UNIT 7 Logarithmic Functions

7.1: Characteristics of Logarithmic Functions with Base 10 and Base e

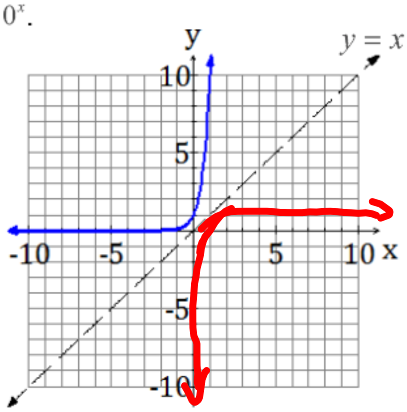
Investigation - Part A: The Common Logarithm

$y = 10^{-2}$
$= 0.01$
$y = 10^{-1}$

1. Complete the table of values for $y = 10^x$.

$y = 10^x$	
x	y
-2	0.01
-1	0.1
0	1
1	10
2	100

$x = 10^y$	
x	y
0.01	-2
0.1	-1
1	0
10	1
100	2



2. How can you use the table to create a table of values for the new function $x = 10^y$?

We can switch the x + y values for our equations

3. Sketch the graph of $x = 10^y$ on the same axes.

4. How are these two functions related?
What is the connection to the line $y = x$?

The graphs are mirrored/reflected in the line $y = x$

5. The equation of the second function, $x = 10^y$ can be rewritten in another form called **logarithmic** form:

$y = \log x$ or $y = \log_{10} x$

6. Compare the characteristics of both functions:

	Exponential	Logarithmic
Domain x	$x \in \mathbb{R}$	$x > 0, x \in \mathbb{R}$
Range y	$y > 0, y \in \mathbb{R}$	$y \in \mathbb{R}$
y-intercept	$y = 1$	none
x-intercept	none	$x = 1$
Increasing/ Decreasing	increasing	increasing
End Behaviour	Q_2 to Q_1	Q_4 to Q_1

7. Use graphing technology to graph the following functions and match them with those provided on the graph below.

DESMOS

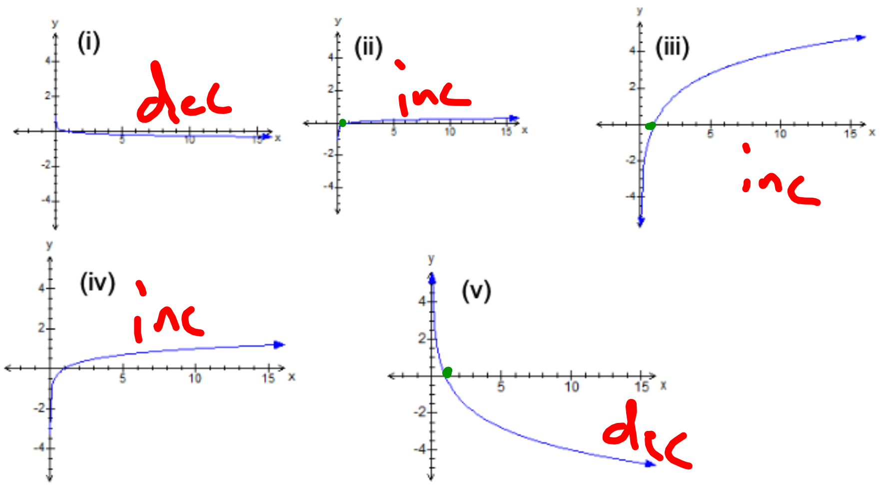
A. $y = \log_{10} x$ iv

B. $y = 4\log_{10} x$ iii

C. $y = -4\log_{10} x$ v

D. $y = \frac{1}{4}\log_{10} x$ ii

E. $y = -\frac{1}{4}\log_{10} x$ i



8. What is the effect on the graph of $y = a\log_{10} x$ if $a > 0$? $a < 0$?

$a > 0$ (positive) increase

$a < 0$ (negative) decrease

9. Does "a" affect the x-coordinate or the y-coordinate? Is this a vertical or a horizontal transformation?

affects the y-coordinate

→ vertical, reflected in x-axis

10. Which point is easily identified from the graph?

x-intercept!

Part B: The Natural Logarithm

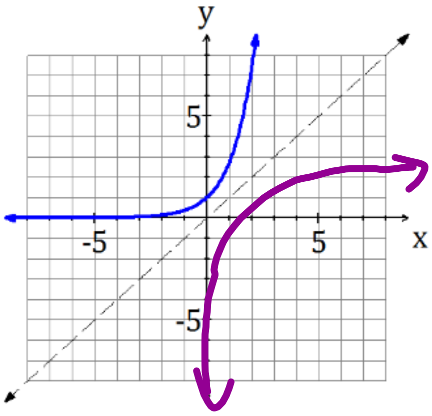
1. Complete the table of values for $y = e^x$ and $x = e^y$.

