

Math 3201

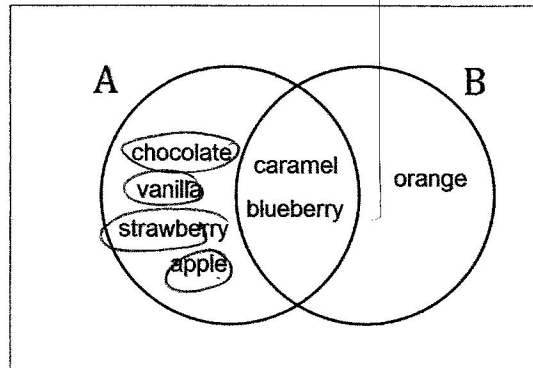
Sample Midterm

PART I

Total Value: 35 marks

Answer all items. Shade the letter of the correct answer on the computer scorable answer sheet or place the letter in the blank provided on your Multiple Choice Answer Sheet.

1. Given the Venn diagram below, how many elements are in the complement of B, $n(B')$?



- (A) 1
 (B) 2
 (C) 4
 (D) 6

2. Which statement is true for sets A, B, and C?

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

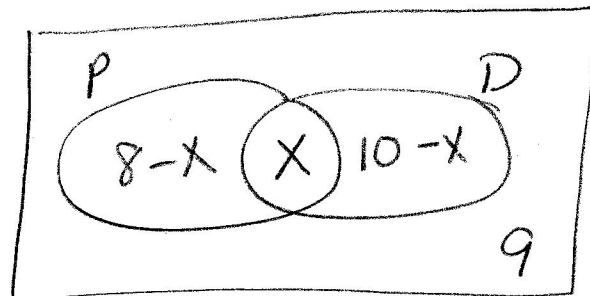
$$B = \{5, 10\}$$

$$C = \{3, 6, 9, 2\}$$

- (A) A is a subset of B, $A \subset B$ ✗
 (B) A is a subset of C, $A \subset C$ ✗
 (C) B is a subset of A, $B \subset A$ ✓
 (D) C is a subset of A, $C \subset A$ ✗

3. There are 25 people in line for a movie. 8 people have popcorn (P), 10 people have a drink (D), 9 people have neither popcorn nor a drink. How many people have both popcorn and a drink, $(P \cap D)$?

- (A) 2
 (B) 7
 (C) 16
 (D) 18



$$8 - x + 10 - x + x + 9 = 25$$

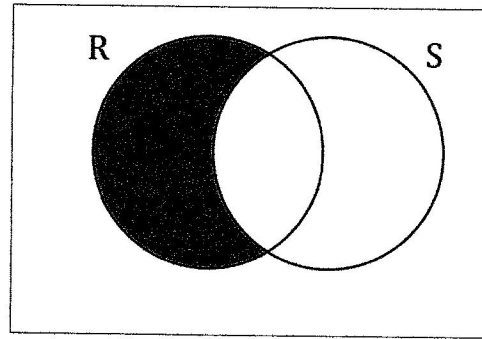
$$27 - x = 25$$

$$27 - 25 = x$$

$$2 = x$$

4. In the Venn diagram below, what does the shaded portion represent?

- (A) $R \setminus S$ (R minus S) ✓
- (B) $R \cup S$ (R or S)
- (C) $S \setminus R$ (S minus R)
- (D) $S \cap R$ (S and R)

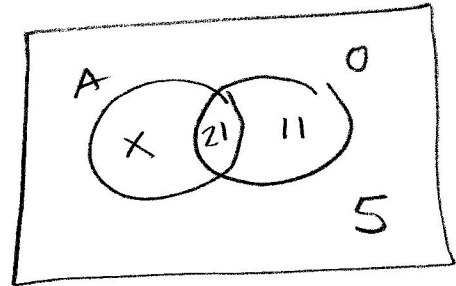


5. Vince asked 50 people if they liked apples or oranges:

- 5 people didn't like apples or oranges
- 21 people liked **both** apples and oranges
- 11 people liked **only** oranges

How many people liked **only** apples?

