Math 1201
Unit 3: Factors & Products

Read *Building On*, *Big Ideas*, and *New Vocabulary*, p. 132 text.

Ch. 3 Notes

§3.1 Factors and Multiples of Whole Numbers (1 class)

Read *Lesson Focus* p. 134 text.

Outcomes

1. Define and give an example of a **prime number**. See notes
2. Define and give an example of a **composite number**. See notes
3. Explain what is meant by the **prime factorization** of a natural number. p. 135
4. Determine the **prime factors** of a whole number. p. 135
5. Determine the **greatest common factor** (GCF) of two or more numbers. p. 136
6. Determine the **least common multiple** (LCM) of two or more numbers. p. 137
7. Solve problems involving GCF and LCM. pp. 138-139

Def$^{\underline{n}}$: A **prime number** is a number with exactly two factors.

E.g.: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73…

Def$^{\underline{n}}$: A **composite number** is a number that is **not** prime.

E.g.: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, …

Def$^{\underline{n}}$: The **prime factorization** of a natural number is the number rewritten as the product of **prime** numbers.

E.g.: $6 = 2\times3$; E.g.: $3300 = 2^2 \times 3 \times 5^2 \times 11$

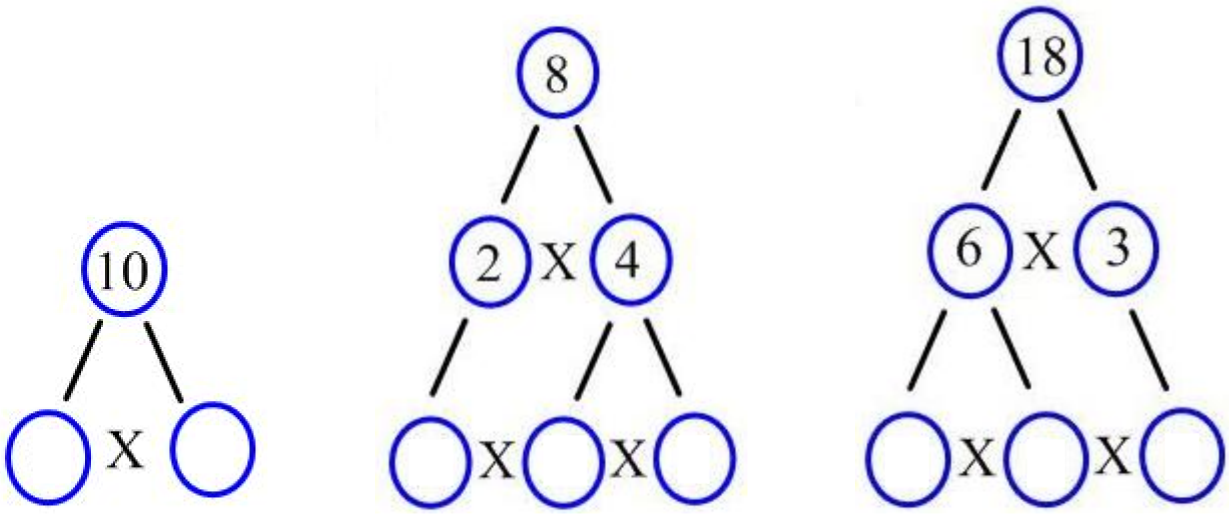
Non E.g.: $12 = 3\times4$

Note that the divisibilty rules can help you find these prime numbers.

Divisible by:	If …	Example
2	Even – ends in 0, 2, 4, 6, or 8.	24
3	Sum of digits is a multiple of 3.	123
4	Last two digits divisble by 4.	216
5	Ends in 0 or 5.	225
6	Divisible by 2 AND by 3.	54
8	Last three digits divisble by 8.	2104
9	Sum of digits is a multiple of 9.	306
10	Ends in 0.	123 456 780

Finding the Prime Factorization of a Natural Number

E.g.:



E.g.: Draw a factor tree and write the prime factorization of 225 $\left[\text{ANS: } 3^2 \times 5^2\right]$

Do #'s 5 a, c, e, 6 c, f p. 140 text in your homework booklet.

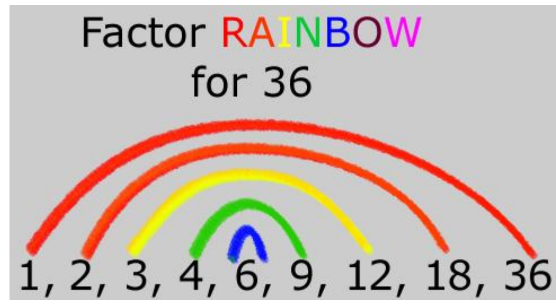
Def $^{\underline{n}}$: The **greatest common factor** (GCF) of two or more numbers is the **largest** number that divides evenly into the given numbers. Note that the GCF cannot be larger than the smallest given number.

E.g.: The GCF of 5 and 25 is 5.
E.g.: The GCF of 14 and 35 is 7.
E.g.: The GCF of 18, 27, and 36 is 9.

Finding the Greatest Common Factor (GCF) of Two or More Numbers

E.g.: Find the GCF of 24 and 36.

Method 1: Factor Rainbows

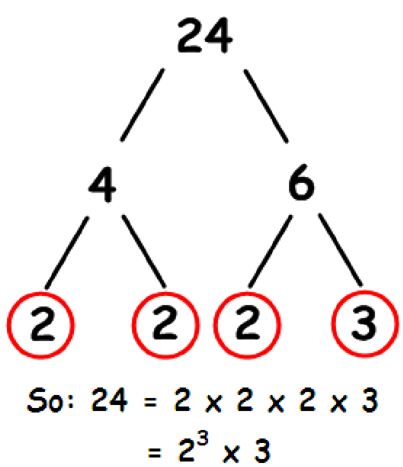


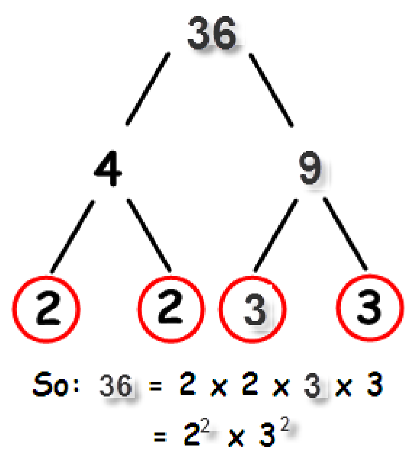
GCF of 24 and 36 is 12.

How do you know when to stop when you are using factor rainbows?

When you get to the square root of the number, you should have all the factors.

Method 2: Prime Factorization





The GCF is the **lowest** power of each prime number. So the GCF of 24 and 36 is $2^2 \times 3^1 = 12$

Method 3: Table

24	
2	**12**
3	8
4	6

36	
2	18
3	**12**
4	9
6	6

The GCF of 24 and 36 is 12

Simplifying Fractions using the GCF

E.g.: Reduce $\frac{1225}{2750}$.

$1225 = 5^2 \times 7^2 = 2^0 \times 5^2 \times 7^2 \times 11^0$

$2750 = 2 \times 5^3 \times 11 = 2 \times 5^3 \times 7^0 \times 11$

The GCF is the **lowest** power of each prime number. So the GCF of 1225 and 2750 is $2^0 \times 5^2 \times 7^0 \times 11^0 = 1 \times 25 \times 1 \times 1 = 25$

So $\frac{1225}{2750} = \frac{1225 \div 25}{2750 \div 25} = \frac{49}{110}$

Do #'s 8 a, c, e, 9 a, 15 a, c, f, 21, p. 140 text in your homework booklet.

Def^n: The **least common multiple (LCM)** of two or more numbers is the **smallest** number that the given numbers divide into evenly. Note that the LCM cannot be smaller than the largest given number.

E.g.: The LCM of 5 and 25 is 25.
E.g.: The LCM of 14 and 35 is 70.
E.g.: The LCM of 18, 27, and 36 is 108.

Finding the Least Common Multiple (LCM) of Two or More Numbers

E.g.: Find the LCM of 24 and 36.

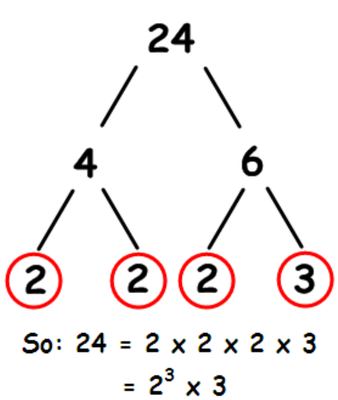
Method 1: List the multiples of each given number until you find a match.

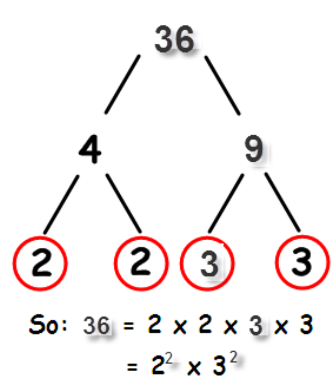
24, 48, 72, 96, …

36, 72, 108, …

The LCM is 72.