Note: e is an irrational number like π where e = 2.71828... $y = (2.71828...)^x$

$y = e^{-2}$
$= 0.1353$

$y = e^x$	
x	y
-2	0.1353
-1	0.3678
0	1
1	2.7182
2	7.389

$x = e^y$	
x	y
0.1353	-2
0.3678	-1
1	0
2.7182	1
7.389	2



2. Sketch the graph of $x = e^y$ on the same axes. How does it compare to $y = e^x$?

reflected in the line $y=x$

3. The equation of the second function, $x = e^y$ can be rewritten in another form called **logarithmic** form:

$y = \ln x$ or $y = a \ln x$

4. Compare the characteristics of both functions:

	Exponential	Logarithmic
Domain	$x \in \mathbb{R}$	$x > 0, x \in \mathbb{R}$
Range	$y > 0, y \in \mathbb{R}$	$y \in \mathbb{R}$
y-intercept	$y = 1$	none
x-intercept	none	$x = 1$
Increasing/ Decreasing	increasing	increasing
End Behaviour	Q_2 to Q_1	Q_4 to Q_1

5. How do the characteristics of the function $y = \ln x$ compare to those of $y = \log_{10} x$? (Does it matter if the base is 10 or e?)

All of our chacteristics are the same!
Values for points are different.

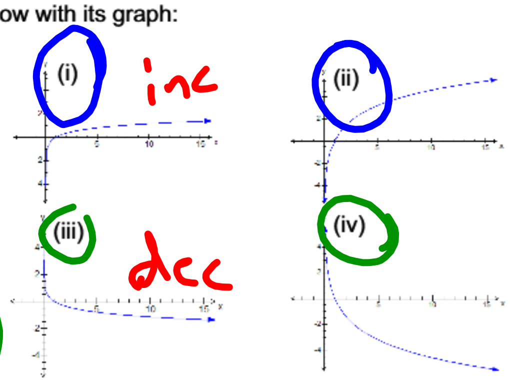
6. Match each function below with its graph:

dec A. $y=-\frac{1}{2}\ln x$ (iii)

inc B. $y=2\ln x$ (ii)

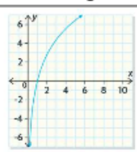
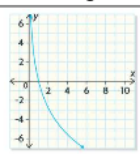
inc C. $y=\frac{1}{2}\ln x$ (i)

dec D. $y=-2\ln x$ (iv)



SUMMARY:

All logarithmic functions of the form $f(x)=a\log x$ and $f(x)=a\ln x$ have the following characteristics:

x- intercept	one (1, 0)	
Number of y - intercepts	none	
	1. Q4 to Q1 or if a > 0 (positive) increasing 	2. Q1 to Q4 if a < 0 (negative) decreasing 
Domain	$\{x / x>0, x\in R\}$	
Range	$\{y / y\in R\}$	

Example 1: (Ex. 1/2, p. 414/5)

Predict the x-intercept, the number of y-intercepts, the domain and the range, and the end behaviour of the following functions:

a) $y = 15\log x$

x-intercept: 1 or (1,0)
y-intercept: none
Domain: $x \mid x>0, x \in \mathbb{R}$
Range: $y \in \mathbb{R}$
End Behaviour: increasing
Q4 to Q1

b) $y = -4\ln x$

x-intercept: 1 or (1,0)
y-intercept: none
Domain: $x \mid x>0, x \in \mathbb{R}$
Range: $y \in \mathbb{R}$
End Behaviour: decreasing
Q1 to Q4

Example 2: (Ex. 3, p. 417)

Which function matches each graph? Provide your reasoning.

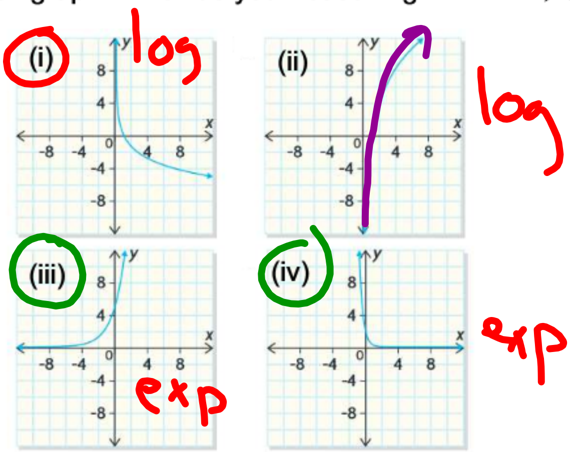
$b>1$ increasing
$0<b<1$ decreasing
$a>0$
x-int

A. $y = 5(2)^x$ (iii)

B. $y = 2(0.1)^x$ (iv)

C. $y = 6\log x$ (ii)

D. $y = -2\ln x$ (i)



Practice:
p. 420 - 425, #2, 3, 5ace, 8

M3201 - Section 7.2

Section 7.2 Evaluating Logarithmic Expressions

A logarithmic function can be expressed as an exponential function and vice versa.