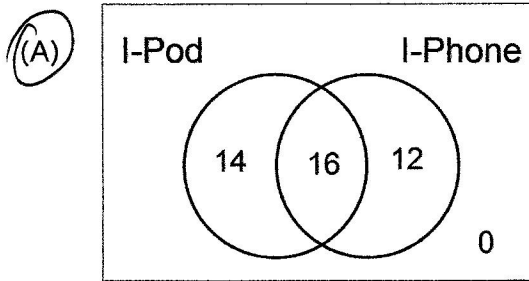
- (A) 10
- (B) 13
- (C) 34
- (D) 39



$$\begin{aligned}x + 21 + 11 + 5 &= 50 \\x + 37 &= 50 \\x &= 50 - 37 \\x &= 13\end{aligned}$$

6. The table below shows the results of 42 people who own Apple products. Which Venn diagram best represents the information in the table?

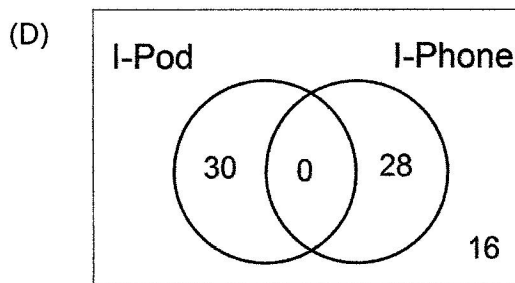
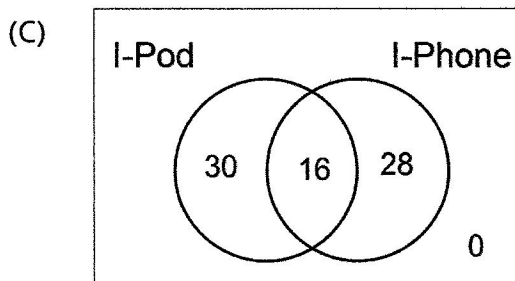
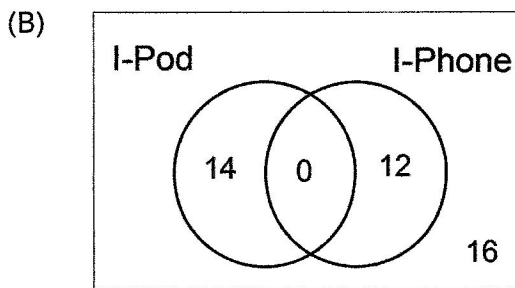
Apple Product	# of People
I-Pod	30
I-Phone	28



$$\text{I-Pod} = 14 + 16 = 30$$

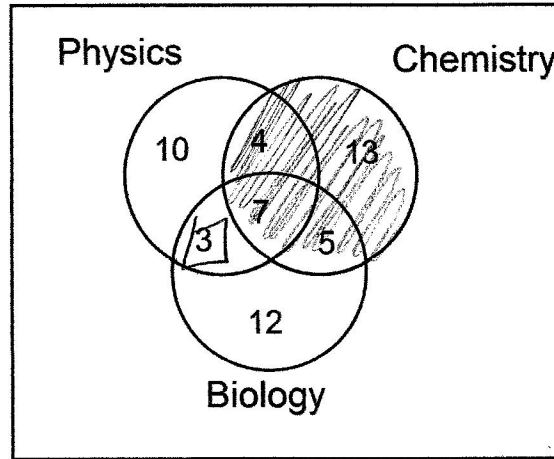
$$\text{I-Phone} = 12 + 16 = 28$$

$$\text{Total} = 14 + 16 + 12 + 0 = 42$$



7. Given the Venn diagram below, how many students are in Physics **AND** Biology but **NOT** Chemistry?

- (A) 41
 (B) 25
 (C) 10
 (D) 3



8. Bob is selecting a new vehicle. He can either get a car or a truck. Each vehicle comes in 4 different colours. Given the selections below, how many choices does he have?

Vehicle	Colour
Car	Red
	Blue
Truck	Grey
	Black

- (A) 48
 (B) 24
 (C) 8
 (D) 6

$$2 \times 4 = 8$$

9. How many ways can the letters in the word **FACTOR** be arranged if you must **start** with a consonant (F, C, T, R) and **end** with a vowel (A, O)?

