# Mathematics 3201 <br> PRE-PUBLIC EXAMINATION 

JUNE 2017

Value: 100 Marks<br>Duration: 3 Hours

## General Instructions

This examination consists of two parts. Part I of the exam consists of selected response questions and Part II consists of constructed response questions.

Selected Response (50 marks)
Select the letter of the correct response from those provided. Place the CAPITAL LETTER of the correct response in the blank space provided on your Selected Response Answer Sheet.

Constructed Response (50 marks)
Answer ALL questions fully and concisely in the space provided. Show all work.

## Student Checklist

The items below are your responsibility. Please ensure that they are completed.

- Write your name on this page.
- Write your name on the Selected Response Answer Sheet.
- Check the exam to ensure there are no missing pages. There are $\underline{19}$ pages in total.


## ALL MATERIALS MUST BE PASSED IN WITH THIS EXAM.

Name: $\qquad$

## PART I

Total Value: 50\%

## Answer all items. Write the UPPERCASE letter of the correct answer in the correct blank on the answer sheet provided.

1. Students in a particular high school were surveyed to determine the subjects in which they were currently enrolled. The table below represents the data that were collected.

| Course Enrolled In | Number or Students |
| :---: | :---: |
| Math only | 28 |
| Art only | 33 |
| Math and Art | 17 |
| Neither course | 20 |

How many students are in the universal set?
(A) 61
(B) 64
(C) 78
(D) 98
2. The Venn diagram shows the number of element in each set. How many elements are in the union of sets A and $\mathrm{B}, n(A \cup B)$ ?
(A) 8
(B) 11
(C) 19
(D) 23

3. $A$ is the set of multiplies of $2, B$ is the set of multiplies of 3 and $C$ is the set of multiplies of 4 . Which statement is true?
(A) A is a subset of $\mathrm{B}, A \subset B$
(B) A is a subset of $\mathrm{C}, A \subset C$
(C) B is a subset of $\mathrm{A}, B \subset A$
(D) C is a subset of $\mathrm{A}, C \subset A$
4. The universal set of the primary and secondary colors contains the colors yellow, blue, red, orange, green, purple and black, as shown below:
$R$ is the set of colors that contains red.
What colors are in the complement of $R, R^{\prime}$ ?
(A) yellow and blue
(B) orange and purple
(C) yellow, green, and blue
(D) orange, blue, and purple

5. In how many different ways can seven members of a band arrange themselves in seven stage positions?
(A) 1
(B) 7
(C) 1440
(D) 5040
6. If 12 runners compete in a race, in how many ways can prizes be awarded for first, second, and third places?
(A) 3
(B) 6
(C) 1320
(D) 1728
7. The school cafeteria offers 3 different meats, 4 different vegetables, 5 different desserts and 2 different drinks. How many different meals containing 1 meat, 1 vegetable, 1 dessert and 1 drink are offered?
(A) 14
(B) 120
(C) 1001
(D) 24024
8. Simplify: $\frac{(n-8) \text { ! }}{(n-6)!}$
(A) $\frac{1}{n^{2}+42}$
(B) $\frac{1}{n^{2}-13 n+42}$
(C) $n^{2}-13 n+42$
(D) $n^{2}+42$
9. In the grid below, a person must travel from $A$ to $B$ by only heading East (E) or South (S). One example of a route is two moves East followed by four moves South (EESSSS). Under these rules, which represents the total number of possible routes that can be taken to get from $A$ to $B$ ?
(A) $\frac{6!}{2!4!}$
(B) $4!2$ !
(C) 6 !