$y = \log_b x \quad \longleftrightarrow \quad x = b^y$

Logarithm = Exponent

The expression $y = \log_b x$ means "the exponent that must be applied to base b to give the value of x.

for example, $\log_2 8 = 3$ since $2^3 = 8$

Two Specific Types of Logarithms:

1. Common Logarithm:
   - Base 10

$y = \log_{10} x \quad \longleftrightarrow \quad x = 10^y$
or
$y = \log x$

2. Natural Loagarithm:
   - Base e

$y = \log_e x \quad \longleftrightarrow \quad x = e^y$
or
$y = \ln x$

NOTE:
One way to convert from Exponential form to Logarithmic form is to remember:

$$Base^{Exponent} = Number \quad \longleftrightarrow \quad \log_{Base} Number = Exponent$$

$B^E = N$ the $\log_B N = E$

"Ben the Log Bunny"

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Example 1:
Convert the following to **exponential form**.

a) $y = \log_3 x$

$x = 3^y$

b) $y = \log x$

$y = \log_{10} x$

$x = 10^y$

c) $y = \ln x$

$y = \log_e x$

$x = e^y$

Example 2:
Convert the following to **logarithmic form**.

a) $x = 10^y$

$y = \log_{10} x$

$y = \log x$

b) $x = 5^y$

$y = \log_5 x$

c) $x = e^y$

$y = \log_e x$

$y = \ln x$

Evaluating a Logarithmic Function

Example 3: Evaluate.

Ask yourself,
2 to what exponent is 8

new

$\log_2 8$

$y = \log_2 8$ Set the logarithmic function equal to y

Unit 6

$2^y = 8$ Write in exponential form

$2^y = 2^3$ Rewrite using common bases

$y = 3$ Since the bases are the same, the exponents must be the same.

→

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Example 4: Evaluate.

a) $\log_3 81$

$y = \log_3 81$

$3^y = 81$

$3^y = 3^4$

$\boxed{y = 4}$

b) $\log_2 16$

$y = \log_2 16$

$2^y = 16$

$2^y = 2^4$

$\boxed{y = 4}$

c) $\log_4 64$

$y = \log_4 64$

$4^y = 64$

$4^y = 4^3$

$\boxed{y = 3}$

d) $\log_8 1$

$y = \log_8 1$

$8^y = 1$

$8^y = 8^0$

$\boxed{y = 0}$

e) $\log_7 7$

$y = \log_7 7$

$7^y = 7$

$7^y = 7^1$

$\boxed{y = 1}$

f) $\log_2 (-4)$

$y = \log_2 (-4)$

$2^y = -4$

$\varnothing$ →

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Example 5: Evaluate.

a) $\log_3\left(\frac{1}{27}\right)$

$y = \log_3\left(\frac{1}{27}\right)$

$3^y = \frac{1}{27}$

$3^y = \frac{1}{3^3}$

$3^y = 3^{-3}$

$\boxed{y = -3}$

b) $\log_{\left(\frac{1}{4}\right)} 64$

$y = \log_{\left(\frac{1}{4}\right)} 64$

$\left(\frac{1}{4}\right)^y = 64$

$4^{-y} = 4^3$

$-y = 3^{x-1}$

$\boxed{y = -3}$

c) $\log_2 \sqrt{8}$

$y = \log_2 \sqrt{8}$

$2^y = \sqrt{8}$

$2^y = 8^{\frac{1}{2}}$

$2^y = 2^{3\left(\frac{1}{2}\right)}$

$2^y = 2^{\frac{3}{2}}$

$\boxed{y = \frac{3}{2}}$

d) $\log_9 \sqrt[5]{81}$

$y = \log_9 \sqrt[5]{81}$

$9^y = \sqrt[5]{81}$

$9^y = 81^{\frac{5}{2}}$

$9^y = 9^{2\left(\frac{5}{2}\right)}$

$9^y = 9^5$

$\boxed{y = 5}$

Example 6: Which expression has the greater value?

a) $\log_2 1 + \log_2\left(\frac{1}{8}\right)$

$y = \log_2 1$

$2^y = 1$

$2^y = 2^0$

$\boxed{y = 0}$

$y = \log_2\left(\frac{1}{8}\right)$

$2^y = \frac{1}{8}$

$2^y = \frac{1}{2^3}$

$2^y = 2^{-3}$

$\boxed{y = -3}$

$0 + -3 = \boxed{-3}$

b) $\log_{\frac{1}{2}} 16 - \log_{\frac{1}{3}} 27$

$y = \log_{\frac{1}{2}} 16$

$\left(\frac{1}{2}\right)^y = 16$

$2^{-y} = 2^4$

$-y = 4$

$\boxed{y = -4}$

$y = \log_{\frac{1}{3}} 27$

$\left(\frac{1}{3}\right)^y = 27$

$3^{-y} = 3^3$

$-y = 3$

$\boxed{y = -3}$

$(-4) - (-3)$

$= -4 + 3$

$\boxed{= -1}$

$-3 < -1$

Greater Value

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Example 7:

