly. Lorna makes monthly payments of $210.45. Complete the following table.

$i = \frac{0.042}{12} = 0.0035$

Month	Payment	Interest	Principal	Balance
				5500
1	210.45	5500 × 0.0035 = 19.25	210.45 − 19.25 = 191.20	5500 − 191.20 = $5308.80
2	210.45	5308.80 × 0.0035 = 18.58	210.45 − 18.58 = 191.87	5308.80 − 191.87 = $5116.93
3	210.45	17.91	192.54	$4924.39
4	210.45	17.24	193.21	$4731.18

2. The interest on the loan shown in the chart below is 5% compounded monthly. How much of the second payment is the interest toward the loan?

Payment Period (month)	Payment ($)	Principal Paid ($)	Balance ($)
0			15,000
1	450	387.50	14,612.50
2	450	389.11	14,223.39
3	450	390.74	13,832.65

$450 − $389.11 = $\boxed{\$60.89}$ ← interest

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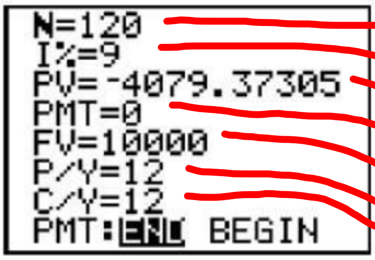
9.3 Paying Back Loans & Amortization Period

Determining the Cost of a Loan Using Technology

There are many online applications that enable people to determine the cost of various loans. These apps enable the user to manipulate different variables such as interest rates, payment periods and compounding frequency as well as observe what impact these variables will have on interest paid and the time taken to pay off a loan. Such apps can be found at various bank web sites such as CIBC, Scotia Bank, TD, and BMO.

There is an app on the TI83 Plus graphing calculator that also enables us to do these things. It is called "Finance". Below are instructions for getting into this app.

Press [APPS], then FINANCE, choose TVM Solver. Press [ENTER]



Screen shot - sample values used.

N = The number of total periods for compounding
I% = the interest rate (usually an annual rate)
PV= Present value
PMT = the periodic payment
FV= Future value
P/Y =Payments periods per year
C/Y =Compounding periods per year

Explanation of what each variable represents.

Notes:
- if a variable is not being used in a question, or it is being solved for, put in zero for its value.
- For our purposes we will always leave PMT at END

To solve for a variable:
- Make sure you put in 0 for its' value.
- Hit "Apps" then "Finance".
- Scroll down to the variable you want to solve for and hit "Enter".

Example 1:

When you were born your grandparents deposited $5,000 in a special account for your 21st birthday. The interest was compounded monthly at 5%. How much will it be worth on your 21st birthday?

N=12 × 21
= 252

(A) Enter the values that you would type in on the TI83 Plus.

N= 252
I%= 5
PV= 5000
PMT= 0
FV= 0
P/Y= 12
C/Y= 12
PMT: END BEGIN

When we enter these values and run the app, the calculator returns the following value:

FV = 14257.12055

(B) How much is the investment worth on your 21st birthday?

$14 257.12

(C) Show how the data entered into the calculator would change if your grandparents not only paid a lump sum of money into an investment, but also added an extra $10 every month.

N= 252
I%= 5
PV= 5000
PMT= 10
FV= 0
P/Y= 12
C/Y= 12
PMT: END BEGIN

FV = 18700.54

(D) What would the investment now be worth on your 21st birthday?

$18700.54

Example 2:

Suppose you find a car for $12,500. You are going to put $3000 down and take a loan for the rest for 4 years? The rate offered to you is 8.3%. What is your car payment each month?

N = 4 × 12
= 48

Complete the following and use a TI83 Plus to solve for the monthly payment.

PV = 12 500 − 3000
= 9500

N= 48
I%= 8.3
PV= 9500
PMT= 0
FV= 0
P/Y= 12
C/Y= 12
PMT: END BEGIN

Solve: PMT = −233.26

Monthly Payment = $233.26

Example 3:

Brittany takes out a loan for $100 000 at 3.25% interest, compounded monthly. She takes 20 years to repay the loan. Ask students to answer to following:

(A) Use a financial application to determine the amount of each monthly payment.

(B) How much interest will she have paid at the end of the 20 years?

N = 20 × 12
= 240

N= 240
I%= 3.25
PV= 100 000
PMT= 0
FV= 0
P/Y= 12
C/Y= 12
PMT: END BEGIN

a) PMT = −567.20

Monthly Payment = $567.20

b) 240 × 567.20
= $136128.00

136128.00
− 100000.00

$36128.00

Amortization Period
This refers to the total amount of time that it takes to pay off a loan. For example, if you take out a five year payment for a new vehicle, then the amortization period for the loan is five years.