10. There are 9 marbles in a bowl: 3 yellow, 2 green and 4 red. If taken out one at a time, in how many different ways can all 9 marbles be taken out of the bowl?
(A) 24
(B) 1260
(C) 9 !
(D) 2903040
11. Given the following probabilities, which event is more likely to occur?
(A) $\quad P(A)=0.28$
(B) $P(B)=\frac{1}{4}$
(C) $P(C)=27 \%$
(D) $P(D)=\frac{1}{5}$
12. A hockey player has 19 attempts on net and scores 7 goals. What are the odds in favour of him scoring a goal?
(A) $7: 12$
(B) $7: 19$
(C) $12: 7$
(D) $19: 7$
13. Which events are dependent?
(A) Drawing a heart from a standard deck of 52 cards, putting it back, and then drawing another heart.
(B) Rolling a 2 and rolling a 3 with a pair of six-sided dice, numbered 1 to 6.
(C) Rolling a standard six-sided die and drawing a king from a standard deck of 52 cards.
(D) Rolling a 4 and having a sum greater than 6 with a pair of six-sided dice, numbered 1 to 6.
14. $A$ and $B$ are mutually exclusive events. The probability that either $A$ or $B$ will occur, $P(A \cup B)$, is $65 \%$. If the probability of $A$ occurring, $P(A)$, is $32 \%$, what is the probability of $B$ not occurring, $P\left(B^{\prime}\right)$ ?
(A) $33 \%$
(B) $35 \%$
(C) $67 \%$
(D) $97 \%$
15. Nine boys and twelve girls have signed up to go on a school trip. Only six students will be selected to go on the trip. What is the probability there will be an equal number of boys and girls on the trip?
(A) 0.17
(B) 0.23
(C) 0.27
(D) 0.34
16. Which expression is equivalent to $\frac{x-2}{x-6}, x \neq 6$ ?
(A) $\frac{2 x-4}{2 x-6}$
(B) $\frac{x^{2}-2 x}{x^{2}-6 x}$
(C) $\frac{3 x-6}{x-6}$
(D) $\frac{5 x-10}{5 x-30}$
17. What are the non-permissible value(s) for $\frac{2 x}{x^{2}-4 x}$ ?
(A) $\{ \pm 2\}$
(B) $\quad\{-4,4\}$
(C) $\{0,-4\}$
(D) $\{0,4\}$
18. What is the simplified form of $\frac{9-x^{2}}{x^{2}-3 x}$ ?
(A) $\frac{3}{x}, x \neq 0,3$
(B) $\frac{x+3}{x}, x \neq 0,3$
(C) $\frac{-(x+3)}{x}, x \neq 0,3$
(D) $\frac{x-3}{x}, x \neq 0,3$
19. What is the simplified form of $\frac{2 y^{2}-4 y}{6 y^{2}-24}$ ?
(A) $\frac{y}{6(y+2)}, y \neq \pm 2$
(B) $\frac{y}{3(y+2)}, y \neq \pm 2$
(C) $\frac{y}{3(y-2)}, y \neq \pm 2$
(D) $\frac{-y}{3(y+2)}, y \neq \pm 2$
20. What is the simplified form of $\frac{7 x^{2}}{15} \div \frac{21 x}{35}$ ?
(A) $\frac{7 x}{9}, x \neq 0$
(B) $\frac{9}{7 x}, x \neq 0$
(C) $\frac{7 x^{3}}{25}, x \neq 0$
(D) $\frac{25}{7 x^{3}}, x \neq 0$
21. What is the simplified form of $\frac{6 x}{2 x+6}-\frac{8 x}{x+3}$ ?
(A) $\frac{5 x}{x+3}, x \neq-3$
(B) $\frac{-5 x}{x+3}, x \neq-3$
(C) $\frac{11 x}{x+3}, x \neq-3$
(D) $\frac{-11 x}{x+3}, x \neq-3$
22. What is the range of the function $y=f(x)$ shown in the graph below?

(A) $\quad\{x \mid x \in R\}$
(B) $\{y \mid y \in R\}$
(C) $\{x \mid x \geq 0, x \in R\}$
(D) $\{y \mid y \geq 0, y \in R\}$
23. Describe the end behaviour of this polynomial function:

The curve extends from quadrant
(A) II to I
(B) II to IV
(C) III to I
(D) III to IV

24. What is the $y$-intercept of the graph of the function $f(x)=5 x^{3}-2 x^{2}+4 x+3$ ?
(A) -2
(B) 3
(C) 4
(D) 5
25. Describe the characteristics of the function shown.
(A) Negative leading coefficient, extends from QIII to QI
(B) Negative leading coefficient, extends from QII to QIV
(C) Positive leading coefficient, extends from QIII to QI
(D) Positive leading coefficient, extends from QII to QIV