- (A) 1152
 (B) 720
 (C) 192
 (D) 16

$$\underline{4} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{2} = 192$$

10. Simplify: $\frac{(n+5)!}{(n+3)!}$

$$\frac{(n+5)(n+4)(n+3)!}{(n+3)!} = (n+5)(n+4)$$
$$= n^2 + 4n + 5n + 20$$
$$= n^2 + 9n + 20$$

- (A) $n^2 + 20$
- (B) $n^2 + 9n + 20$
- (C) $\frac{1}{n^2 + 20}$
- (D) $\frac{1}{n^2 + 9n + 20}$

11. Evaluate: $\frac{100!}{98!}$

$$= \frac{100 \times 99 \times \cancel{98!}}{\cancel{98!}} = 9900$$

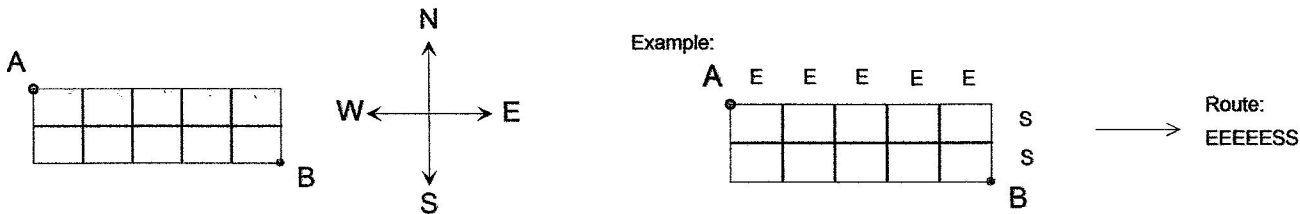
- (A) 970200
- (B) 9900
- (C) 199
- (D) 2

12. A teacher has 5 different math books. How many ways can any 3 of them be arranged on a shelf?

- (A) 60
- (B) 12
- (C) 10
- (D) 6

$$\underline{5} \times \underline{4} \times \underline{3}$$
$$= 60$$

13. In the grid below, a person must travel from A to B by only heading East (E) or South (S). One example of a route is shown representing 5 moves East followed by 2 moves South (EEEESS). Under these rules, which represents the total number of possible routes that can be taken to get from A to B?



- (A) 7!
 (B) $2! \times 5!$
 (C) $\frac{2!5!}{7!}$
 (D) $\frac{7!}{2!5!}$

$$\frac{7!}{2!5!}$$

14. John, Sam, Alice and Brianna are to be arranged in a line from left to right. In how many ways can they be arranged?

- (A) 24
 (B) 12
 (C) 4
 (D) 1

$$4 \times 3 \times 2 \times 1 = 24$$

15. Which expression is equivalent to $\binom{8}{5}$?

- (A) $\frac{5!}{8!}$
 (B) $\frac{5!3!}{8!}$
 (C) $\frac{8!}{5!}$
 (D) $\frac{8!}{5!3!}$

$$nCr = \frac{n!}{r!(n-r)!}$$

$$8C5 = \frac{8!}{5!(8-5)!}$$

$$= \frac{8!}{5!3!}$$

16. A Math teacher must choose 3 girls and 2 boys to attend a math competition. In how many ways can this be done if there are 10 girls and 8 boys who are interested in attending?

3 girls AND 2 boys

- (A) 1028160
 (B) 40320
 (C) 8568
 (D) 3360

$$\begin{aligned}
 & 10C_3 \times 8C_2 \\
 & = 120 \times 28 \\
 & = 3360
 \end{aligned}$$

17. A chess club in high school must consist of 7 members.

- There must be exactly 4 Level III students.
- There must be exactly 2 Level II students.
- There must be exactly 1 Level I student.

If 10 Level III's, 12 Level II's, and 8 Level I's wish to participate, how many different chess clubs are possible?

4 Level III AND 2 Level II AND 1 Level I

$$10C_4 \times 12C_2 \times 8C_1$$

- (A) ${}_{30}C_7$
 (B) ${}_{10}C_4 \times {}_{12}C_2 \times {}_8C_1$
 (C) ${}_{30}P_7$
 (D) ${}_{10}P_4 \times {}_{12}P_2 \times {}_8P_1$

18. The odds of randomly selecting a red marble from a bag of colored marbles are 5:8. What is the probability of selecting a red marble?

