Section 1.1 – Patterns in Division

Divisibility refers to whether or not a number is divisible by another number. If a number **divides evenly into another number** (no remainder), then it is divisible by that number.

For example, $36 \div 9 = 4$. 36 is divisible by 4 since 9 divides evenly into 36 (there is no remainder).

Divisibility by 10

Consider the following numbers. Circle the numbers that are divisible by 10.

44 50 62

75 90 38 40

10 88 120

How do we know if a number is divisible by 10?

Rule: ____

Divisibility by 2

Consider the following numbers. Circle the numbers that are divisible by 2.

34 99 59

52 78 67 32

52 46 31

How do we know if a number is divisible by 2?

Rule: ____

Divisibility by 5

Consider the following numbers. Circle the numbers that are divisible by 5.

80 49 61

25 40 57 55

78 10 15

How do we know if a number is divisible by 5?

Rule: ____

Example 1:

Circle the numbers that are **divisible by both 2 and by 5**.

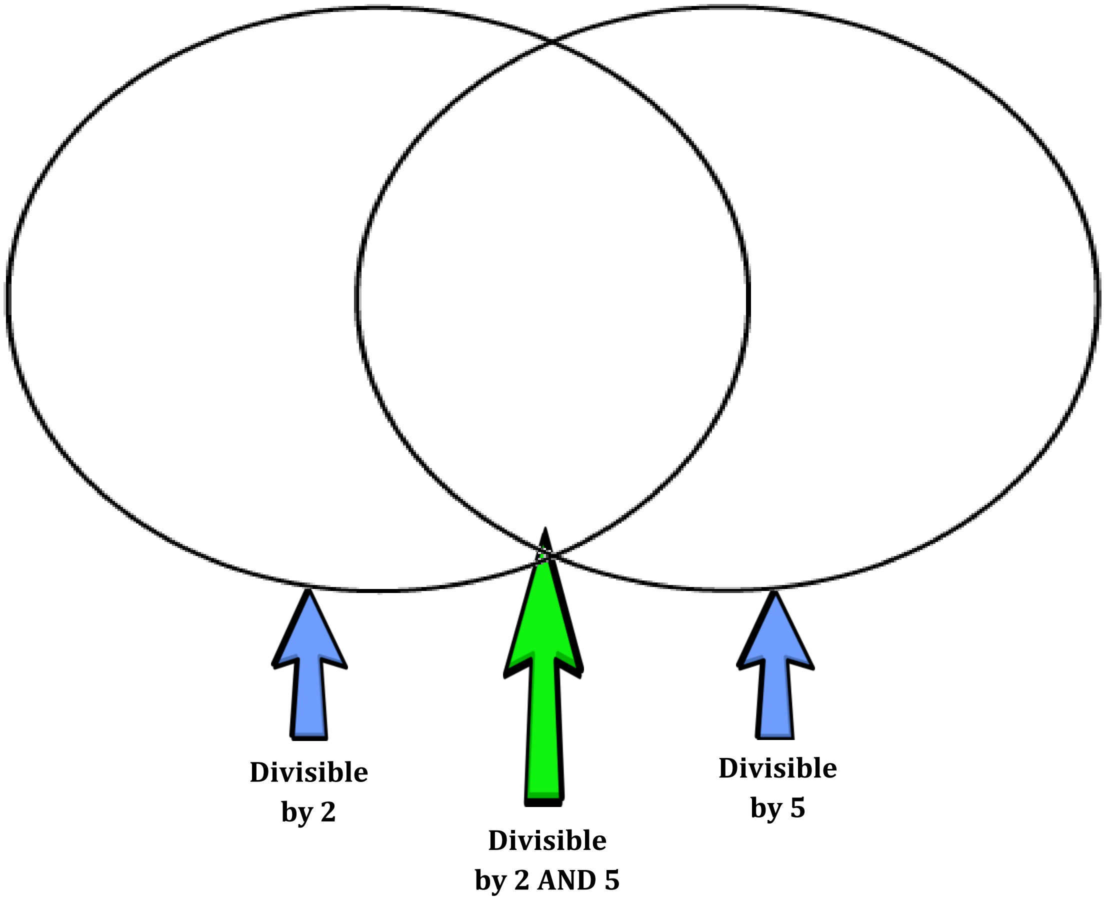
54 20 33

75 40 48 65

22 10 15

Venn diagrams are diagrams that use circles to represent groups and to show the relationship between the groups.