26. Which function passes through the point $(2,7)$ ?
(A) $y=-x^{3}+2 x^{2}-3 x+1$
(B) $y=-x^{3}-2 x^{2}-3 x-1$
(C) $y=x^{3}-2 x^{2}+3 x+1$
(D) $y=x^{3}+3 x^{2}+2 x+1$
27. What is the leading coefficient of the polynomial function $f(x)=x^{2}(x-2 x+10)$ ?
(A) -2
(B) -1
(C) 1
(D) 10
28. Which function is an increasing exponential function?
(A) $\quad f(x)=5\left(\frac{1}{2}\right)^{x}$
(B) $\quad f(x)=\frac{2}{3}\left(\frac{5}{2}\right)^{x}$
(C) $\quad f(x)=4(0.75)^{x}$
(D) $\quad f(x)=0.5(1)^{x}$
29. Which exponential function best represents the graph shown?
(A) $\quad f(x)=\left(\frac{1}{2}\right)^{x}$
(B) $\quad f(x)=(2)^{x}$
(C) $\quad f(x)=2\left(\frac{1}{2}\right)^{x}$
(D) $\quad f(x)=2(2)^{x}$

30. An investment of $\$ 750$ is known to double in value approximately every 5 years and is modeled by the function $A(t)=750(2)^{\frac{t}{5}}$. $A(t)$ represents the value of the investment and $t$ represents the time in years. How much time will it take for the investment to be valued at \$12000?
(A) 3 years
(B) 4 years
(C) 15 years
(D) 20 years
31. Solve for $x$ : $\quad 25^{x+1}=5^{4 x-3}$
(A) $-\frac{5}{2}$
(B) -2
(C) 2
(D) $\frac{5}{2}$
32. Which is described by the function $(x)=100(0.90)^{\frac{t}{5}}$ ?
(A) Investment of \$100 appreciating 10\% every 5 years.
(B) Investment of $\$ 100$ appreciating $90 \%$ every 5 years.
(C) Investment of $\$ 100$ depreciating $10 \%$ every 5 years.
(D) Investment of $\$ 100$ depreciating $90 \%$ every 5 years.
33. What response is true of the exponential function given in the table?

| $x$ (years) | 0 | 2 | 4 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ (amount) | 20 | 60 | 180 | 540 | 1620 |


|  | Initial Amount | Amount Growth |
| :---: | :---: | :---: |
| (A) | 20 | doubles every two years |
| (B) | 20 | triples every two years |
| (C) | 60 | doubles every three years |
| (D) | 60 | triples every three years |

34. Which graph best represents $y=\log _{2} x$ ?
(A)

(B)

(C)

(D)

35. What is the logarithmic form of $e^{x}=12$ ?
(A) $\ln 12=x$
(B) $\ln x=12$
(C) $\log _{x} 12=e$
(D) $\log _{12} x=e$
36. Evaluate: $\log _{5} 12$
(A) 0.65
(B) 1.23
(C) 1.46
(D) 1.54
37. Evaluate: $\log _{2}\left(\frac{1}{128}\right)$
(A) -64
(B) -7
(C) 7
(D) 64
38. What is $\log _{5} 24-3 \log _{5} 2$ written as a single logarithm?
(A) $\quad \log _{5} 3$
(B) $\quad \log _{5} 4$
(C) $\quad \log _{5} 16$
(D) $\quad \log _{5} 18$
39. Which situation can be modeled by $A(t)=10\left(\frac{1}{2}\right)^{\frac{t}{6}}$ ?
(A) 6 g of a radioactive substance has a half-life of 10 years.
(B) 6 g of a radioactive substance increases by $50 \%$ every 10 years.
(C) 10 g of a radioactive substance has a half-life of 6 years.
(D) 10 g of a radioactive substance increases by $50 \%$ every 6 years.
40. What is $210^{\circ}$ in radians?
(A) $\frac{6}{7}$
(B) $\frac{7}{6}$
(C) $\frac{6 \pi}{7}$
(D) $\frac{7 \pi}{6}$
41. What is the amplitude for the graph shown below?
(A) -4
(B) 2
(C) 3
(D) 6