Method 2: Prime Factorization





The LCM is the **highest** power of each prime number. So the LCM of 24 and 36 is $2^3 \times 3^2 = 72$

Adding and Subtracting Fractions using LCM

E.g.: Simplify $\frac{3}{5}+\frac{5}{18}-\frac{7}{3}$

$5=5^1$

$18=2\times 3^2$

$3=3^1$

The LCM is the **highest** power of each prime number. So the LCM of 5, 18 and 3 is $2^1\times 3^2\times 5=90$

So $\frac{3}{5}+\frac{5}{18}-\frac{7}{3}=\frac{3\times 18}{5\times 18}+\frac{5\times 5}{18\times 5}-\frac{7\times 30}{3\times 30}=\frac{54}{90}+\frac{25}{90}-\frac{210}{90}=\frac{54+25-210}{90}=\frac{79-210}{90}=-\frac{131}{90}$

Do #'s 10 a, c, e, 11 a, b, 12, 14, 16 e, f, p. 140 text in your homework booklet.

Problem Solving Using GCF and LCM

E.g.: A land developer buys a parcel of land that measures 600m by 720m and wants to divide the land into square lots. What is the side length of the largest possible square lot?

The side length of the largest possible square lots must be the GCF of 600 and 720.

$600=3\times 8\times 25=3^1\times 2^3\times 5^2$

$720=16\times 5\times 9=2^4\times 5^1\times 3^2$

The GCF of 600 and 720 is $2^3\times 3^1\times 5^1=120$. So the largest possible square lot that can be used to subdivide the land has side length 120m.

Do # 17, p. 140 text in your homework booklet.

E.g.: Grenfell House has 54 members while Coaker House has 72 members. The houses must line up so that both houses have the same number of columns. What is the largest possible number of columns?

The largest number of columns is the GCF of 54 and 72.

$54=2\times 27=2^1\times 3^3$

$72=8\times 9=2^3\times 3^2$

The GCF of 54 and 72 is $2^1\times 3^2=18$. So the largest number of columns is 18.

Do # 13, p. 140 text in your homework booklet.

E.g.: What are the dimensions of the smallest square that can be covered using floor tiles that are 8cm wide and 10cm long, if the tiles cannot be cut?

The side length of the smallest possible square is the LCM of 8 and 10.

$8 = 2^3$

$10 = 2^1 \times 5^1$

The LCM of 8 and 10 is $2^3 \times 5^1 = 40$. So the smallest possible square lot that can be tiled has side length 40cm.

Do # 19 a, p. 140 text in your homework booklet.

E.g.: Graphing calculators come in boxes which are 5cm by 14cm by 32cm. What is the side length of the smallest possible **cubic** shipping carton that could be filled using these calculators?

The side length of the smallest possible carton is the LCM of 5, 14, and 32.

$5 = 5^1$

$14 = 2^1 \times 7^1$

$32 = 2^5$

The LCM of 5, 14, and 32is $2^5 \times 5^1 \times 7^1 = 1120$. So the smallest possible square carton that can be filled has side length 1120cm.

Do # 22, p. 140 text in your homework booklet.

§3.2 Perfect Squares, Perfect Cubes, and Their Roots (1 class)

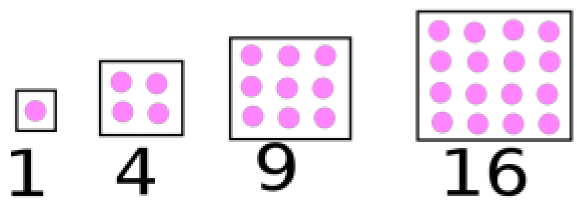
Read *Lesson Focus* p. 142 text.

Outcomes

1. Define and give an example of a **perfect square**. p. 143
2. Explain the relationship between a perfect square and the area of a square. p. 143
3. Explain the relationship between the square root of a perfect square and the side length of a square. p. 143
4. Define and give an example of a **perfect cube**. p. 144
5. Explain the relationship between a perfect cube and the volume of a cube. p. 144
6. Explain the relationship between the cube root of a perfect cube and the side length of a cube. p. 144
7. Define and give an example of a **radical**. p. 145
8. Identify the **index** of a radical. p. 145
9. Identify the **radicand** of a radical. p. 145
10. Solve problems involving radicals. p.

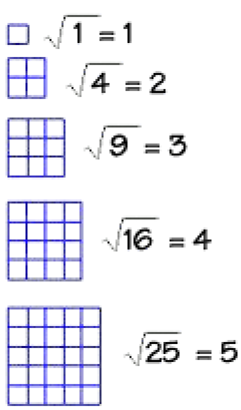
Def^{n}: A **perfect square** is any natural number that can be used to represent the area of a square with a side length that is also a whole number.

E.g.:



Def^{n}: The **square root** of a number $\left(\sqrt[2]{n} = \sqrt{n}\right)$ is a whole number that can be multiplied by itself to get a perfect square. Note that the square root is the side length of the square.

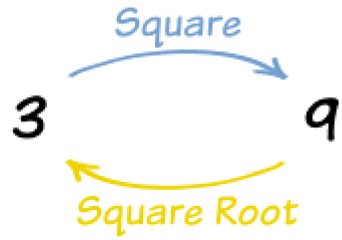
E.g.:



Summary

1. Perfect Square = Area of a Square
2. Square Root = Side Length of the Square

Squares	Square Roots
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$



3 squared is 9, so a **square root of 9 is 3**

Do #'s 7, 17 a, p. 147 text in your homework booklet.

Finding the Square Root of a Number

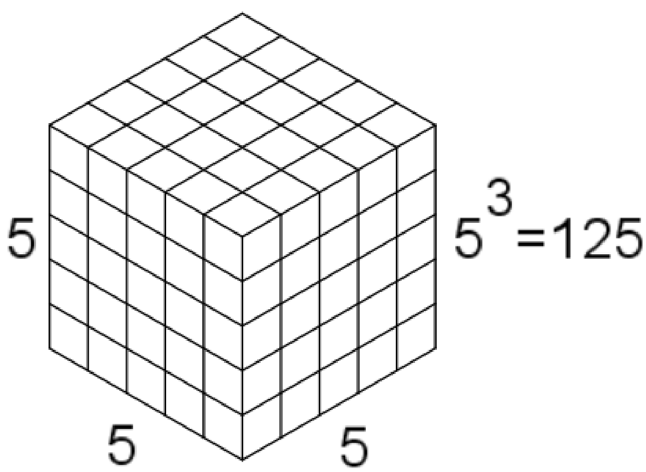
You should know the square roots of perfect squares up to 144. For others, use a calculator.

E.g.: $\sqrt[2]{36} = \sqrt{36} = 6$; $\sqrt[2]{169} = \sqrt{169} = 13$; $\sqrt[2]{400} = \sqrt{400} = 20$; $\sqrt[2]{1225} = \sqrt{1225} = 35$

Do #'s 4 a, c, e, p. 146 text in your homework booklet.

Def^n: A **perfect cube** is any natural number that can be used to represent the volume of a cube with a side length that is also a whole number.

E.g.: 125 is a perfect cube since it is the volume of a cube with side length 5.



Def^{n}: The **cube root** of a number $\left(\sqrt[3]{n}\right)$ is a whole number that can be multiplied by itself three times to get a perfect cube. Note that the cube root is the side length of the cube.

E.g.: $\sqrt[3]{125} = 5$ because 5 is the side length of the cube with volume 125units^3.

Summary

1. Perfect Cube = Volume of a Cube
2. Cube Root = Side Length of the Cube

Do #'s 8, 17 b, p. 147 text in your homework booklet.

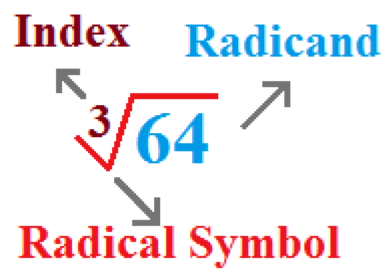
Finding the Cube Root of a Number

You should know the cube roots of perfect cubes up to 125. For others, use a calculator.

E.g.: $\sqrt[3]{216} = 6$; $\sqrt[3]{512} = 8$; $\sqrt[3]{1331} = 11$; $\sqrt[3]{9261} = 21$

Do #'s 5 a, c, e, p. 146 text in your homework booklet.

Note that the **square root** of a number $\left(\sqrt[2]{n} = \sqrt{n}\right)$ and the **cube root** of a number $\left(\sqrt[3]{n}\right)$ are both radicals. A radical typically has three parts (see right).