We can use a Venn Diagram to show the numbers divisible by 2 and 5.



Note: A number that is not divisible by either number is placed on the outside of the diagram.

Divisibility by 4 and 8

Trying to figure out which numbers are divisible by 4 and 8 can be a little more difficult. However, we can develop a rule to help us quickly figure it out without having to complete long division.

On the hundreds chart below, place a circle around all the numbers divisible by 4.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

On the chart below, continue the pattern.

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

What do you notice? Write a rule for numbers divisible by 4.

Rule: ____

Example1:

Which of the following numbers is divisible by 4? Justify your answer.

24	321	436	2048

Example 2:

Using the digits 0-9, replace the □ in each number with all the possibilities that will make each number divisible by 4.

a) 13□

b) 14□8

c) 234□

d) 15□52

Divisibility by 8 is similar to the rule for divisibility by 4 but, instead of the last two digits, we look at the last three.

Rule: <u>A number is divisible by 8 if the last three digits are divisible by 8.</u>

Example 1:

Explain which of the following are divisible by 8. Show how you know.

a) 5872

b) 12 168

c) 3 024

Section 1.2 – More Patterns in Division

Divisibility by 3

Complete the chart below. The first one is done for you.

Number	Divisible by 3?	Sum of Digits	Sum of Digits Divisible by 3?
30	yes	9+0=9	yes
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48			
49			
50			
51			

What do you notice??

Rule: ____

Divisibility by 9

Complete the chart below. The first one is done for you

Number	Divisible by 9?	Sum of Digits	Sum of Digits Divisible by 9?
18	yes	9+0=9	Yes
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			

What do you notice?