42. What are the period and the midline for the function $f(x)=3 \sin 0.5\left(x-30^{\circ}\right)+2$ ?

|  | Period | Midline |
| :--- | :--- | :--- |
| (A) | $180^{\circ}$ | $\mathrm{y}=2$ |
| (B) | $180^{\circ}$ | $\mathrm{y}=3$ |
| (C) | $720^{\circ}$ | $\mathrm{y}=2$ |
| (D) | $720^{\circ}$ | $\mathrm{y}=3$ |

43. What is the range of the function: $y=2 \cos 3\left(x-30^{\circ}\right)-2$ ?
(A) $\{y \mid-4 \leq y \leq 0, y \in R\}$
(B) $\{y \mid-2 \leq y \leq 2, y \in R\}$
(C) $\{y \mid 0 \leq y \leq 4, y \in R\}$
(D) $\{y \mid y \in R\}$
44. Which function has a horizontal shift of $60^{\circ}$ to the right and amplitude of 4 ?
(A) $y=0.25 \sin \left(x-60^{\circ}\right)$
(B) $y=0.25 \sin \left(x+60^{\circ}\right)$
(C) $y=4 \sin \left(x-60^{\circ}\right)$
(D) $y=4 \sin \left(x+60^{\circ}\right)$
45. The graph of the function $y=4 \sin 6\left(x+45^{\circ}\right)-1$ has its midline increased by 2 and its period tripled. Which represents the new equation?
(A) $y=4 \sin 2\left(x+45^{\circ}\right)-3$
(B) $y=4 \sin 2\left(x+45^{\circ}\right)+1$
(C) $y=4 \sin 18\left(x+45^{\circ}\right)-3$
(D) $y=4 \sin 18\left(x+45^{\circ}\right)+1$
46. Sherry borrowed $\$ 3000$ from the bank at an annual interest rate of $8 \%$ compounded monthly. If she pays the loan off in 2 years, how much interest does she pay the bank, to the nearest dollar?
(A) $\$ 240$
(B) $\$ 499$
(C) $\$ 519$
(D) $\$ 1920$
47. The equation $A=4500(1.015)^{t}$ represents a bank loan that is compounded monthly. What is the annual interest rate?
(A) $0.015 \%$
(B) $0.18 \%$
(C) $1.5 \%$
(D) $18 \%$
48. At the end of three years, Jackie pays off her loan with a single payment of $\$ 2150$. If the loan had an annual interest rate of $12 \%$ compounded semi-annually, what was the principal amount she borrowed, to the nearest dollar?
(A) $\$ 1089$
(B) $\$ 1516$
(C) $\$ 1530$
(D) $\$ 1805$
49. How long, in years, did it take Dave to repay his truck loan if he made 143 bi-weekly payments?
(A) 2.8 years
(B) 5.5 years
(C) 6.0 years
(D) 11.9 years
50. Which compounding period would result in the greatest amount of interest paid?
(A) Bi-weekly
(B) Daily
(C) Monthly
(D) Semi-Annually

Answer ALL items in the space provided. Show ALL workings
Value
3
51. In a school of 95 students:

- 5 students took English, Physics and Chemistry
- 15 students took Physics and English
- 8 students took Physics and Chemistry
- 20 students took Physics
- 49 students took English
- 35 students took Chemistry
- 19 students take neither of the three courses

Complete the Venn diagram shown below and determine the number of students who took English and Chemistry.

52.(a) In how many ways can a disk jockey arrange a program of two different instrumentals and eight different vocals, if the program must begin and end with vocals?