Solving Problems Involving Square Roots & Cube Roots

E.g.: Use factoring to determine if 729 is a perfect square or a perfect cube.

$$\begin{array}{c} 729 \\ 3\times 243 \\ 3\times 3\times 81 \\ 3\times 3\times 3\times 27 \\ 3\times 3\times 3\times 3\times 9 \\ 3\times 3\times 3\times 3\times 3\times 3 \\ 3^6 = \left(3^2\right)^3 = 9^3 \end{array} \qquad \begin{array}{c} 729 \\ 3\times 243 \\ 3\times 3\times 81 \\ 3\times 3\times 3\times 27 \\ 3\times 3\times 3\times 3\times 9 \\ 3\times 3\times 3\times 3\times 3\times 3 \\ 3^6 = \left(3^3\right)^2 = 27^2 \end{array}$$

Therefore 729 is a perfect cube because $9^3 = 729$ AND it is a perfect square because $27^2 = 729$.

Do #'s 6 a, c, e, p. 146 text in your homework booklet.

E.g.: A cube has a surface area of 726in^2. What is its volume?

We know that a cube has 6 sides with equal area so if we let s be a side length of the cube, then

$$6s^2 = 726$$

$$\frac{\cancel{6}s^2}{\cancel{6}} = \frac{726}{6}$$

$$s^2 = 121$$

$$s = \pm\sqrt{121}$$

$$s = \pm 11$$

$$s = 11\text{in}$$

So the volume of the cube must be $(11)^3 = 1331\text{in}^3$

Do # 10, p. 147 text in your homework booklet.

Section 3.3 Common Factors of a Polynomial (2 classes)

Read *Lesson Focus* p. 150

Outcomes:

1. Define and give an example of a **monomial**. See notes
2. Define and give an example of a **polynomial**. See notes
3. Identify the **terms** of a polynomial. See notes
4. Define and give an example of a **binomial**. See notes
5. Define and give an example of a **trinomial**. See notes
6. Explain what is meant by **factoring**. See notes
7. Explain, using examples, the relationship between factoring and multiplication of polynomials. p. 151
8. Determine the common factors in the terms of a polynomial and express the polynomial in factored form. pp. 152-154
9. Factor a polynomial and verify by multiplying the factors. pp. 152-154

Def$^{\underline{n}}$: A **monomial** is a constant (a number) or the product of a constant and one or more variables.

E.g.: $2,\quad -15.67,\quad \frac{3}{5},\quad x \text{ or } 1x,\quad -4r^2,\quad 3.2567xy,\quad -11x^7$

Def$^{\underline{n}}$: A **polynomial** is a monomial or the sum or difference of two or more monomials. Each monomial in the polynomial is referred to as a **term**.

E.g.: $2,\quad -15.67x+4,\quad \frac{3}{5}y^3+7y-13,\quad 4-x,\quad -4r^2+7r^6,\quad 3.2567xy-7x,\quad 25x^2-121y^2$

Def$^{\underline{n}}$: A **binomial** is a polynomial with **two** terms.

E.g.: $2-4y,\quad -15.67x+4,\quad \frac{3}{5}y^3-13,\quad 4-x,\quad -4r^2+7r^6,\quad 3.2567xy-7x,\quad 25x^2-121y^2$

Def$^{\underline{n}}$: A **trinomial** is a polynomial with **three** terms.

E.g.:

$2-7x+14x^2,\quad -15.67x+4-11y,\quad \frac{3}{5}y^3+7y-13,\quad 4-x+xy,\quad -4r^2+7r^6-11r,\quad 25x^2-121y^2-11$

Def$^{\underline{n}}$: **Factoring** a polynomial means to write the polynomial as a **product**.

E.g. $22=(2)(11),\quad 4x+6=2(2x+3)$

A Comparison of Factoring and Multiplying Factors

Recall that in Grade 9, you multiplied a monomial by a binomial using the **distributive property** $\left(a\left(b+c\right)=ab+ac\right)$. This process is known as **expanding**. In this chapter you will also be expected to work backwards to **factor** a polynomial.

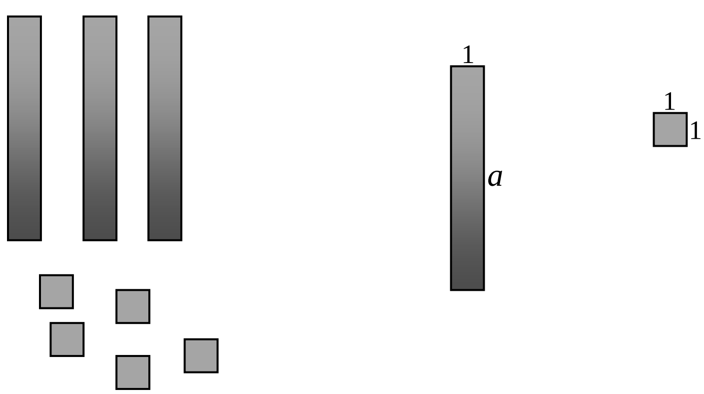
In Arithmetic	
Multiplying integers to obtain a product (**Expanding**).	Writing a number as a product of integer factors. (**Factoring**)
E.g.: $(3)(5)=15$	E.g.: $15=(3)(5)$
E.g.: $(-2)(3)(11)=-66$	E.g.: $-66=(-2)(3)(11)$
In Algebra	
Multiplying a monomial by a binomial to obtain a product (**Expanding** using the distributive property).	Writing an expression as the product of a monomial and a binomial using the GCF. (**Factoring** using the distributive property).
E.g.: $2(a+5)=2a+2(5)=2a+10$	E.g.: $2a+10=2(a)+2(5)=2(a+5)$
E.g.: $-3(11-r)=-3(11)-3(-r)=-33+3r$	E.g.: $-33+3r=-3(11)-3(-r)=-3(11-r)$

Factoring a Binomial

Method 1: Algebra Tiles

E.g.: Factor $4a+6$ (See http://nlvm.usu.edu/en/nav/frames_asid_189_g_4_t_2.html?open=activities&from=category_g_4_t_2.html)

Recall that we can represent $4a$ using four rectangular rods/strips, each of which has a length of *a* and a width of 1. We can represent 6 using six square unit tiles, each with side length 1 (see below). We will let shaded tiles represent positive values and white tiles represent negative values.



To factor $4a+6$ using algebra tiles, we must arrange these tiles to form a rectangle with area $4a+6$ (see below).

We find the length and width of the rectangle to get the factors of $4a+6$. The width of the rectangle is 2 and the length of the rectangle is $2a+3$. We use that fact that the area of a rectangle is the product of the width and the length to factor $4a+6$.

$$4a+6=2(2a+3)$$

Note that we can **verify** that our factors are correct by expanding using the distributive property.

$$2(2a+3)=2(2a)+2(3)=4a+6$$

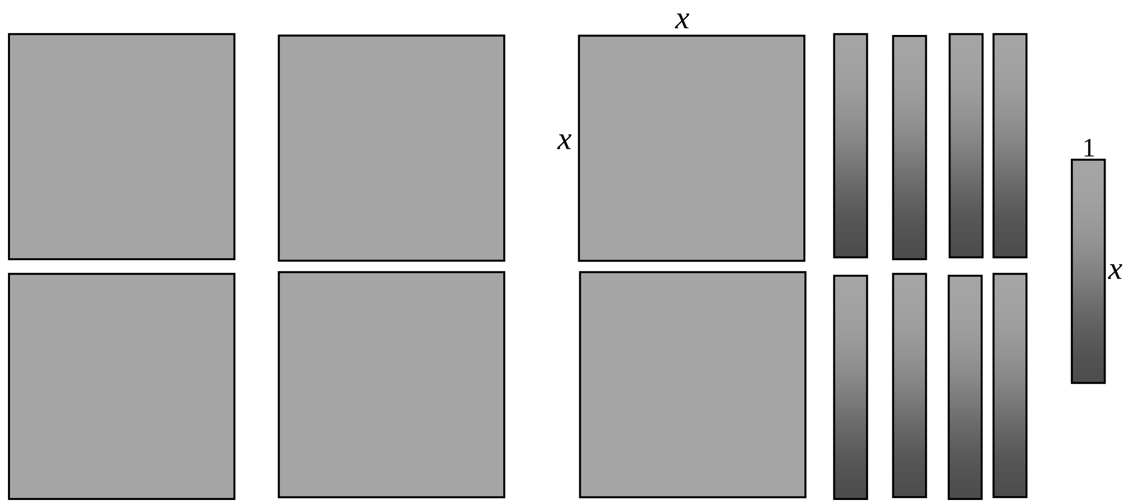
Method 2: Using the GCF and the Distributive Property

Step 1: Write the prime factorization for each term.	$4a=2^2\times a^1=2^2\times a^1\times 3^0$ $6=2^1\times 3^1=2^1\times 3^1\times a^0$
Step 2: Take the **lowest** power for each factor and multiply them to find the GCF.	$\text{GCF}=2^1\times 3^0\times a^0=2\times 1\times 1=2$
Step 3: Write each term as the product of the GCF and another monomial.	$4a=2(2a)$ $6=2(3)$
Step 4: Use the distributive property to rewrite the binomial as a **product**.	$4a+6=2(2a)+2(3)=2(2a+3)$