Evaluate each using a calculator to two decimal places.

a) $\log 85$

1.93

b) $\ln 23$

3.14

Example 8: (Ex. 2, p. 429)
Determine the value of y in each exponential equation.

a) $81 = 10^y$

$\log_{10} 81 = y$

$\log 81 = y$

$\boxed{1.91 = y}$

b) $25 = e^y$

$\log_e 25 = y$

$\ln 25 = y$

$\boxed{y = 3.22}$

Example 9:

Mary evaluated $\log(-3.24)$ on her calculator and an error message was displayed. Explain why an error message occurred.

Can't take the log of a negative number.

Practice: p. 436, #5 - 13

M3201 - Section 7.2

Solving Problems Involving Logarithmic Scales

Many real life situations have values that vary greatly. A logarithmic scale with powers of 10 can be used to make comparisons between large and small values more manageable.

Three examples of logarithmic scales are:

1. The Richter scale - used to measure the magnitude of an earthquake.
2. The pH scale - used to measure the acidity of a solution.
3. The Decibel scale - used to measure sound level.

Example 1: See p. 433 for pH scale; 0 (acidic) - 14 (basic); pH = 7 is neutral

The pH, p(x), of a solution can be determined using the formula $p(x) = -\log x$, where the concentration of hydrogen ions, x, is measured in mol/L.

a) The hydrogen ion concentration of the solution is 0.0001 mol/L. x ☆

Calculate the pH of the solution.

$P(x) = -\log x$

$P(0.0001) = -\log 0.0001$

$P(0.0001) = 4$

b) Calculate the hydrogen ion concentration of lemon juice (pH = 2).

$2 = -\log x$

$-2 = \log x$

$-2 = \log_{10} x$

$10^{-2} = x$

$\boxed{x = 0.01}$

c) How many time more acidic is Solution A, with a pH of 1.6, than Solution B, with a pH of 2.5? Round your answer to the nearest tenth.

$1.6 = -\log x$

$-1.6 = \log_{10} x$

$10^{-1.6} = x$

$\boxed{x = 0.025}$ Soln A

$2.5 = -\log x$

$-2.5 = \log_{10} x$

$10^{-2.5} = x$

$\boxed{x = 0.0032}$ Soln B

$\frac{\text{Soln A}}{\text{Soln B}} = \frac{0.025}{0.0032} = \boxed{7.8}$

Soln A is 7.8 times more acidic than Soln B!

M3201 - Section 7.2

Example 2:

The magnitude of an earthquake, y, can be determined using $y = \log x$ where x is the amplitude of the vibrations measured using a seismograph. An increase of one unit in magnitude results in a 10-fold increase in the amplitude. Answer the following questions using the table below.

Location	Magnitude
Chernobyl, 1987	4
Haiti, January 12, 2012	7
Northern Italy, May 20, 2012	6

a) How many times as intense was the earthquake in Haiti compared to the one in Chernobyl? ☆

Chernobyl

$y = \log x$

$4 = \log_{10} x$

$10^4 = x$

$\boxed{x = 10\,000}$

Haiti:

$y = \log x$

$7 = \log_{10} x$

$10^7 = x$

$\boxed{x = 10\,000\,000}$

$\frac{\text{Haiti}}{\text{Chernobyl}} = \frac{10\,000\,000}{10\,000}$

$\boxed{= 100}$

Haiti was 100 times more powerful.

b) How many times as intense was the earthquake in Haiti compared to the one in Northern Italy?

Haiti: 10 000 000

Italy: $y = \log x$

$6 = \log_{10} x$

$10^6 = x$

$\boxed{x = 1\,000\,000}$

$\frac{\text{Haiti}}{\text{NI}} = \frac{10\,000\,000}{1\,000\,000}$

$\boxed{= 10}$ 10 times more powerful

c) If a recent earthquake was half as intense as the one in Haiti, what would be the approximate magnitude? ☆

$\frac{10\,000\,000}{2} = 5\,000\,000$

$y = \log_{10} 5\,000\,000$

$= \boxed{6.7}$

M3201 - Section 7.2

Example 3:

Sound levels are measured in decibels. The decibel scale is logarithmic and is defined by the equation $\beta = 10(\log I + 12)$ where β is the sound level in decibels, db, and I is the sound intensity in watts per square metre, W/m².
What is the sound level, to the nearest decibel, of each sound?