3 52.(b) Algebraically solve for $n$ : $\quad{ }_{n} C_{2}=45$
52.(c) Five singers, four dancers, and three comics have auditioned to perform in a talent show, but only six acts can be included. How many different arrangements of six acts are possible if each act must have two singers, two dancers, and two comics?
53.(a) In beach volleyball, the probability that Hannah's jump serve will be out of bounds is $60 \%$ if it is windy. If it is not windy, the probability that her jump serve is out of bounds is $20 \%$. If there is a $45 \%$ chance that it will be windy on game day, what is the probability that Hannah's jump serve will be out of bounds?
(b) A piggy bank contains five loonies and eight toonies. It is shaken until two coins drop out. What is the probability that two loonies will fall out?
54. (a) Simplify and state the restrictions.
$\frac{2 x-x^{2}}{5 x^{3}} \div \frac{7 x^{2}-28}{10 x^{3}-50 x^{2}}$
54. (b) It takes Bob 2 hours to clean the house and it takes Sonya 3 hours. How long will it take them to clean the house if they work together?
55. (a) Sketch two possible graphs that are different, yet both cubic functions with positive leading coefficients and positive y-intercepts. Explain why the graphs you have sketched are different.

Graph 1:


Graph 2:

(b) Determine the following characteristics for the function graphed below:

Degree: $\qquad$
Sign of leading coefficient: $\qquad$
End behaviour: $\qquad$
y-intercept: $\qquad$

56.(b) A father invested $\$ 500$ for his son, who was born in 2012, in an account that pays 8\% interest a year compounded monthly. In the same year, another father bought his son a hockey card for $\$ 250$ that appreciates at a rate of $20 \%$ per year. Write a function that models each father's investment. Which investment will be worth more in the year 2022?
57. (a) Algebraically solve for $x: 3^{x+2}=5^{x-1}$
57. (b) The pH scale is used to measure the acidity of a solution. The $\mathrm{pH}, p(x)$, can be determined using the formula $p(x)=-\log x$, where the concentration of hydrogen ions, $x$, is measured in mol/L.
(ii) How much more acidic is a solution with a pH of 2.1 than a solution with a pH of 2.9? Express answer to the nearest tenth.
58. Use the sinusoidal function shown below to answer the questions that follow:

i) Determine the amplitude, period, equation of midline and range.
ii) Use the information from part (i) to determine a function that represents the graph in the form $y=a \sin b(x)+d$
59. Samantha wants to purchase a new computer for $\$ 1500$. She has two options to finance her purchase.

Option A: Pay an initial administration fee of $\$ 45$ and the taxes of $\$ 195$ immediately. No other payments are required for 1 year. After that year, payments of $\$ 54.61$ are required for 36 months.

Option B: Borrow a principal of $\$ 1500$ plus the taxes of $\$ 195$ from the bank and make 48 monthly payments of $\$ 43.34$.

Which option is least expensive?

## Math 3201 Information Sheet

$$
\begin{array}{ll}
n(A \cup B \cup C)=n(A)+n(B)+n(C)-n(A \cap B)-n(B \cap C)-n(A \cap C)+n(A \cap B \cap C) \\
n(A \cup B)=n(A)+n(B)-n(A \cap B) & { }_{n} P_{r}=\frac{n!}{(n-r)!} \\
{ }_{n} C_{r}=\binom{n}{r}=\frac{n!}{(n-r)!r!} & \frac{n!}{a!b!c!\ldots} \\
P(A \cup B)=P(A)+P(B)-P(A \cap B) & x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
P(A \cap B)=P(A) \cdot P(B \mid A) & A=P(1+r t)
\end{array}
$$

$\qquad$

| 1. | 11. | 21. | 31. | 41. |
| :---: | :---: | :---: | :---: | :---: |
| 2. | 12. | 22. | 32. | 42. |
| 3. | 13. | 23. | 33. | 43. |
| 4. - | 14. | 24. | 34. | 44. |
| 5. | 15. | 25. | 35. | 45. |
| 6. | 16. | 26. | 36. | 46. |
| 7. - | 17. | 27. | 37. | 47. |
| 8. - | 18. | 28. | 38. | 48. |
| 9.- | 19. | 29. | 39. | 49. |
| 10. | 20._ | 30._ | 40. | 50. |