- (A) $\frac{3}{13}$
 (B) $\frac{3}{8}$
 (C) $\frac{5}{13}$
 (D) $\frac{5}{8}$

$$\begin{aligned}
 & faw = 5 \\
 & unfaw = 8 \\
 & total = 13
 \end{aligned}$$

$$p = \frac{5}{13}$$

19. Twelve adults were surveyed and asked how many children they had. The results are below:

1, 3, 0, 2, 2, 0, 1, 1, 3, 2, 0, 0

If one adult is chosen from this group, what is the probability that they will have at least one child?

- (A) 66.67%
 (B) 33.33%
 (C) 1.88%
 (D) 1.25%

$$\frac{8}{12} = 66.67\%$$

20. A hockey player has taken 21 shots and scored 6 goals. What are the odds against the player scoring a goal on their next shot?

- (A) 2:5
 (B) 2:7
 (C) 5:2
 (D) 7:2

total = 21
 fav = 6
 unfav = 21 - 6
 = 15

odds against = (unfav : fav)
 = (15 : 6)
 = (5 : 2)

21. Which experiment involves independent events?
- (A) driving while texting and having an accident
 (B) rolling a die and flipping a coin ✓
 (C) rolling a die and getting an even number
 (D) studying math and passing the test

22. A standard six sided die is rolled twice. What is the probability of rolling a 3 and then rolling a 5?

(A) $\frac{1}{6} + \frac{1}{6}$

$\frac{1}{6} \times \frac{1}{6}$

(B) $\frac{1}{3} + \frac{1}{5}$

(C) $\frac{1}{6} \times \frac{1}{6}$

(D) $\frac{1}{3} \times \frac{1}{5}$

23. The student council has 7 members (3 males and 4 females). Four members are going to be chosen at random to attend a conference. Which expression represents the probability of at least 2 males being selected?

(A) $\frac{{}_3P_1 \times {}_4P_3 + {}_3P_2 \times {}_4P_2}{{}_7P_4}$

2 males AND 2 Females
OR
3 males AND 1 Female
OR

(B) $\frac{{}_3P_2 \times {}_4P_2 + {}_3P_3 \times {}_4P_1}{{}_7P_4}$

~~4 males AND 0 Females~~

(C) $\frac{{}_3C_1 \times {}_4C_3 + {}_3C_2 \times {}_4C_2}{{}_7C_4}$

Total $= {}_7C_4$

(D) $\frac{{}_3C_2 \times {}_4C_2 + {}_3C_3 \times {}_4C_1}{{}_7C_4}$

Fav = $({}_3C_2 \times {}_4C_2) + ({}_3C_3 \times {}_4C_1)$

24. The probability of Sean going to the hockey game on Friday night is 0.4. The probability he will go to the dance on Friday night is 0.5. The probability he will go to either the game or the dance is 0.8. What is the probability he will go to the dance and the hockey game?

$P(H \cup D)$

- (A) 0.90
 - (B) 0.20
 - (C) 0.16
 - (D) 0.10
- $P(H) = 0.4$
 $P(D) = 0.5$
 $P(H \cup D) = 0.8$
 $P(H \cap D) = ?$

$$P(H \cup D) = P(H) + P(D) - P(H \cap D)$$

$$0.8 = 0.4 + 0.5 - P(H \cap D)$$

$$0.8 = 0.9 - P(H \cap D)$$

$$0.8 - 0.9 = -P(H \cap D)$$

$$\underline{-0.1} = \underline{-P(H \cap D)}$$

25. Bob randomly selects 2 cards, without replacement, from the deck of 30 cards shown below. What is the probability of him selecting 1 red and then 1 blue?