a) a conversation at 50 cm, if $\underbrace{I = 2\times10^{-4}}_{I = 0.0002}$ W/m^2 ☆

$$\beta = 10(\log 0.0002 + 12)$$
$$= 10(-3.70 + 12)$$
$$= 10(8.3)$$
$$\boxed{\beta = 83}$$

b) rustle of leaves, if $I = 1\times10^{-11}$ W/m^2 → 0.00000000001

$$\beta = 10(\log(10^{-11}) + 12)$$
$$= 10(-11 + 12)$$
$$= 10(1) \quad = \boxed{10}$$

c) siren at 30 m, if $I = 1\times10^{-2}$ W/m^2

$$\beta = 10(\log 10^{-2} + 12)$$
$$= 10(-2 + 12)$$
$$= 10(10)$$
$$= \boxed{100}$$

Practice:
p. 437- 438, #14bde, 15ace, 16ab,17, 19, 20

M3201 - Section 7.3

Section 7.3 - Laws of Logarithms

1. Product law: $\log_b(m \times n) = \log_b m + \log_b n$

example, $\log_3(9\times 27) = \log_3 9 + \log_3 27$

$y=\log_3 9$
$3^y = 9$
$3^y = 3^2$
$\boxed{y=2}$

$y=\log_3 27$
$3^y = 27$
$3^y = 3^3$
$\boxed{y=3}$

$y=\log_3(9\times 27)$
$y=\log_3(243)$
$3^y = 243$
$3^y = 3^5 \rightarrow \boxed{y=5}$

Remember to evaluate:
$\log_3 9 = y$
$3^y = 9$
$3^y = 3^2$
$y = 2$

2. Quotient Law: $\log_b(m \div n) = \log_b m - \log_b n$

example, $\log_2(256 \div 32) = \log_2 256 - \log_2 32$

$y=\log_2 256$
$2^y = 256$
$2^y = 2^8$
$\boxed{y=8}$

$y=\log_2 32$
$2^y = 32$
$2^y = 2^5$
$\boxed{y=5}$

$3 = 8-5$
$3 = 3$

Note the similarities between the **laws of logarithms** and the **laws of exponents.**

$a^n \times a^m = a^{n+m}$

$a^n \div a^m = a^{n-m}$

$(a^n)^m = a^{nm}$

3. Power Law: $\log_b(m^n) = n\log_b m$

example $\log_2 4^3 = 3\log_2 4$

$y=\log_2 64$
$2^y = 64$
$2^y = 2^6$
$\boxed{y=6}$

$y=\log_2 4$
$2^y = 4$
$2^y = 2^2$
$\boxed{y=2}$

$3\log_2 4$
$=3(2)$
$=6$

Quotient Rule ext.

$y=\log_2(256\div 32)$
$y=\log_2(8)$
$2^y = 8$
$2^y = 2^3$
$\boxed{y=3}$

M3201 - Section 7.3

Example 1:

Write as a single logarithm, then evaluate.

a) $\log_2 5 + \log_2 6.4$

$\log_2(5 \times 6.4)$

$y = \log_2(32)$

$2^y = 32$

$2^y = 2^5$

$\boxed{y = 5}$

b) $\log 12 - \log 6$

$\log(12 \div 6)$

$y = \log 2$

$\boxed{y = 0.30}$

Calculator!

c) $\log_3 27^5$

$\cancel{\log_3}$(super long #)

$= 5(\log_3 27)$

$y = \log_3 27$

$3^y = 27$

$3^y = 3^3$

$\boxed{y = 3}$

$= 5(3)$

$\boxed{= 15}$

d) $2\log_3 6 + \log_3 0.75$

$= \log_3 6^2 + \log_3 0.75$

$= \log_3 36 + \log_3 0.75$

$= \log_3(36 \cdot 0.75)$

$y = \log_3 27$

$3^y = 27$

$3^y = 3^3$

$\boxed{y = 3}$

e) $\log_2 \sqrt{80} - \log_2 \sqrt{5}$

$= \log_2\left(\frac{\sqrt{80}}{\sqrt{5}}\right)$

$= \log_2\left(\sqrt{\frac{80}{5}}\right)$

$= \log_2(\sqrt{16})$

$y = \log_2 4$

$2^y = 4$

$2^y = 2^2$

$\boxed{y = 2}$

f) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

$\log_3 6^2 - \log_3 64^{1/2} + \log_3 2$

$\log_3 36 - \log_3 \sqrt{64} + \log_3 2$

$\log_3 36 - \log_3 8 + \log_3 2$

$\log_3\left(\frac{36}{8} \cdot 2\right)$

$\log_3\left(\frac{72}{8}\right)$

$\log_3 9$

$y = \log_3 9$

$3^y = 9$

$3^y = 3^2$

$\boxed{y = 2}$

M3201 - Section 7.3

Your Turn:

Write as a single logarithm, then evaluate.