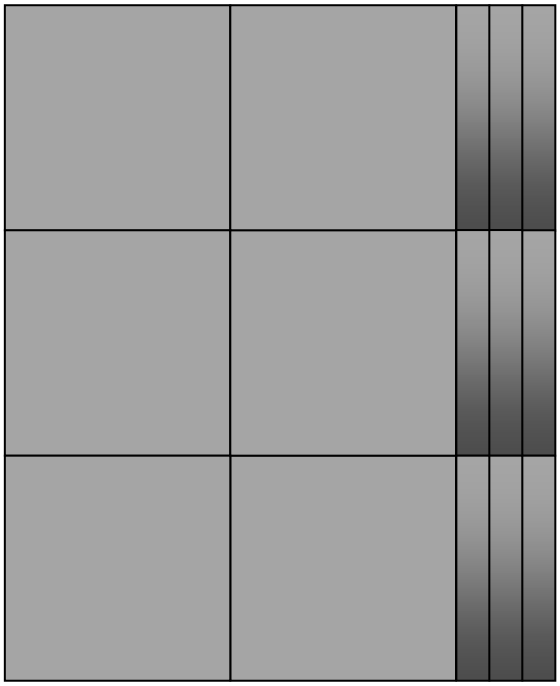
E.g.: Factor $6x^2+9x$ (See http://nlvm.usu.edu/en/nav/frames_asid_189_g_4_t_2.html?open=activities&from=category_g_4_t_2.html

Method 1: Algebra Tiles

Recall that we can represent $9x$ using nine rectangular rods/strips, each of which has a length of x and a width of 1. We can represent $6x^2$ using six large square tiles, each with side length x (see below).



To factor $6x^2 + 9x$ using algebra tiles, we must arrange these tiles to form a rectangle with area $6x^2 + 9x$ (see below).



We find the length and width of the rectangle to get the factors of $6x^2 + 9x$. The width of the rectangle is $3x$ and the length of the rectangle is $2x+3$. We use that fact that the area of a rectangle is the product of the width and the length to factor $6x^2 + 9x$.

$$6x^2 + 9x = (3x)(2x+3)$$

Method 2: Using the GCF and the Distributive Property

Step 1: Write the prime factorization for each term.	$6x^2 = 2^1 \times 3^1 \times x^2 = 2^1 \times 3^1 \times x^2$ $9x = 3^2 \times x^1 = 2^0 \times 3^2 \times x^1$
Step 2: Take the **lowest** power for each factor and multiply them to find the GCF.	$\text{GCF} = 2^0 \times 3^1 \times x^1 = 1 \times 3 \times x = 3x$
Step 3: Write each term as the product of the GCF and another monomial.	$6x^2 = 3x(2x)$ $9x = 3x(3)$
Step 4: Use the distributive property to rewrite the binomial as a **product**.	$6x^2 + 9x = 3x(2x) + 3x(3) = (3x)(2x+3)$

E.g.: Tara came to school early for her first class and decided that she would get some of her homework done. One of her homework questions asked her to factor $8x^2 - 32x$ completely. Her workings are given below.

$$8x^2 - 32x = 4x(2x - 8)$$

Did Tara factor $8x^2 - 32x$ completely? If not, explain why she did not factor completely and write the complete factorization.

Tara did not factor completely. The GCF of $8x^2$ and $-32x$ is $8x$ and not $4x$. The complete factorization is $8x^2 - 32x = 8x(x-4)$

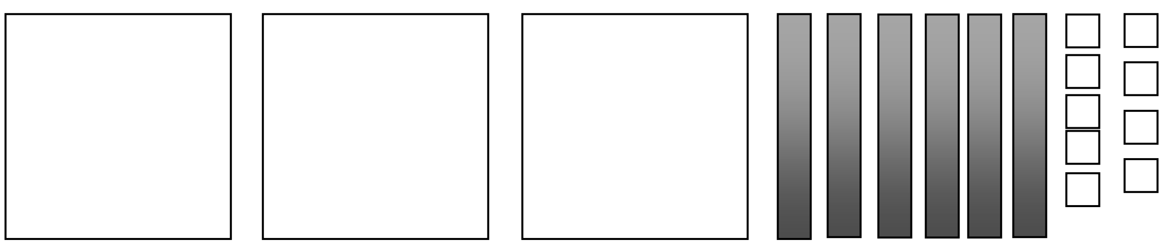
Do #'s 4 a, b, 5, 7 a, c, e, 8, 11, 17 a, 18 a, pp. 155-156 text in your homework booklet.

Factoring a Trinomial

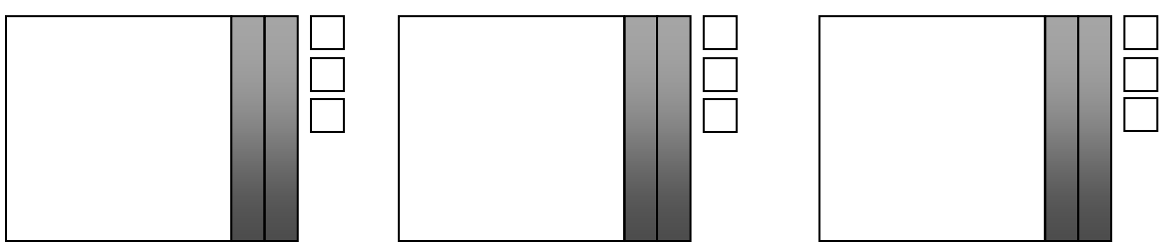
E.g.: Factor $-3z^2 + 6z - 9$

Method 1: Algebra Tiles

Recall that we can represent $-3z^2$ using three large white square tiles, each with side length *z*. We can represent $6z$ using six rectangular rods/strips, each of which has a length of *z* and a width of 1. We can represent -9 using nine square white unit tiles, each with side length 1 (see below).



To factor $-3z^2 + 6z - 9$ using algebra tiles, we must arrange these tiles to form equal groups (see below).



We formed 3 equal groups and each group contains $-z^2 + 2z - 3$. So,

$$-3z^2 + 6z - 9 = 3(-z^2 + 2z - 3)$$

Method 2: Using the GCF and the Distributive Property

Step 1: Write the prime factorization for each term.	$-3z^2 = -1\times 3^1 \times z^2 = -1\times 3^1 \times z^2 \times 2^0$ $6z = 2^1 \times 3^1 \times z^1 = 2^1 \times 3^1 \times z^1 \times -1^0$ $-9 = -1\times 3^2 = -1\times 3^2 \times 2^0 \times z^0$
Step 2: Take the **lowest** power for each factor and multiply them to find the GCF.	$\text{GCF} = -1^0 \times 2^0 \times 3^1 \times z^0 = 1\times 1\times 3\times 1 = 3$
Step 3: Write each term as the product of the GCF and another monomial.	$-3z^2 = 3\left(-z^2\right)$ $6z = 3\left(2z\right)$ $-9 = 3\left(-3\right)$
Step 4: Use the distributive property to rewrite the binomial as a **product**.	$-3z^2 + 6z - 9 = 3\left(-z^2\right) + 3\left(2z\right) + 3\left(-3\right) = 3\left(-z^2 + 2z - 3\right)$

Do #'s 4 c, 9 a, b, c, 10 a, d, e, 12, 14 a, c, pp. 155-156 text in your homework booklet.

Factoring Polynomials in More than One Variable

E.g.: Factor $-20c^4d - 30c^3d^2 - 25cd$

Step 1: Write the prime factorization for each term.	$-20c^4d = -1\times 2^2 \times 5^1 \times c^4 \times d^1 = -1\times 2^2 \times 5^1 \times c^4 \times d^1 \times 3^0$ $-30c^3d^2 = -1\times 2^1 \times 3^1 \times 5^1 \times c^3 \times d^2 = -1\times 2^1 \times 3^1 \times 5^1 \times c^3 \times d^2$ $-25cd = -1\times 5^2 \times c^1 \times d^1 = -1\times 2^0 \times 3^0 \times 5^2 \times c^1 \times d^1$
Step 2: Take the **lowest** power for each factor and multiply them to find the GCF.	$\text{GCF} = -1\times 2^0 \times 3^0 \times 5^1 \times c^1 \times d^1 = -1\times 1\times 1\times 5\times c\times d = -5cd$
Step 3: Write each term as the product of the GCF and another monomial.	$-20c^4d = -5cd\left(4c^3\right)$ $-30c^3d^2 = -5cd\left(6c^2d\right)$ $-25cd = -5cd\left(5\right)$
Step 4: Use the distributive property to rewrite the binomial as a **product**.	$-20c^4d - 30c^3d^2 - 25cd = -5cd\left(4c^3 + 6c^2d + 5\right)$

E.g.: Factor $51m^2n+39mn^2-72mn$

Step 1: Write the prime factorization for each term.	$51m^2n=3\times17\times m^2\times n=3\times17\times m^2\times n\times2^0\times13^0$ $39mn^2=3\times13\times m\times n^2=3\times13\times m\times n^2\times2^0\times17^0$ $72mn=2^3\times3^2\times m\times n=2^3\times3^2\times m\times n\times13^0\times17^0$
Step2: Take the **lowest** power for each factor and multiply them to find the GCF.	$\text{GCF}=2^0\times3\times13^0\times17^0\times m\times n=3mn$
Step 3: Write each term as the product of the GCF and another monomial.	$51m^2n=3mn(17m)$ $39mn^2=3mn(13n)$ $72mn=3mn(24)$
Step 4: Use the distributive property to rewrite the binomial as a **product**.	$51m^2n+39mn^2-72mn=3mn(17m+13n-24)$

Do #'s 16 a, c, d, e, f, p. 156 text in your homework booklet.