- (A) $\frac{5}{19}$
- (B) $\frac{10}{87}$
- (C) $\frac{1}{9}$
- (D) $\frac{1}{100}$

$$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$$

$$\frac{10}{30} = \frac{1}{3} \times \frac{10}{29}$$

$$P(H \cap D) = 0.1$$

Card Colour	Cards
red	0 1 2 3 4 5 6 7 8 9
blue	0 1 2 3 4 5 6 7 8 9
green	0 1 2 3 4 5 6 7 8 9

26. What are the non-permissible values for the rational expression $\frac{7x}{3(2-x)(4x+1)}$?

- (A) $\{-\frac{1}{4}, 2\}$
- (B) $\{-\frac{1}{4}, 0, 2\}$
- (C) $\{-2, \frac{1}{4}\}$
- (D) $\{-2, 0, \frac{1}{4}\}$

$$2-x \neq 0 \Rightarrow 2 \neq x$$

$$4x+1 \neq 0 \Rightarrow \frac{4x}{4} \neq \frac{-1}{4} \Rightarrow x \neq -\frac{1}{4}$$

$$\{-\frac{1}{4}, 2\}$$

27. Which expression is equivalent to $\frac{x+10}{x+4}$, $x \neq -4$?

(A) $\frac{x+5}{x+2}$ X

(B) $\frac{x^2+10x}{x^2+4x}$ \rightarrow NPV = $\frac{x(x+10)}{x(x+4)}$
 $x \neq 0, x \neq -4$

(C) $\frac{3x+10}{3x+4}$

(D) $\frac{5x+50}{5x+20}$

$\frac{5}{5} \left(\frac{x+10}{x+4} \right) = \frac{5x+50}{5x+20}$

N.P.V $5x+20 \neq 0$
 $5(x+4) \neq 0$
 $x+4 \neq 0$
 $x \neq -4$

28. Which rational expression has non-permissible values of 0 and 5?

(A) $\frac{4}{x(x-5)}$ $x \neq 0$ $x-5 \neq 0$
 $x \neq 5$

(B) $\frac{4x}{(x-5)}$

(C) $\frac{(x-5)}{4x}$

(D) $\frac{x(x-5)}{4}$

29. Simplify: $\frac{3x+12}{x^2-16} = \frac{3(x+4)}{(x-4)(x+4)} = \frac{3}{x-4}$

(A) $\frac{3}{x+4}$, $x \neq \pm 4$

(B) $\frac{3}{x-4}$, $x \neq \pm 4$

(C) $\frac{3(x+4)}{(x+4)(x-4)}$, $x \neq \pm 4$

(D) $\frac{3(x+4)}{(x-4)(x-4)}$, $x \neq \pm 4$

30. Simplify: $\frac{x^2}{x^2-3x} = \frac{x^2}{x(x-3)} = \frac{x}{x-3}$

(A) $-3x, x \neq 0, 3$

(B) $-\frac{1}{3x}, x \neq 0, 3$

(C) $\frac{1}{1-3x}, x \neq 0, 3$

(D) $\frac{x}{x-3}, x \neq 0, 3$

31. Simplify: $\frac{x+5}{x+6} \cdot \frac{x+6}{x^2-25} \Rightarrow \frac{x+5}{x+6} \cdot \frac{x+6}{(x-5)(x+5)}$

(A) $x-5, x \neq -6, \pm 5$

(B) $\frac{x+5}{x^2-25}, x \neq -6, \pm 5$

(C) $\frac{1}{x-5}, x \neq -6, \pm 5$

(D) $\frac{1}{x+5}, x \neq -6, \pm 5$

$$= \frac{\cancel{(x+5)} \cancel{(x+6)}}{\cancel{(x+6)} (x-5) \cancel{(x+5)}} = \frac{1}{x-5}$$

32. Simplify: $\frac{6x}{20} \div \frac{2x^3}{5}$

$$\frac{6x}{20} \times \frac{5}{2x^3}$$

$$= \frac{30x}{40x^3}$$

(A) $\frac{3x^4}{25}, x \neq 0$

(B) $\frac{3x^3}{25}, x \neq 0$

(C) $\frac{3x^2}{4}, x \neq 0$

(D) $\frac{3}{4x^2}, x \neq 0$

$$= \frac{3}{4x^2}$$

33. Simplify: $\frac{8x-2}{x+1} + \frac{3x+4}{x+1}, x \neq -1$

Already got a LCD

(A) $\frac{11x+2}{2x+2}$

(B) $\frac{11x-2}{2x+2}$

(C) $\frac{11x+2}{x+1}$

(D) $\frac{11x-2}{x+1}$

$$= \frac{8x-2+3x+4}{x+1}$$

$$= \frac{8x+3x-2+4}{x+1}$$

$$= \frac{11x+2}{x+1}$$

34. Simplify: $\frac{-7x}{x+4} - \frac{3x}{2x+8}$, $x \neq -4$

(A) $\frac{-4x}{2(x+4)}$ $\hookrightarrow \frac{-7}{x+4} - \frac{3x}{2(x+4)}$ LCD = $2(x+4)$