Rule: ____

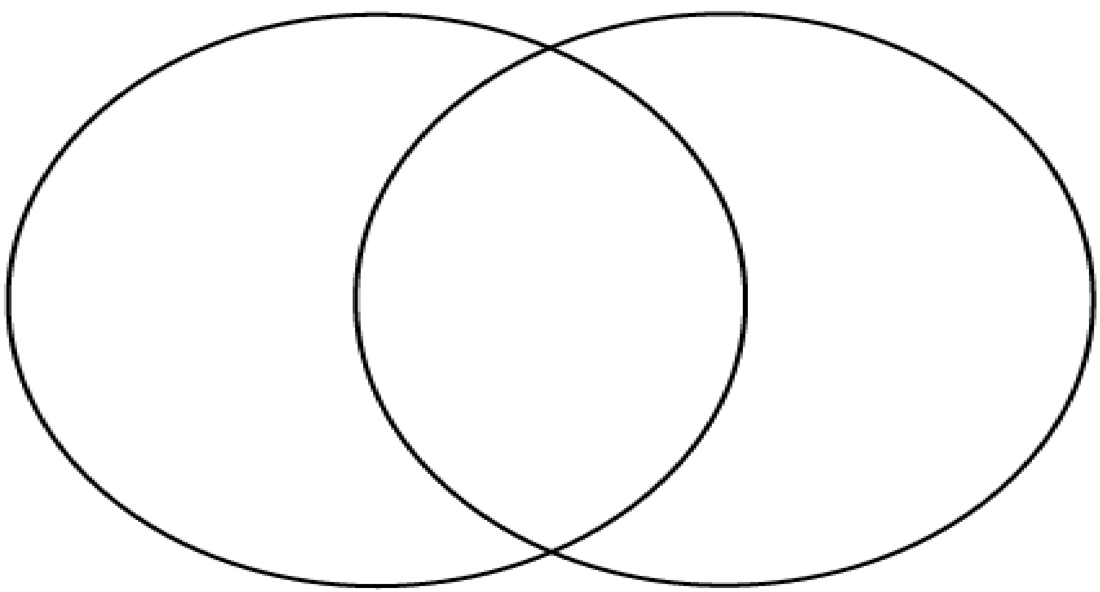
<u>Divisibility by 6</u>

Sort the following numbers and place them in the Venn diagram.

12, 21, 36, 42, 56, 61, 74, 88, 93, 135, 246, 453, 728

Divisible by 2	Divisible by 3	Divisible by Neither

Let's place the numbers in a Venn diagram!



What do you notice about the numbers in the overlapping region of the diagram?

Write a rule for divisibility by 6:

Rule: ____

We can also sort numbers using a ***Carroll diagram.***

	Divisible by 2	Not Divisible by 2
Divisible by 3		
Not Divisible by 3		

Let's sort these numbers: 1, 11, 15, 20, 24, 35, 47, 98, 100

	Divisible by 5	Not Divisible by 5
Divisible by 2		
Not Divisible by 2		

<u>Section 1.3 – Algebraic Expressions</u>

Joe makes $3 on every chocolate bar he sells. How much money he earns, depends on how many bars he sells each week.

We can express this situation as **3*b*.**

- This means $3 \times b$ (3 times b, since he gets $3 per chocolate bar)
- "b" represents the number of chocolate bars
- "b" is called a **<u>variable</u>**

Variable:____

____

____

$3b$ is called an **<u>expression</u>**.

Expression____

____

____

In the expression $3b$, **3** is called the **<u>numerical coefficient</u>**.

Numerical coefficient: ____

____

____

In the expression 3b + 5, **5** is called the **constant.**

Constant Term: ____

____

____

Example 1:

In each expression, identify the variable, numerical coefficient and constant term.

a) **3*r*+7**

Variable: ___________

Numerical Coefficient: ___________

Constant Term: ___________

b) **4*h* – 1.3**

Variable: ___________

Numerical Coefficient: ___________

Constant Term: ___________

c) **19 – 6*w***

Variable: ___________

Numerical Coefficient: ___________

Constant Term: ___________

d) **½ *d*+3**

Variable: ___________

Numerical Coefficient: ___________

Constant Term: ___________

e) $\mathbf{5.4}\boldsymbol{k}$

Variable: ___________

Numerical Coefficient: ___________

Constant Term: ___________

f) **c – 8**

Variable: ___________

Numerical Coefficient: ___________

Constant Term: ___________

Example 2:

Write expressions for the following:

a) Five more than a number ____

b) Three less than a number ____

c) Six times a number ____

d) Three more than two times a number ____

e) A number divided by twenty ____

f) One hundred divided by a number ____

g) Seven subtracted from four times a number ____

h) Twelve time a number is added to fifteen ____

i) Nine more than triple a number ____

Just as we can write expressions for sentences, we can write sentences (words) for expressions:

Example 3:

a) $13p$ ____

b) $m + 12$ ____

c) $p/2$ ____

d) $3k + 6$ ____

e) 16 – n/2 ____

We can evaluate an expression for a given value, by "plugging" a value in where you see the variable.

Example 4:

Evaluate each expression assuming that $n = 4$.

a) $4n = 4\ (4) = 16$

b) $12/n =$

c) $14 - n =$

d) $n + 8 =$

e) $2n + 7 =$

f) $28 - 24/n =$

Worksheet

Part A: What words match with the mathematical operations?

Math Symbol	Words
+	
-	
÷	
X	

Part B: Write English phrases for the following mathematical expressions.