Section 3.4 Modeling Trinomials as Binomial Products (part class)

Read *Lesson Focus* p. 157

Outcomes:

1. Model a trinomial using algebra tiles and, **if possible**, rearrange the tiles to form a rectangle. pp. 157-158
2. Define and give an example of a **perfect square trinomial**. See notes

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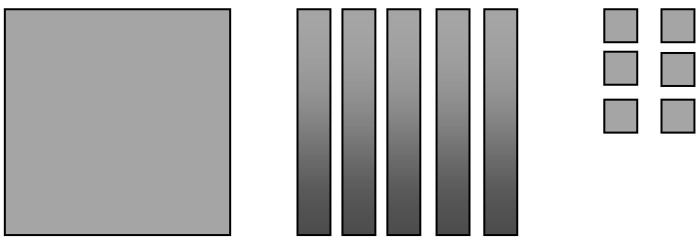
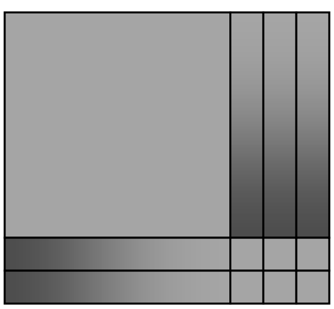
E.g.: Model z^2+5z+6 using algebra tiles. If possible, rearrange the algebra tiles to form a rectangle.

We can represent z^2 using a large square tile, each with side length *z*. We can represent $5z$ using five rectangular rods/strips, each of which has a length of *z* and a width of 1. We can represent 6 using six square unit tiles, each with side length 1 (see below).



To factor z^2+5z+6 using algebra tiles, we must arrange these tiles to form a rectangle with area z^2+5z+6 (see below).

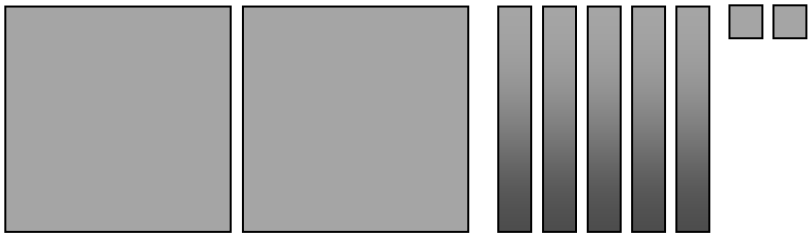


The width of this rectangle is $z+2$ and the length of the rectangle is $z+3$. Therefore we can factor z^2+5z+6 and write

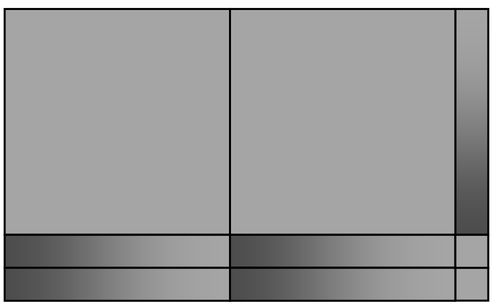
$$z^2+5z+6=(z+2)(z+3)$$

E.g.: Model $2r^2 + 5r + 2$ using algebra tiles. If possible, rearrange the algebra tiles to form a rectangle.

We can represent $2r^2$ using two large square tile, each with side length *r*. We can represent $5r$ using five rectangular rods/strips, each of which has a length of *r* and a width of 1. We can represent 2 using two square unit tiles, each with side length 1 (see below).



To factor $2r^2 + 5r + 2$ using algebra tiles, we must arrange these tiles to form a rectangle with area $2r^2 + 5r + 2$ (see below).

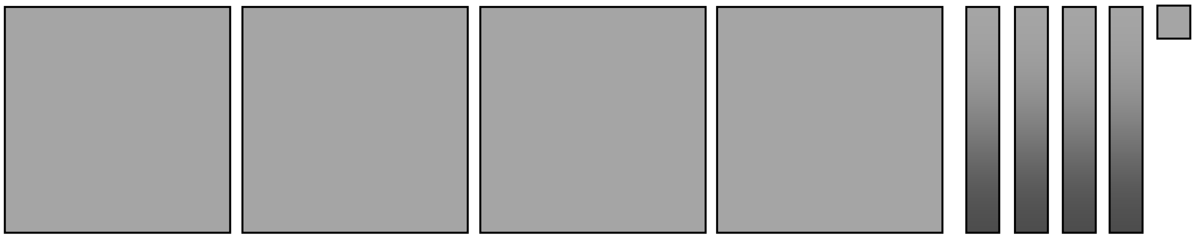


The width of this rectangle is $r + 2$ and the length of the rectangle is $2r + 1$. Therefore we can factor $2r^2 + 5r + 2$ and write

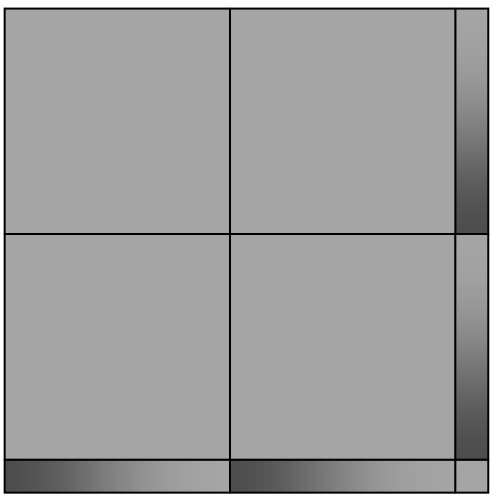
$$2r^2 + 5r + 2 = (2r + 1)(r + 2)$$

E.g.: Model $4n^2+4n+1$ using algebra tiles. If possible, rearrange the algebra tiles to form a rectangle.

We can represent $4n^2$ using four large square tile, each with side length *n*. We can represent $4n$ using four rectangular rods/strips, each of which has a length of *n* and a width of 1. We can represent 1 using one square unit tile, each with side length 1 (see below).



To factor $4n^2+4n+1$ using algebra tiles, we must arrange these tiles to form a rectangle with area $4n^2+4n+1$ (see below).



Note that our rectangle is actually a square. The width of this square is $2n+1$ and the length of the square is $2n+1$. Therefore we can factor $4n^2+4n+1$ and write

$$4n^2+4n+1=(2n+1)(2n+1)=(2n+1)^2$$

Def$^{\underline{n}}$: Any trinomial that can be written in the form $(ax+b)^2$ is called a **perfect square trinomial**. For a perfect square trinomial, the algebra tiles can be arranged into a square

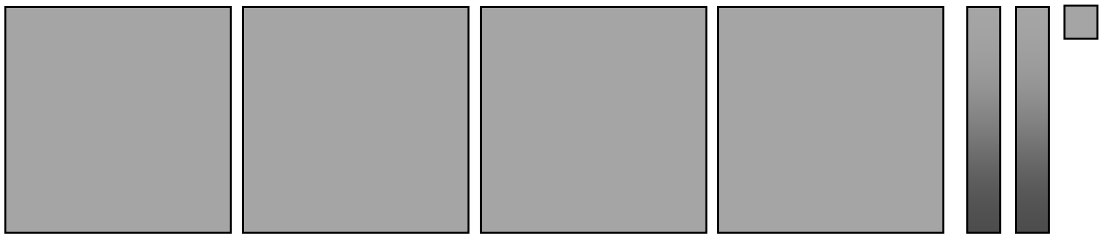
E.g.: $4n^2+4n+1$ is a perfect square trinomial because $4n^2+4n+1=(2n+1)^2$. Each side of the square is $2n+1$.

E.g.: $9r^2+12r+4$ is a perfect square trinomial because $9r^2+12r+4=(3r+2)^2$. Each side of the square is $3r+2$.