If you take on a short amortization period, this will do two things:
- Increase your regular payments (ie. larger monthly payments)
- Decrease the total interest paid.

Thus, it is often to a customers benefit to decrease an amortization period, provided the larger payments are manageable for the customer.

Example 4:
Compare the monthly payments on a mortgage of $300000 at an interest rate of 3.5% compounded monthly, with no down payment, for amortization periods of (A) 25 years and (B) 20 years.

(A)	(B)
N= 300	N= 240
I%= 3.5	I%= 3.5
PV= 300 000	PV= 300000
PMT= 0	PMT= 0
FV= 0	FV= 0
P/Y= 12	P/Y= 12
C/Y= 12	C/Y= 12
PMT: END BEGIN	PMT: END BEGIN

$N = 25 \cdot 12 = 300$

$N = 20 \times 12 = 240$

PMT: −1501.87 monthly

PMT: −1739.88

$300 \times 1501.87 = \boxed{\$450\ 561.00}$

$240 \times 1739.88 = \boxed{\$417\ 571.20}$

(C) How much will you pay overall for each option?

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(D) How much interest gets paid in each case?

25yr: 450 561.00 - 300 000
= $150 561.00

20yr: 417 571.20 - 300 000
= $117 571.20

(E) What are the monthly payments for each option?

25yr: $1501.87

20yr: $1739.88

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9.4 Credit Options

There are various forms of credit available to customers. These include bank loans, credit cards and special promotions that have various conditions. When trying to decide which form is best for them, customers should compare credit options with varying interest rates, compounding periods, annual fees and special limited time offers such as "no interest" periods. Amortization tables, spreadsheets and financial applications should be used to determine monthly payments, total cost, total interest, etc., to ultimately determine the most financially sound investment.

It is important for customers to consider the advantages and disadvantages of using ***line of credit***, ***instore financing options*** and ***credit cards*** for purchasing. When making decisions about credit, customers should consider the following points:

- Which borrowing method generally has the lowest interest rate?
- Are you planning on paying the balance off in full at the end each month or carrying forward a balance? How should this factor into your decision?
- Are there any promotions that you can take advantage of?
- Are there any 'hidden' fees?
- Should you pay more than the minimum required payment? Which borrowing option is best for large purchases?

Example 1:
Brad and Marie decide to order a home gym online. The order totaled $2668 and the shipping cost is $347. They can afford to pay $200 each month. Which credit card should they use?

- Brad's credit card charges 13.9%, compounded daily, with an annual fee of $75.
- Marie's credit card charges 19.8%, compounded daily.

The following amortization tables shows the repayment on both credit cards:

Brad:

#/Year	Date	Payment	Interest	Principal	Balance
Loan:	12/01/12				3,015.00
1/01	01/01/13	200.00	37.20	162.80	2,927.20
2/01	02/01/13	200.00	35.24	164.76	2,762.44
3/01	03/01/13	200.00	30.02	169.98	2,592.46
4/01	04/01/13	200.00	31.21	168.79	2,423.67
5/01	05/01/13	200.00	28.23	171.77	2,251.90
6/01	06/01/13	200.00	27.11	172.89	2,079.01
7/01	07/01/13	200.00	24.22	175.78	1,903.23
8/01	08/01/13	200.00	22.91	177.09	1,726.14
9/01	09/01/13	200.00	20.78	179.22	1,546.92
10/01	10/01/13	200.00	18.02	181.98	1,364.94
11/01	11/01/13	200.00	16.43	183.57	1,181.37
12/01	12/01/13	200.00	13.76	186.24	995.13
Y-T-D 2013	12/31/13	2,400.00	305.12	2,094.87	
Running	12/31/13	2,400.00	305.13	2,094.87	
13/02	01/01/14	200.00	11.98	188.02	807.11
14/02	02/01/14	200.00	9.72	190.28	616.83
15/02	03/01/14	200.00	6.70	193.30	423.53
16/02	04/01/14	200.00	5.10	194.90	228.63
17/02	05/01/14	231.29	2.66	228.63	0.00
Y-T-D 2014	12/31/14	1,031.29	36.16	995.13	
Running	12/31/14	3,431.29	341.29	3,090.00	

Marie:

#/Year	Date	Payment	Interest	Principal	Balance
Loan:	12/01/12				3,015.00
1/01	01/01/13	200.00	51.83	148.17	2,866.83
2/01	02/01/13	200.00	49.28	150.72	2,716.11
3/01	03/01/13	200.00	42.14	157.86	2,558.25
4/01	04/01/13	200.00	43.98	156.02	2,402.23
5/01	05/01/13	200.00	39.95	160.05	2,242.18
6/01	06/01/13	200.00	38.55	161.45	2,080.73
7/01	07/01/13	200.00	34.61	165.39	1,915.34
8/01	08/01/13	200.00	32.93	167.07	1,748.27
9/01	09/01/13	200.00	30.06	169.94	1,578.33
10/01	10/01/13	200.00	26.25	173.75	1,404.58
11/01	11/01/13	200.00	24.15	175.85	1,228.73
12/01	12/01/13	200.00	20.44	179.56	1,049.17
-D 2013	12/31/13	2,400.00	434.17	1,965.82	
Running	12/31/13	2,400.00	434.17	1,965.83	
13/02	01/01/14	200.00	18.04	181.96	867.21
14/02	02/01/14	200.00	14.91	185.09	682.12
15/02	03/01/14	200.00	10.58	189.42	492.70
16/02	04/01/14	200.00	8.47	191.53	301.17
17/02	05/01/14	306.18	5.01	301.17	0.00

(A) After the third payment, which credit card option appears to be better? Do you think this will always be the case?

Add up interest for first 3 months:

Brad: \$102.46 + \$75 = 177.46

Marie: \$143.25

(B) At the end of the sixth month, which credit card appears to be better? Discuss your findings.

Brad: \$189.19 + \$75 = \$264.19

Marie: \$325.73

(C) Overall, which credit card was the better choice and why?

Brad's card is better, pay less interest!

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9.5 Variables Involved in Financing

Part 1: Appreciation vs. Depreciation

Appreciation and depreciation both deal with asset value over time. **Appreciation** means that an asset increases in value, while **depreciation** means it decreases in value.

Assets such as real estate and bonds usually gain value over time while other assets such as vehicles may decrease in value. There are situations where assets can appreciate or depreciate.

Example 1:

Houses usually appreciate over time, but there are economic circumstances that may cause them to depreciate. What could some of these factors be?

- economic downturn
- socio-economic factors of neighborhood deteriorate
- population decline
- crime rate ↑
- job losses
- disaster

Example 2:

Vehicles can appreciate or depreciate in value over time. Explain.

New cars depreciate in value until they become antique (restored)

→ Law of supply + demand

Part 2: Renting, Leasing and Buying

When deciding to rent, lease or buy, completing a **cost-and-benefit analysis** is essential in determining which option is best. To make an informed decision, you should consider:

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- affordable monthly payments
- interest rates
- amount of down-payment
- total end cost including interest
- personal benefits such as convenience and flexibility
- appreciation and depreciation
- amount of disposable income for example money left when bills are all paid
- the importance of building equity for example the portion of the house that you own once the mortgage is paid off
- initial fees and possible penalties for example repaying a mortgage early, lawyers fees, property tax, etc.

Renting vs. Leasing

Renting and **Leasing** are terms used with respect to real estate. Both are used in context to using a land, building, etc. in return for a payment. The two differ in terms of the time period, payment and type of contract. Renting is generally shorter term, the terms of the agreement are easier to change, and the agreement can be cancelled at any time with no to little penalty. Leasing applies more to long term agreements and involves a more detailed and rigid contract. It is more difficult to break a lease before the end date since the agreement is often in writing and involves penalties for early termination.

Example 3:

Sarah is going to university in the fall and her parents are trying to decide whether to buy a house or rent an apartment for the 5 years she will be there. The house they are considering buying costs $200,000 and requires a down payment of $10,000. The bank will provide a 20year mortgage, for the remainder of the cost, at 3% with payments every month. The house they are considering renting is $1400 per month and requires an initial damage deposit of $700.

(A) What is the monthly payment for the mortgage? Hint: Using TI83 Plus to calculate PMT

N= 240
I%= 3
PV= 190 000
PMT= 0
FV= 0
P/Y= 12
C/Y= 12
PMT: END BEGIN

$n = 20 \times 12$
$= 240$

$PV = 200\,000 - 10\,000$
$= 190\,000$

PMT: -1053.74

Monthly payment (mortgage): $1053.74

(B) What is the total amount spent in the first five years if you purchase the house?

$\$1053.74 \times 12 \times 5 = \$63\,224.40$

months years

$+\ 10\,000$

$\$73\,224.40$

(C) What is the monthly payment for renting?

$1400

(D) What is the total cost for renting for 5 years?

$\$1400 \times 12 \times 5 = 84\,000$

$+\ 700$

$\$84\,700$

(E) If we only consider monthly costs, which option would be best?

Buying!

(F) What other factors should a person consider when making this decision?

- downpayment
- responsibility of home ownership
  - ↳ extra costs of maintaining a house
- short term

Factors that should be considered when deciding whether to buy, rent or lease.

Buy	Rent	Lease
• build equity • long term investment • renovate the house • maintenance costs	• can be short term • can be contract free	• long term investment • includes a contract • must pay in full if the person decides to move before the end of the contract
	• do not own • cannot change the property • rent can change after 12 months • owner's rules • no maintenance costs	