Expression	Sentence
$7n$	
$8 + n$	
$a - 7$	
$10 \div m$	

$2n + 5$	
$3c - 2$	

Part C: Translate each English expression or equation into mathematical form.

English	Expression
1. Double a number	
2. A number increased by six.	
3. A number decreased by four.	
4. The sum of a number and ten.	
5. Seven times a number.	
6. Seven less than a number.	
7. Half of a number increased by nine.	
8. A number increased by seven is fourteen.	
9. Three times a number plus six is twenty-four.	
10. One-quarter of a number equals eighteen.	
11. A number divided by five and then decreased by eight.	
12. Seven decreased by a number	

Question: Which of the above are **equations**? How do you know?

Part D: For each algebraic expression, identify the **numerical coefficient**, the **variable**, and the **constant term**.

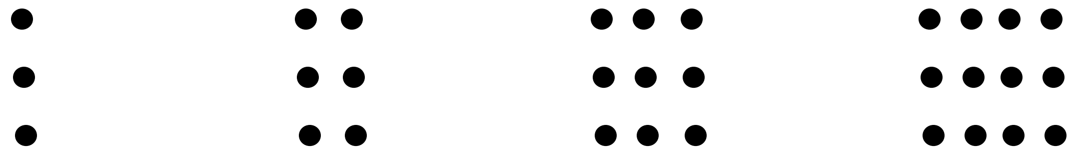
Algebraic Expression	**Numerical Coefficient**	**Variable**	**Constant Term**
$3x + 4$			
$7 - 2h$			
$g + 10$			
$5d$			
$2w - 25$			

Part E: Evaluate each expression by replacing the *variable* with the given number.

$x + 10$ ($x = 3$)	$2h - 4$ ($h = 3$)	$10 + 3n$ ($n = 9$)	$\frac{f}{3} - 6$ ($f = 18$)

Section 1.4 –Relationships in Patterns

Consider the pattern:



1 2 3 4

We can show this in a table:

Diagram # *(d)*	1	2	3	4	5
# of circles *(c)*					

What do you notice?

What relationship do you see between the diagram number and the number of dots?

In Words:

Algebraically:

3*d* is a ____ because the variable, _____________, is related to the number of circles and vice versa.

Example:

How is each term related to the term number? Write a relation for each.

a)

Term Number	1	2	3	4	5	6
Term	2	4	6	8	10	12

b)

Term Number	1	2	3	4	5	6
Term	3	4	5	6	7	8

Section 1.5 –Patterns and Relationships in Tables

We can represent a relation using an input/output table. We enter numbers in the input column, do what the relation tells us, and write the result under the output.

These tables come in handy when we want to graph our relations.

Examples: Complete each table and explain how the output is related to the input.

a)

Input n	Output 3n+2
1	5
2	8
3	11
4	14
5	17

3(1)+2 = 3+2 = 5

3(2)+2 = 6+2 = 8

The output is two more than three times the input.

b)

Input p	Output 12-p
1	11
2	10
3	9
4	8
5	7

12-1 = 11

12-2 = 10

The output is twelve minus the input.

c)

Input k	Output 3+5k
1	8
2	13
3	18
4	23
5	28

3+5(1) = 3+5=8

3+5(2)=3+10=13

The output is *three more than 5 times the input*

We can also write the relation using algebra when we are given the table.

Examples: Write a relation for each table.

a)

Input j	Output
1	6
2	10
3	14
4	18
5	22

b)

Input a	Output
1	9
2	18
3	27
4	36
5	45

c)

Input h	Output
1	4
2	5
3	6
4	7
5	8

Section 1.6 – Graphing Relations

We can use graphs to show the relationship between two quantities.