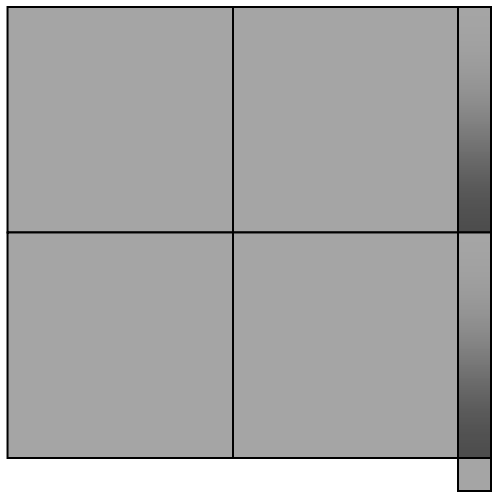
E.g.: x^2+2x+1 is a perfect square trinomial because $x^2+2x+1=(x+1)^2$. Each side of the square is $x+1$.

E.g.: Model $4n^2+2n+1$ using algebra tiles. If possible, rearrange the algebra tiles to form a rectangle.

We can represent $4n^2$ using four large square tile, each with side length n. We can represent $2n$ using two rectangular rods/strips, each of which has a length of n and a width of 1. We can represent 1 using one square unit tile, each with side length 1 (see below).



To factor $4n^2+2n+1$ using algebra tiles, we must arrange these tiles to form a rectangle with area $4n^2+2n+1$ (see below).



However, **it is not possible to arrange the algebra tiles to form a rectangle**. This tells us that $4n^2+2n+1$ **CANNOT BE FACTORED.**

Do #'s 1 a, b, c, 2 a, b, c, p. 158 text in your homework booklet.

Section 3.6 Polynomials of the Form ax^2+bx+c (4 classes)

Read *Lesson Focus* p. 168

Outcomes:

1. Model the multiplication of two binomials. pp. 169-171
2. Relate the multiplication of two binomials to an area model. pp. 169-171
3. Model the factoring of a trinomial in the form ax^2+bx+c. pp. 172-176
4. Relate the factorization of a trinomial to an area model. pp. 172-176
5. Identify and explain errors in the factorization of a polynomial. p. 178

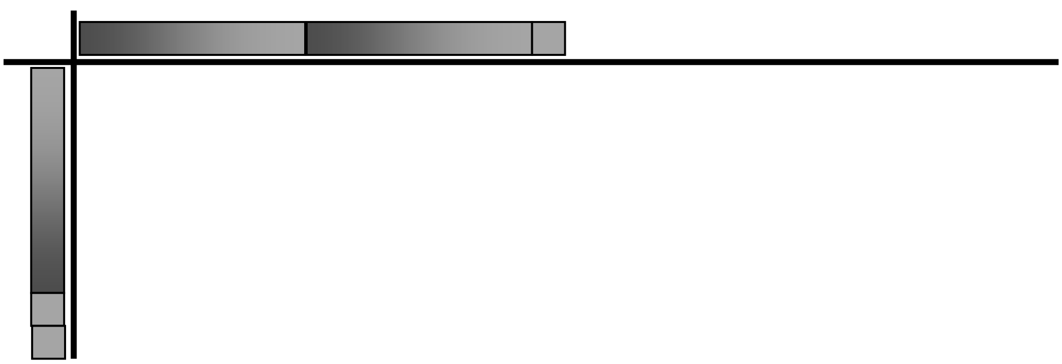
Multiplying (Expanding) Two Binomials with Positive Terms (2 classes)

E.g.: Expand $(2x+1)(x+2)$

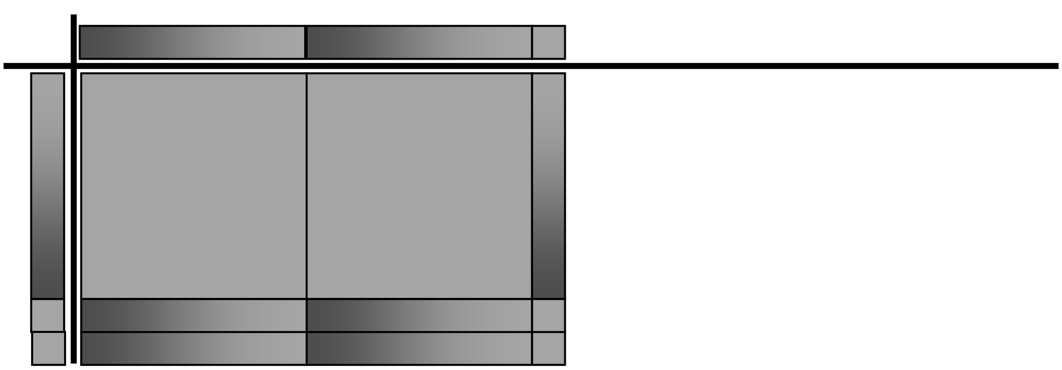
Recall that $(2x+1)(x+2)$ represents a rectangle with length $2x+1$ and width $x+2$. When we expand $(2x+1)(x+2)$, we are finding the area of this rectangle.

Method 1: Using algebra tiles (See http://nlvm.usu.edu/en/nav/frames_asid_189_g_4_t_2.html)

Step 1: Model $2x+1$ and $x+2$ using algebra tiles.



Step 2: Model a rectangle with length $2x+1$ and width $x+2$.



Step 3: Complete the expansion equation.

Since the rectangle has area $2x^2+5x+2$ we know that $(2x+1)(x+2)=2x^2+5x+2$

Do #'s 5, 7 c, d, (7a is done in the notes), p. 177 text in your homework booklet.

Method 2: Box Method (Area Model)

E.g.: Expand $(2x+1)(x+2)$

Step 1: Draw a rectangle broken into 4 smaller rectangles and use the terms of the binomials to represent the dimensions of the smaller rectangle.

	$2x$	1
$1x$		
2		

Step 2: Find the area of each smaller rectangle.

	$2x$	1
$1x$	$2x^2$	x
2	$4x$	2

Step 3: Add the area of each smaller rectangle to find the area of the large rectangle. Combine any like terms.

$2x^2+4x+x+2=2x^2+5x+2$

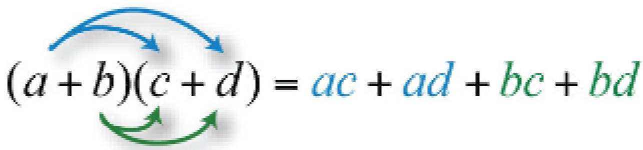
Step 4: Complete the expansion equation.

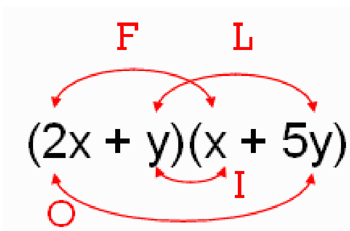
$(2x+1)(x+2)=2x^2+5x+2$

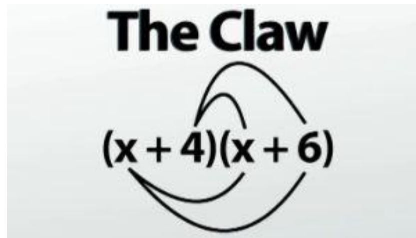
Do #'s 6 a, c, e, (Use Box/Area Method, not algebra tiles), 9 a, 10 a, 8 a, p. 177 text in your homework booklet.

Method 3: The Distributive Property (FOIL/ “THE CLAW”)

This method makes use of the fact that:







E.g.: Expand $(2x+1)(x+2)$

Step 1: FOIL

First (F): $(2x)(1x)=2x^2$

Outside (O): $(2x)(2)=4x$

Inside (I): $(1)(x)=1x=x$

Last (L): $(1)(2)=2$

Step 2: Add and combine like terms.

$2x^2+4x+1x+2=2x^2+5x+2$

Step 3: Complete the expansion equation.

$(2x+1)(x+2)=2x^2+5x+2$

Do #’s 6 b, d, f, (Use FOIL/Claw Method, not algebra tiles), p. 177 text in your homework booklet.

Multiplying (Expanding) Two Binomials with Negative Terms

If a binomial has negative terms, it can be difficult to model the product of the binomials. Therefore, if either binomial has negative terms, we will use the Box/Area method or the FOIL/Claw method.
E.g.: Simplify $(6t-9)(7-5t)$

Box Method (Area Model)

Step 1: Draw a rectangle broken into 4 smaller rectangles and use the terms of the binomials to represent the dimensions of the smaller rectangle.

	$6t$	-9
7		
$-5t$		

Step 2: Find the area of each smaller rectangle.

	$6t$	-9
7	$42t$	-63
$-5t$	$-30t^2$	$45t$

Step 3: Add the area of each smaller rectangle to find the area of the large rectangle. Combine any like terms.

$-30t^2+42t+45t-63=-30t^2+87t-63$

Step 4: Complete the expansion equation.