(B) $\frac{-10x}{2(x+4)} = \left(\frac{-7}{x+4} \cdot \frac{2}{2} \right) - \frac{3x}{2(x+4)}$

(C) $\frac{-11x}{2(x+4)} = \frac{-14x}{2(x+4)} - \frac{3x}{2(x+4)}$

(D) $\frac{-17x}{2(x+4)} = \frac{-14x - 3x}{2(x+4)} = \frac{-17x}{2(x+4)}$

35. A student simplifies $\frac{3}{x-4} - \frac{2}{x}$, $x \neq 0, 4$. In which step does she make an error?

Step 1 $\left(\frac{3}{x-4} \cdot \frac{x}{x} \right) - \left(\frac{2}{x} \cdot \frac{x-4}{x-4} \right)$ ✓

Step 2 $\frac{3x - (2x-8)}{x(x-4)}$ ✓

Step 3 $\frac{3x - 2x + 8}{x(x-4)}$ ✗

Step 4 $\frac{x-8}{x(x-4)}$

- (A) 1
- (B) 2
- (C) 3
- (D) 4

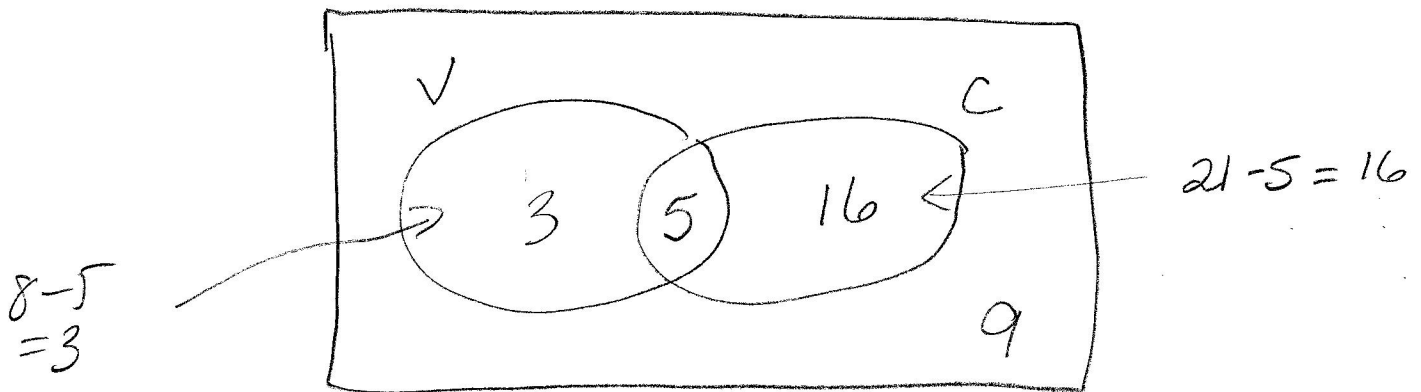
PART II - Total Value: 30 marks

Answer **ALL** items in the space provided. Show **ALL** workings.

Value

- 3 36(a). Jessica surveyed her classmates to see if they liked Vanilla ice cream or Chocolate ice cream.
- 8 people liked Vanilla ice cream
 - 5 people liked both Vanilla ice cream and Chocolate ice cream
 - 9 people liked neither type of ice cream
 - 21 people liked Chocolate ice cream

Draw a Venn diagram to determine how many people she surveyed in total.