a) $\log 12 + \log 2$

$\log(12 \cdot 2)$

$\log 24$

$= 3.18$

calculator

b) $\log_5 100 - \log_5 4$

$\log_5(100 \div 4)$

$y = \log_5 25$

$5^y = 25$

$5^y = 5^2$

$\boxed{y = 2}$

c) $\log_3 18 + \log_3\left(\frac{3}{2}\right)$

$\log_3\left(18 \cdot \frac{3}{2}\right)$

$y = \log_3(27)$

$3^y = 27$

$3^y = 3^3$

$\boxed{y = 3}$

d) $\log_5 40 - 3\log_5 2$

$\log_5 40 - \log_5 2^3$

$\log_5 40 - \log_5 8$

$\log_5(40 \div 8)$

$y = \log_5 5$

$5^y = 5$

$5^y = 5^1$

$y = 1$

e) $3\log_6(2) + \log_6(27)$

$\log_6 2^3 + \log_6 27$

$\log_6 8 + \log_6 27$

$\log_6(8 \cdot 27)$

$y = \log_6 216$

$6^y = 216$

$6^y = 6^3$

$\boxed{y = 3}$

f) $\log_5(2.5) + 2\log_5(10) - \log_5(2)$

$\log_5(2.5) + \log_5 10^2 - \log_5 2$

$\log_5(2.5) + \log_5 100 - \log_5 2$

$\log_5(2.5 \cdot 100) - \log_5 2$

$\log_5(250) - \log_5 2$

$\log_5(250 \div 2)$

$y = \log_5(125)$

$5^y = 125$

$5^y = 5^3$

$\boxed{y = 3}$

M3201 - Section 7.3

Example 4: Error Analysis

Simplify: $\log_5 36 + 2\log_5 3$

Student 1:

$\log_5 36 + 2\log_5 3$ ✓

$\log_5 36 + \log_5 3^2$ ✓

$\log_5 36 + \log_5 6$ X

$\log_5 (36 \times 6)$

$\log_5 324$

Student 2:

$\log_5 36 + 2\log_5 3$ ✓

$\log_5 36 + \log_5 3^2$ ✓

$\log_5 36 + \log_5 9$ ✓

$\log_5 (36 \div 9)$ X

$\log_5 4$

$\log_5 36 + 2\log_5 3$

$\log_5 36 + \log_5 3^2$

$\log_5 36 + \log_5 9$

$\log_5 (36 \cdot 9)$

$\boxed{\log_5 324}$

Example 5:

Express $\log 6$ as a:

a) sum of two logs.

$2 \times 3 = 6$

$\log 6 = \log(2 \cdot 3) = \boxed{\log 2 + \log 3}$

b) difference of two logs.

$12 \div 2 = 6$

$\log 6 = \log(12 \div 2)$

$= \boxed{\log 12 - \log 2}$

M3201 - Section 7.3

Example 6:

What is $4\log A + \log B - 2\log C$ expressed as a single log?

$\log A^4 + \log B - \log C^2$

$\log (A^4 \cdot B) - \log C^2$

$\log \left(\frac{A^4 B}{C^2}\right)$

Example 7:

Write an equivalent expression for $\log\left(\frac{A^2 B}{C^3}\right)$.

$\log \left(\frac{A^2 B}{C^3}\right)$

$\log A^2 B - \log C^3$

$\log A^2 + \log B - \log C^3$

$2\log A + \log B - 3\log C$

Practice:
p. 446-447, #1 - 7, 10 - 16 + Worksheet

M3201- Section 7.4

Section 7.4: Solving Exponential Equations using Logarithms

Recall in Unit 6 we solved exponential equations by writing with the same base and then equating the exponents.

For example,
$$2^x = 8$$
$$2^x = 2^3$$
$$x = 3$$

The same equation can be solved using logarithms.

> You can solve an exponential equation by taking the logarithms of both sides of the equation.

$2^x = 8$

$2^x = 2^3$

$x = 3$

For example,
$$2^x = 8$$
$$\log 2^x = \log 8 \quad \textbf{(Take the logarithm of both sides)}$$
$$x \log 2 = \log 8 \quad \textbf{(Apply the power law)}$$
$$x = \frac{\log 8}{\log 2}$$
$$x = 3$$

Example 1: Solve the following, $2^x = 7$

> NOTE: This example cannot be solved by writing with the same base. It has to be solved using the "new" way! ☆