Consider the example below:

1. Triangles are used to create the pattern below:

Diagram 1

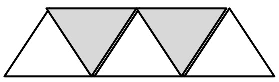


Diagram 2

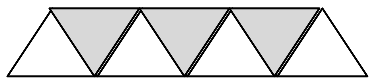
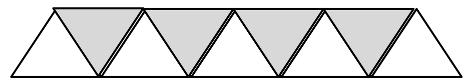
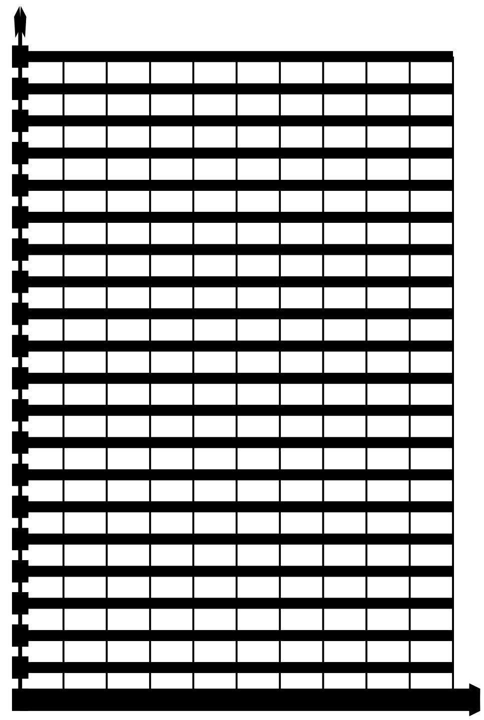


Diagram 3



Complete the table and graph the relation:

Diagram Number (n)	Number of Shaded Triangles (t)
1	
2	
3	
4	
5	
6	



** Remember, the input goes on the bottom (horizontal axis) and the output goes on the side (vertical axis).

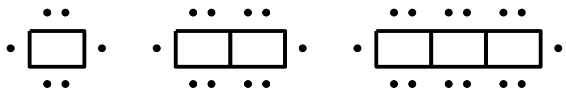
Write a relation to show how the number of squares is related to the diagram number, *n*.

When the points on a graph fall in a straight line, the relation is called a

____.

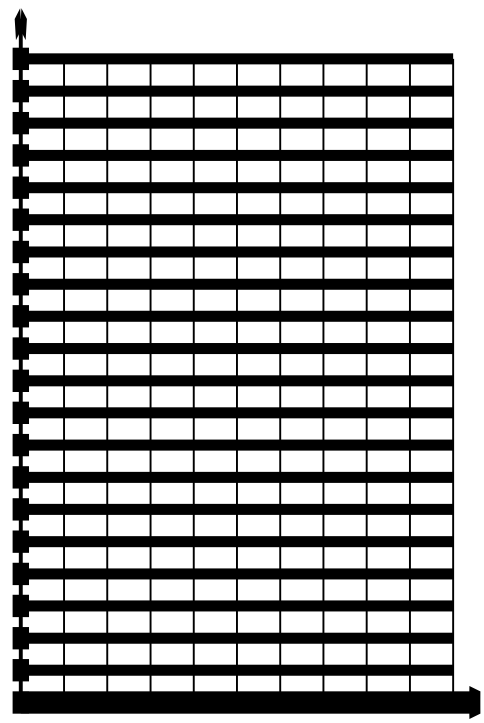
Try the following examples:

2. Sheila was having a party and could arrange the table and chairs as follows:



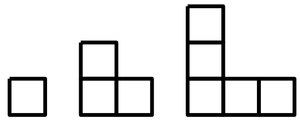
Complete the table and graph the relation.