$(6t-9)(7-5t)=-30t^2+87t-63$

Do #'s 9 b, d, f, (Use Box/Area Method), 10 b, d, f, (Use Box/Area Method), 8 b, p. 177 text in your homework booklet.

E.g.: Simplify $(6t-9)(7-5t)$

FOIL/Claw Method

Step 1: FOIL

First (F): $(6t)(7)=42t$

Outside (O): $(6t)(-5t)=-30t^2$

Inside (I): $(-9)(7)=-63$

Last (L): $(-9)(-5t)=45t$

Step 2: Add and combine like terms.

$-30t^2+42t+45t-63=-30t^2+87t-63$

Step 3: Complete the expansion equation.

$(6t-9)(7-5t)=-30t^2+87t-63$

Do #'s 9 c, e, (Use FOIL/Claw Method), 10 c, e, (Use FOIL/Claw Method), 8 c, p. 177 text in your homework booklet.

Working Backwards: Factoring a Trinomial in the form ax^2+bx+c (2 classes)

Now let's work backwards. Instead of expanding/simplify the binomials to find trinomial, let's start with the trinomial and find the binomials that were multiplied together to get the trinomial. IN ORDER TO FACTOR EFFICIENTLY, YOU MUST KNOW YOUR MULTIPLICATION AND ADDITION FACTS.

We will begin by **expanding** $(2t+3)(3t+1)$ and looking for any relationships between the coefficients of the terms.

Line 1: $(2t+3)(3t+1)$

Line 2: $=6t^2+2t+9t+3$

Line 3: $=6t^2+11t+3$

Q: How are the 2 and the 9 in Line 2 related to the 11 in Line 3?

A: Do you see that $2+9=11$?

Q: How are the 2 and the 9 in Line 2 related to the 6 and the 3 in Line 3?

A: Do you see that $2 \times 9 = 6 \times 3$?

***** Now suppose we are working backwards and factoring $6t^2 + 11t + 3$. Do you see that the first thing we would have to do is find two numbers with a sum of 11 and a product of $6 \times 3 = 18$? ***** We would then rewrite (decompose) the middle term using these two numbers and complete the factoring. Factoring in this way is called **factoring by decomposition**.

E.g.: Factor $6t^2 + 11t + 3$ completely.

Step 1: Find two numbers with a sum of 11 and a product of $6 \times 3 = 18$.	The numbers are 2 and 9.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$6t^2 + 2t + 9t + 3$
Step 3: Group the first two terms and the last two terms using parentheses.	$\left(6t^2 + 2t\right)\left(+9t + 3\right)$
Step 4: remove a common factor $\left[2t\right]$ from the first group and a sign and a common factor $\left[+3\right]$ from the second group. Note that the binomials inside both parentheses must be the same $\left(3t + 1\right)$.	$2t\left(3t + 1\right) + 3\left(3t + 1\right)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$\left(3t + 1\right)\left(2t + 3\right)$
Step 6: Verify your factoring by expanding	$\left(3t + 1\right)\left(2t + 3\right) = 6t^2 + 9t + 2t + 3 = 6t^2 + 11t + 3$

E.g.: Factor $4g^2 + 11g + 6$ completely.

Step 1: Find two numbers with a sum of 11 and a product of $4 \times 6 = 24$.	The numbers are 8 and 3.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$4g^2 + 8g + 3g + 6$
Step 3: Group the first two terms and the last two terms using parentheses.	$\left(4g^2 + 8g\right)\left(+3g + 6\right)$
Step 4: remove a common factor $\left[4g\right]$ from the first group and a sign and a common factor $\left[+3\right]$ from the second group. Note that the binomials inside both parentheses must be the same $\left(g + 2\right)$.	$4g\left(g + 2\right) + 3\left(g + 2\right)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$\left(g + 2\right)\left(4g + 3\right)$
Step 6: Verify your factoring by expanding	$\left(g + 2\right)\left(4g + 3\right) = 4g^2 + 3g + 8g + 6 = 4g^2 + 11g + 6$

E.g.: Factor $6m^2 - 7m - 10$ completely.

Step 1: Find two numbers with a sum of -7 and a product of $6 \times -10 = -60$.	The numbers are -12 and 5.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$6m^2 - 12m + 5m - 10$
Step 3: Group the first two terms and the last two terms using parentheses.	$\left(6m^2 - 12m\right)\left(+5m - 10\right)$
Step 4: remove a common factor $\left[6m\right]$ from the first group and a sign and a common factor $\left[+5\right]$ from the second group. Note that the binomials inside both parentheses must be the same $\left(m-2\right)$.	$6m\left(m-2\right)+5\left(m-2\right)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$\left(m-2\right)\left(6m+5\right)$
Step 6: Verify your factoring by expanding	$\left(m-2\right)\left(6m+5\right) = 6m^2 + 5m - 12m - 10 = 6m^2 - 7m - 10$

You may have wondered how you might come up with the numbers -12 and 5 in the last example. If you cannot determine the numbers by inspection, use a table and list the factors of -60 until you find two factors that add up to -7.

Factors of -60	Sum of Factors
1 and -60	-59
2 and -30	-28
3 and -20	-17
4 and -15	-11
5 and -12	-7

E.g.: Find the value(s) that can replace $\square$ so that $12t^2 + \square t + 10$ is factorable.

The sum of the factors of $12 \times 10 = 120$ will make $12t^2 + \square t + 10$ factorable.

Factors of 120	Sum of Factors
1 and 120	121
2 and 60	62
3 and 40	43
4 and 30	34
5 and 24	29
6 and 20	26
8 and 15	23
10 and 12	22
-1 and -120	-121
-2 and -60	-62
-3 and -40	-43
-4 and -30	-34
-5 and -24	-29
-6 and -20	-26
-8 and -15	-23
-10 and -12	-22

So if $\square$ is replaced by $\pm 22, \pm 23, \pm 26, \pm 29, \pm 34, \pm 43, \pm 62, \pm 121$, $12t^2 + \square t + 10$ is factorable.

Do # 20 a, b, c, e, p. 178 text in your homework booklet.

E.g.: Factor $6k^2 - 11k - 35$ completely.

Step 1: Find two numbers with a sum of -11 and a product of $6 \times -35 = -210$.	<table><tr><th>Factors of -210</th><th>Sum of Factors</th></tr><tr><td>1 and -210</td><td>-209</td></tr><tr><td>2 and -105</td><td>-103</td></tr><tr><td>3 and -70</td><td>-67</td></tr><tr><td>5 and -42</td><td>-37</td></tr><tr><td>6 and -35</td><td>-29</td></tr><tr><td>7 and -30</td><td>-23</td></tr><tr><td>10 and -21</td><td>-11</td></tr></table> The numbers are 10 and -21.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$6k^2 + 10k - 21k - 35$
Step 3: Group the first two terms and the last two terms using parentheses.	$\left(6k^2 + 10k\right)\left(-21k - 35\right)$
Step 4: remove a common factor $\left[2k\right]$ from the first group and a sign and a common factor $\left[-7\right]$ from the second group. Note that the binomials inside both parentheses must be the same $\left(3k+5\right)$.	$2k\left(3k+5\right) - 7\left(3k+5\right)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$\left(3k+5\right)\left(2k-7\right)$
Step 6: Verify your factoring by expanding	$\left(3k+5\right)\left(2k-7\right) = 6k^2 - 21k + 10k - 35 = 6k^2 - 11k - 35$

Do #'s 14, 15 b, d, f, h, 16, 17, 19 b, d, f, h, j, p. 178 text in your homework booklet.

Combining Methods of Factoring: GCF and Decomposition

You will also be asked to factor trinomials in which the terms have a common factor and the trinomial that remains after the common factor is removed can also be expressed as the product of two binomials.

****** Whenever you factor, if possible, remove a common factor first. ******

E.g.: Factor $24h^2 - 20h - 24$ completely.

Step 1: Determine the GCF of the terms and rewrite the trinomial as a product of the GCF and another trinomial.	GCF = 4 $24h^2 - 20h - 24 = 4\left(6h^2 - 5h - 6\right)$
Step 2: Find two numbers with a sum of -5 and a product of $6 \times -6 = -36$.	<table><tr><th>Factors of -36</th><th>Sum of Factors</th></tr><tr><td>1 and -36</td><td>-35</td></tr><tr><td>2 and -18</td><td>-16</td></tr><tr><td>3 and -12</td><td>-9</td></tr><tr><td>4 and -9</td><td>-5</td></tr></table>The numbers are 4 and -9.
Step 3: Rewrite the middle term of the trinomial using the two numbers found in Step 2.	$4\left(6h^2 - 5h - 6\right) = 4\left(6h^2 + 4h - 9h - 6\right)$
Step 4: Group the first two terms and the last two terms using parentheses.	$4\left[\left(6h^2 + 4h\right)\left(-9h - 6\right)\right]$
Step 5: remove a common factor $[2h]$ from the first group and a sign and a common factor $[-3]$ from the second group. Note that the binomials inside both parentheses must be the same $(3h+2)$.	$4\left[2h\left(3h+2\right) - 3\left(3h+2\right)\right]$
Step 6: The common binomial from each group becomes one factor and the remaining terms form another factor.	$4\left[\left(3h+2\right)\left(2h-3\right)\right]$
Step 7: Verify your factoring by expanding	$4\left[\left(3h+2\right)\left(2h-3\right)\right] = 4\left[6h^2 - 9h + 4h - 6\right]$ $= 4\left(6h^2 - 5h - 6\right) = 24h^2 - 20h - 24$

E.g.: Factor $6x^2 - 21x + 9$ completely.