$2^x = 7$

$\log 2^x = \log 7$

$\frac{x \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 7}{\log 2}$

$\boxed{x = 2.81}$

→

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Example 2:

a) Evaluate each of the following:

i) $\log_2 8$

$y = \log_2 8$

$2^y = 8 \rightarrow 2^y = 2^3$

$\boxed{y = 3}$

ii) $\log_2 16$

$y = \log_2 16$

$2^y = 16 \rightarrow 2^y = 2^4$

$y = 4$

b) Based on those answers, estimate $\log_2 9$. ☆

Between 3-4, closer to 3

~ 3.2

c) Evaluate: $\log_2 9$ ☆

$y = \log_2 9$

$2^y = 9$

$\log 2^y = \log 9$

$\frac{y \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 9}{\log 2}$

$\boxed{y = 3.17}$

Example 3: Evaluate $\log_2 100$ to three decimal places. ☆

$y = \log_2 100$

$2^y = 100$

$\log 2^y = \log 100$

$\frac{y \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 100}{\log 2}$

$\boxed{y = 6.64}$

→

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Change of base formula:

$$\log_b x = \frac{\log x}{\log b}$$

Example 4: Evaluate to 3 decimal places.

a) $\log_4 120$

$\log_4 120 = \frac{\log 120}{\log 4}$

$= \boxed{3.45}$

b) $\log_3 15$

$= \frac{\log 15}{\log 3}$

$= \boxed{2.46}$

Example 5: Solve each of the following

a) $2^{x+1} = 32$

i) Using common bases

$2^{x+1} = 32$

$2^{x+1} = 2^5$

$x+1 = 5$

$x = 5-1$

$\boxed{x=4}$

ii) Using Logarithms ☆

$2^{x+1} = 32$

$\log 2^{x+1} = \log 32$

$\frac{(x+1)\cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 32}{\log 2}$ → Use calculator

$x+1 = 5$

$x = 5-1$

$\boxed{x=4}$

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b) $3^{x-1} = 20$

$\log 3^{x-1} = \log 20$

$\frac{(x-1)\cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 20}{\log 3}$

$x-1 = \frac{\log 20}{\log 3}$

$x = \frac{\log 20}{\log 3} + 1$

$x = 2.73 + 1$

$\boxed{x = 3.73}$

c) $\frac{\cancel{4}(3)^x}{\cancel{4}} = \frac{24}{4}$

$3^x = 6$

$\log 3^x = \log 6$

$\frac{x\cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 6}{\log 3}$

$\boxed{x = 0.778}$

d) $2^{x-1} = 3^{x+1}$

$\log 2^{x-1} = \log 3^{x+1}$

$\frac{(x-1)\cancel{\log 2}}{\cancel{\log 2}} = \frac{(x+1)\log 3}{\log 2}$

$x-1 = (x+1)1.58$

$x-1 = 1.58x + 1.58$

$x - 1.58x = 1.58 + 1$

$\frac{\cancel{-0.58}x}{\cancel{-0.58}} = \frac{2.58}{-0.58}$

$\boxed{x = -4.45}$

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Your Turn:

Solve for x: $2^{x+1} = 5^{x-1}$

$$\log 2^{x+1} = \log 5^{x-1}$$

$$\frac{(x+1)\cancel{\log 2}}{\cancel{\log 2}} = \frac{(x-1)\log 5}{\log 2}$$

$$x+1 = (x-1)2.32$$

$$x+1 = 2.32x - 2.32$$

$$1x - 2.32x = -2.32 - 1$$

$$\frac{-1.32x}{-1.32} = \frac{-3.32}{-1.32}$$

$$\boxed{x = 2.52}$$

Practice:
p. 455 - 458, #1ace,2ab,3ab,5ab,6ac,16

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Word Problems:

In the last unit we completed questions involving half-life, doubling life, compound interest, and depreciation where we could solve the exponential equations by writing with the same base. We will now revisit those questions with one difference - we will not be able to write with the same base therefore we will have to take the log of both sides to finish solving the problem.

Example 6:

If a $1000 deposit is made at a bank that pays 12% per year, compounded annually, how long will it take for the investment to reach $2000.

$A = P(1+i)^n \longrightarrow 2000 = 1000(1+0.12)^n$

$12\% = 0.12$

$i = 0.12$

$P = 1000$

$A = 2000$

$\frac{2000}{1000} = \frac{\cancel{1000}(1+0.12)^n}{\cancel{1000}}$

$2 = (1.12)^n$

$\log 2 = \log 1.12^n$

$\frac{\log 2}{\log 1.12} = \frac{n \cancel{\log 1.12}}{\cancel{\log 1.12}}$

$\boxed{n = 6.12}$

Example 7: (ex. 3, p. 451)

Jahmal works in a laboratory that uses radioactive substances. The laboratory received a shipment of 500 g of radioactive radon-222. Only 13.417 g of the radon-222 remained 19.0 days later. Determine the half-life of radon-222 algebraically using logarithms.