Number of tables (*n*)	**Number of people**
1	
2	
3	
4	
5	



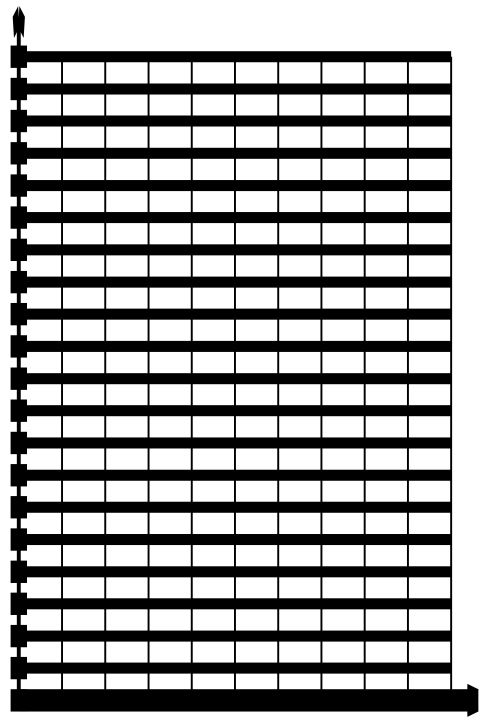
Write a relation to show how the number of squares is related to the diagram number, *n*.

3. Square tiles are used to create the pattern below.



Complete the table and graph the relation.

Diagram number(*n*)	**Number of squares**
1	
2	
3	
4	
5	



Write a relation to show how the number of squares is related to the diagram number, *n*.

Section 1.7 –Reading and Writing Equations

Let's be math detectives!!

Example 1:

I am thinking of a number. If you multiply it by 3 and add 4, you will get 13. What is the number?
(Clue: Write the algebraic equation first!)
What is an algebraic equation?

It is a ____ describing the

relationship , using an ______________________________, between two expressions.

Examples:

$2n + 1 = 9$, then n = ________

$3p + 4 = 19$, then p = ________

Example 2:

I am thinking of a number. If you multiply it by 5 and subtract 4, the answer is 21. What is the number?

Example 3:

Katelyn bought 3 CD's. Each CD cost the same amount. The total cost is $36.00.

A) Write the algebraic equation

B) What is the cost of one CD?

Example 4:

Write an equation for each sentence:

A) Three more than a number **is** 15.

B) A number subtracted from 5, **is** 1.

C) Eight added to three times a number **is** 26.

Example 5:

Write a sentence for each equation:

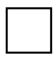
A) $2n + 4 = 16$

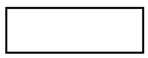
B) $3n - 5 = 10$

C) $2n - 8 = 10$

Section 1.8 – Solving Equations Using Algebra Tiles

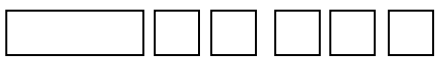
We can use algebra tiles to represent an expression or an equation.

 = +1, called a **unit tile**.

 = x, called a **variable tile**.

Example:

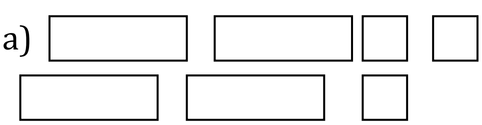
a) $n + 5$ can be represented as

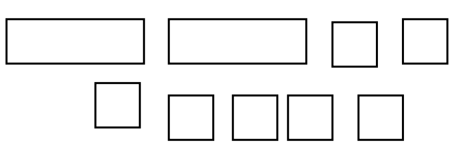


b) $2n + 1$

c) $3n + 2$

What expression is represented by the following?

a) 

b) 

We can solve algebraic expressions using tiles:

For example: $3n + 3 = 6$

We want to get the variable tiles (long tiles) on one side by themselves.

To do this, we take away 3 tiles on the left side....**but to keep the equation balanced**, we **must** take away three tiles on the right side too.

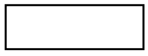
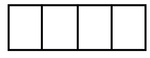
<u>Example:</u> Use tiles to solve each:

a) $7 + x = 15$

b) $4x = 16$

c) $5x + 3 = 13$

Remember:

 $= x$ so,  $= \frac{x}{4}$

If we have $\frac{x}{4} = 5$, we want $1x$.

So, we need ____to complete a whole.

Try: $\frac{x}{3} = 2$