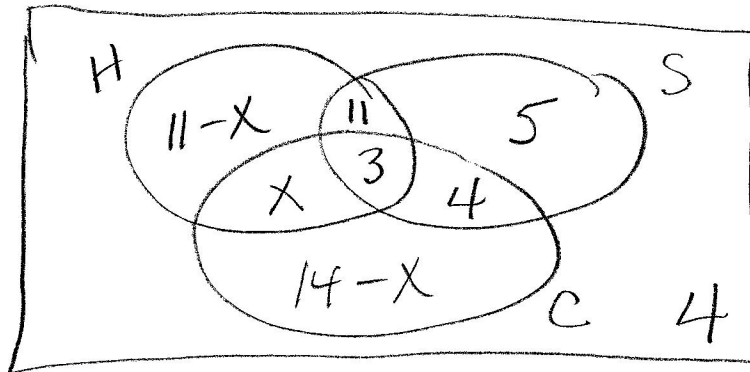


$$3 + 5 + 16 + 9 = 33$$

3 36(b). The first 50 customers at a movie theatre on Saturday night were surveyed about the types of movies they liked:

- ~~25~~ liked Horror movies
- ~~23~~ liked Sci-Fi movies
- 21 liked Comedy movies
- 3 liked all three types of movies
- 14 liked Horror and Sci-Fi movies
- 7 liked Sci-Fi and Comedy movies
- 4 didn't like any of the three types of movies

Determine the number of people who liked Horror **AND** Comedy but **NOT** Sci-Fi.



$$11-x + 11 + 5 + x + 3 + 4 + 14-x + 4 = 50$$

$$52 - x = 50$$

$$52 - 50 = x$$

$$2 = x$$

2 people liked Horror + Comedy, but not Sci-Fi

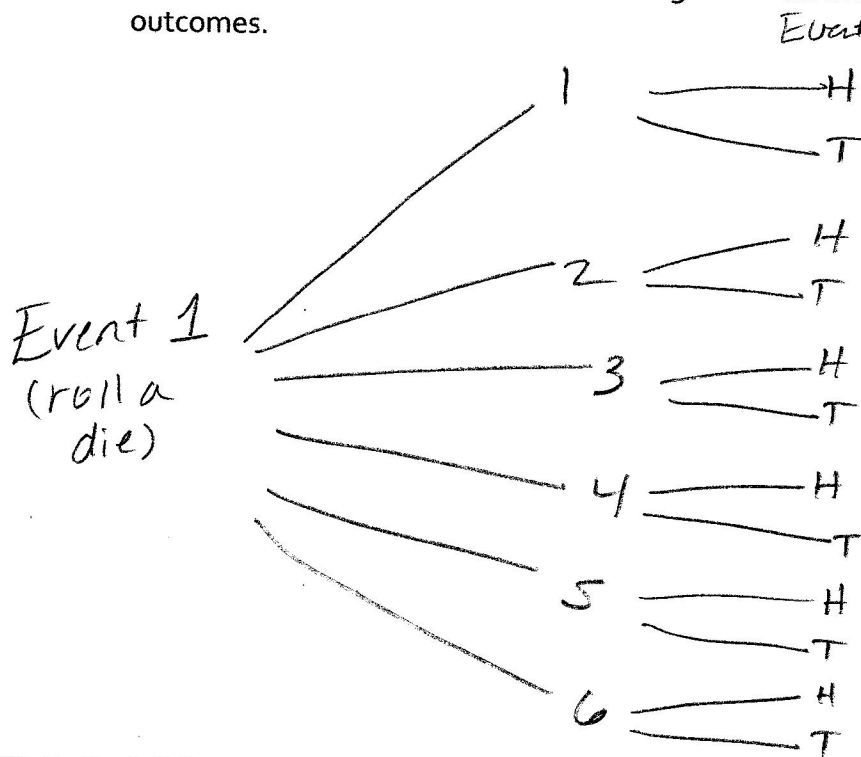
Value

- 2 37(a). How many different arrangements of the letters in the word **ADDRESSED** are possible?

$$\begin{aligned} \frac{\text{total}}{\text{repetitions}} &= \frac{9!}{2! \cdot 2! \cdot 3!} \\ &= \frac{362880}{24} \\ &= \del{45360} \\ &= 15120 \end{aligned}$$



- 2 37(b). David rolls a 6-sided die, numbered 1 to 6, and flips a coin with heads and tails. Use a graphic organizer such as a tree diagram or table to illustrate all possible outcomes.