Recall: The half-life equation is: $A = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$A = 13.417$

$A_o = 500$

$t = 19$

$h = ?$

$\frac{13.417}{500} = \frac{\cancel{500}\left(\frac{1}{2}\right)^{\frac{19}{h}}}{\cancel{500}}$

$0.0268 = \left(\frac{1}{2}\right)^{\frac{19}{h}}$

$\log 0.0268 = \log\left(\frac{1}{2}\right)^{\frac{19}{h}}$

$\frac{\log 0.0268}{\log(\frac{1}{2})} = \frac{\frac{19}{h}\cancel{\log(\frac{1}{2})}}{\cancel{\log(\frac{1}{2})}}$

$\frac{5.22}{1} = \frac{19}{h}$

$\frac{\cancel{5.22}h}{\cancel{5.22}} = \frac{19}{5.22}$

$h = \frac{19}{5.22} = \boxed{3.64}$

or $5.22 = \frac{19}{h}$

$h = \frac{19}{5.22}$

$\boxed{h = 3.64}$

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Example 8: Error Analysis #8, p. 457

Dave thought that he could also solve the exponential equation in Example 1 by taking the logarithm of each side in the first step. However, he made an error in his solution. Correct Dave's error, and complete his solution.

Dave's Solution

$A = P(1 + i)^n$ $P = 3215$ $i = 0.024$ $A = 5000$ The number of compounding periods, n, is unknown.	I substituted the given values into the compound interest formula.
$5000 = 3215(1.024)^n$	
$\log 5000 = \log(3215(1.024)^n)$	I took the common logarithm of each side of the equation.
$\log 5000 = n \log(3215(1.024))$	I used the power law of logarithms to rewrite the equation.
$n = \dfrac{\log 5000}{\log(3215(1.024))}$	I isolated n.
$n = 1.051...$	
It will take 2 years for the balance to reach \$5000.	The interest is compounded annually, so I rounded up to 2 years. This answer is different from my first answer, and it seems much too small.

$$\frac{5000}{3215} = \frac{\cancel{3215}(1.024)^n}{\cancel{3215}}$$

$$1.56 = 1.024^n$$

$$\log 1.56 = \log 1.024^n$$

$$\frac{\log 1.56}{\log 1.024} = \frac{n \cancel{\log 1.024}}{\cancel{\log 1.024}}$$

$$18.75 = n$$

$$\boxed{n = 18.75}$$

$$n \simeq 19$$

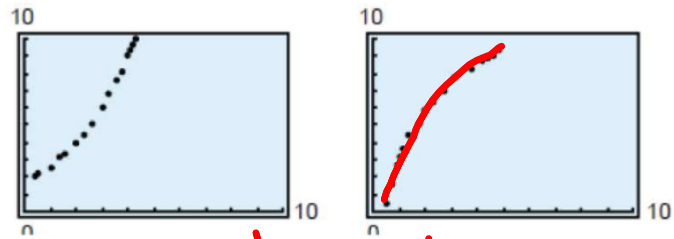
Practice:
p. 457, #10, 11, 12, 13 + Worksheet

M3201 - Section 7.5

Section 7.5: Modelling Data Using Logarithmic Functions

NOTE:
- Domain of a logarithmic function: all positive real numbers
- Logarithmic regressions are mostly used for phenomena that grow quickly at first then slow down over time but the growth continues to increase without bound.
- Exponential regressions are typically used on phenomena where the growth begins slowly then increases very rapidly as time increases.

Example 1:
Which graph is exponential and which is logarithmic?



Exponential log

Example 2:
Create a scatterplot of the data to determine if we should use exponential or logarithmic regression.

x	0.5	0.7	0.9	1.0	1.2	1.4	1.8	2.0	2.3	2.7	3.2	3.8
y	0.5	1.6	2.7	3.1	3.7	4.4	5.1	5.8	6.4	7.0	7.7	8.3

Exp log →

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Example 3:

The flash on most digital cameras requires a charged capacitor in order to operate. The percent charge, Q, remaining on a capacitor was recorded at different times, t, after the flash had gone off.

The t.5 flash duration represents the time until a capacitor has only 50% of its initial charge. The t.5 flash duration also represents the length of time that the flash is effective, to ensure that the object being photographed is properly lit.

a) Construct a scatter plot for the given data.
b) Determine a logarithmic model for the data.
c) Use your logarithmic model to determine the t.5 flash duration to the nearest hundredth of a second.

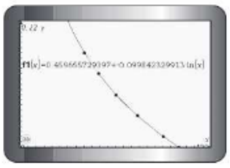
Percent Charge, Q (%)	Time, t (s)
100.00	0
90.26	0.01
73.90	0.03
60.51	0.05
49.54	0.07
40.56	0.09

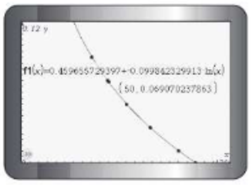
Rico's Solution

a)


b)


The equation is $y = 0.459... - 0.099...(\ln x)$.



c)


At about 0.07 s, the t.5 flash duration has been reached.

NOTE: Most graphing calculators and spreadsheets provide the equation of the logarithmic regression function in the form:

$$y = a + b\ln x$$

$$y = 0.459 - 0.99\ln x$$

Practice:
p. 466-471, #2,3,4,7