- 3 37(c). How many 6 person committees can be formed from a group of 4 teachers and 30 students if there must be **at least** 3 teachers?

3 teachers AND 3 students
OR

4 teachers AND 2 students

$$\begin{aligned} & (4C_3 \times 30C_3) + (4C_4 \times 30C_2) \\ &= (4 \times 4000) + (1 \times 435) \\ &= 16240 + 435 \\ &= 16675 \end{aligned}$$

Value

- 3 37(d). Algebraically solve for n : ${}_nP_2 = 42$

$${}_nP_2 = 42$$

$$\frac{n!}{(n-2)!} = 42$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$$

$$n^2 - n - 42 = 0$$

$$(n-7)(n+6) = 0$$

$$\boxed{n=7} \quad n \neq -6$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nP_2 = \frac{n!}{(n-2)!}$$

factor OR use
quadratic formula

- 3 38(a). What is the probability of Bob and Chris standing next to each other when considering the total possible arrangements of Andrew, Bob, Chris, Dave, and Eric in a line?

fav.

$$\left[\frac{BC}{4!} \times _ \times _ \times _ \times _ \right] \times 2! = 48$$

$$\text{Total} = 5! = 120$$

$$P = \frac{\text{fav.}}{\text{total}} = \frac{48}{120} = \boxed{\frac{2}{5}}$$

- 3 38(b). John likes to jog. If the weather is nice he is 90% likely to jog. If it is raining he is only 30% likely to jog. The forecast for tomorrow indicates a 60% chance of rain. What is the probability that he will jog tomorrow?

2 options jogging AND raining
OR
jogging AND fine weather

$$\begin{aligned} P &= (0.30 \times 0.60) + (0.90 \times 0.40) \\ &= 0.18 + 0.36 \\ &= 0.54 \end{aligned}$$

Value

- 3 38(c). Statistics Canada reports that 92% of all Canadians regularly watch videos online, and of these people, 60% watch the videos on a mobile device. What is the probability of a Canadian person regularly watching videos on a device that is **NOT** mobile?

$$\text{prob. not mobile} = 100\% - 60\% = 40\%$$

$$0.92 \times 0.40 = \boxed{0.368}$$

- 3 39(a). Simplify and state the non-permissible values:

$$\frac{10x+40}{10+2x} \div \frac{3(x+4)}{x^2-25} = \frac{10x+40}{10+2x} \times \frac{x^2-25}{3(x+4)}$$

$$= \frac{10(x+4)}{2(5+x)} \times \frac{(x-5)(x+5)}{3(x+4)}$$

$$= \frac{10 \cancel{(x+4)} (x-5)(x+5)}{2(3)(\cancel{5+x})(\cancel{x+4})}$$

$$= \frac{\cancel{10} (x-5)}{\cancel{6}}$$

$$= \frac{5(x-5)}{3}$$

Restrictions

$$10+2x \neq 0$$

$$\frac{2x}{2} \neq \frac{-10}{2}$$

$$x \neq -5$$

$$3(x+4) \neq 0$$

$$x+4 \neq 0$$

$$x \neq -4$$

$$x^2-25 \neq 0$$

$$(x-5)(x+5) \neq 0$$

$$x-5 \neq 0 \quad x+5 \neq 0$$

$$x \neq 5 \quad x \neq -5$$

$$\therefore x \neq -5, -4, 5$$

2

39(b). Simplify:

$$\frac{3}{n-6} - \frac{2}{n+4}, n \neq 6, -4$$

$$LCD = (n-6)(n+4)$$

$$= \frac{3}{n-6} \cdot \frac{(n+4)}{(n+4)} - \frac{2}{n+4} \cdot \frac{(n-6)}{(n-6)}$$

$$= \frac{3n+12}{(n-6)(n+4)} - \frac{2n-12}{(n-6)(n+4)}$$

$$= \frac{3n+12 - (2n-12)}{(n-6)(n+4)}$$

$$= \frac{3n+12 - 2n+12}{(n-6)(n+4)}$$

$$= \frac{n+24}{(n-6)(n+4)}$$