Step 1: Determine the GCF of the terms and rewrite the trinomial as a product of the GCF and another trinomial.	GCF = 3 $6x^2 - 21x + 9 = 3\left(2x^2 - 7x + 3\right)$
Step 2: Find two numbers with a sum of -7 and a product of $2 \times 3 = 6$.	The numbers are -1 and -6.
Step 3: Rewrite the middle term of the trinomial using the two numbers found in Step 2.	$3\left(2x^2 - 7x + 3\right) = 3\left(2x^2 - 1x - 6x + 3\right)$
Step 4: Group the first two terms and the last two terms using parentheses.	$3\left[\left(2x^2 - 1x\right)\left(-6x + 3\right)\right]$

Step 5: remove a common factor $[x]$ from the first group and a sign and a common factor $[-3]$ from the second group. Note that the binomials inside both parentheses must be the same $(2x-1)$.	$3\left[x(2x-1)-3(2x-1)\right]$
Step 6: The common binomial from each group becomes one factor and the remaining terms form another factor.	$3\left[(2x-1)(x-3)\right]$
Step 7: Verify your factoring by expanding	$3\left[(2x-1)(x-3)\right]=3\left(2x^2-6x-1x+3\right)$ $=3\left(2x^2-7x+3\right)=6x^2-21x+9$

Do #'s 18 a, c, e, p. 178 text in your homework booklet.

Section 3.5 Polynomials of the Form $x^2 + bx + c$ (2 classes)

Read *Lesson Focus* p. 159

Outcomes:

1. Model the multiplication of two binomials. pp. 169-171
2. Relate the multiplication of two binomials to an area model. pp. 169-171
3. Model the factoring of a trinomial in the form $x^2 + bx + c$. pp. 172-176
4. Relate the factorization of a trinomial to an area model. pp. 172-176
5. Identify and explain errors in the factorization of a polynomial. p. 178

Multiplying Two Binomials (1 class)

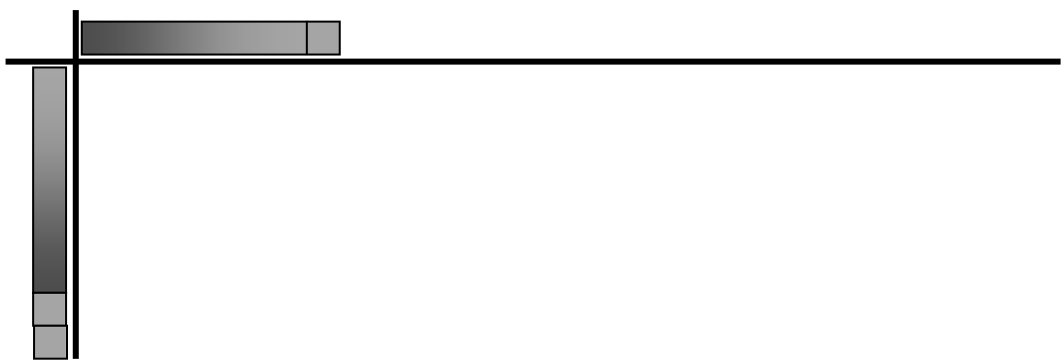
Just as in section 3.6, we can multiply binomials using algebra tiles, the Box/Area method, and the FOIL/Claw method.

E.g.: Expand $(x+1)(x+2)$ (See http://nlvm.usu.edu/en/nav/frames_asid_189_g_4_t_2.html)

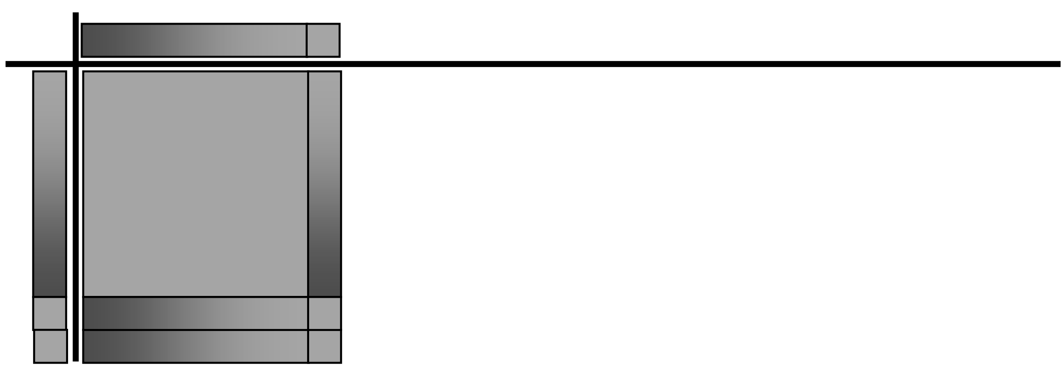
Recall that $(x+1)(x+2)$ represents a rectangle with length $x+1$ and width $x+2$. When we expand $(x+1)(x+2)$, we are finding the area of this rectangle.

Method 1: Using algebra tiles (Positive terms only)

Step 1: Model $x+1$ and $x+2$ using algebra tiles.



Step 2: Model a rectangle with length $x+1$ and width $x+2$.



Step 3: Complete the expansion equation.

Since the rectangle has area x^2+3x+2 we can write the multiplication sentence
$(x+1)(x+2)=x^2+3x+2$

Do #'s 4, 6 a, c, d, p. 166 text in your homework booklet.

Method 2: Box Method (Area Model)

E.g.: Expand $(x+1)(x+2)$

Step 1: Draw a rectangle broken into 4 smaller rectangles and use the terms of the binomials to represent the dimensions of the smaller rectangle.

	$1x$	1
$1x$		
2		

Step 2: Find the area of each smaller rectangle.

	$1x$	1
$1x$	$1x^2$	$1x$
2	$2x$	2

Step 3: Add the area of each smaller rectangle to find the area of the large rectangle. Combine any like terms.

$1x^2+2x+1x+2=x^2+3x+2$

Step 4: Complete the expansion equation.

$(x+1)(x+2)=x^2+3x+2$

Do #'s 9 a, c, 12 a, c, e, g, pp. 166-167 text in your homework booklet.

Method 3: The Distributive Property (FOIL/ "THE CLAW")

E.g.: Expand $(x+1)(x+2)$

Step 1: FOIL

First (F): $(1x)(1x)=1x^2$

Outside (O): $(1x)(2)=2x$

Inside (I): $(1)(x)=1x=x$

Last (L): $(1)(2)=2$

Step 2: Add and combine like terms.

$1x^2+2x+1x+2=x^2+3x+2$

Step 3: Complete the expansion equation.

$(x+1)(x+2)=x^2+3x+2$

Do #'s 12 b, d, f, h, (Use FOIL/Claw Method, not a rectangle), 10, pp. 166-167 text in your homework booklet.

Working Backwards: Factoring a Trinomial in the form x^2+bx+c (1 class)

Now let's work backwards. Instead of expanding/simplify the binomials to find trinomial, let's start with the trinomial and find the binomials that were multiplied together to get the trinomial. IN ORDER TO FACTOR EFFICIENTLY, YOU MUST KNOW YOUR MULTIPLICATION AND ADDITION FACTS.

E.g.: Factor $t^2+13t+12=1t^2+13t+12$ completely.

Step 1: Find two numbers with a sum of 13 and a product of $1\times12=12$.	The numbers are 1 and 12.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$1t^2+1t+12t+12$
Step 3: Group the first two terms and the last two terms using parentheses.	$(1t^2+1t)(+12t+12)$
Step 4: remove a common factor $[1t]$ from the first group and a sign and a common factor $[+12]$ from the second group. Note that the binomials inside both parentheses must be the same $(t+1)$.	$t(t+1)+12(t+1)$

Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$(t+1)(t+12)$
Step 6: Verify your factoring by expanding	$(t+1)(t+12)=t^2+12t+1t+12=t^2+11t+12$

E.g.: Factor $y^2+17y+60$ completely.

Step 1: Find two numbers with a sum of 17 and a product of $1\times 60=60$.	The numbers are 5 and 12.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$y^2+5y+12y+60$
Step 3: Group the first two terms and the last two terms using parentheses.	$(y^2+5y)(+12y+60)$
Step 4: remove a common factor $[y]$ from the first group and a sign and a common factor $[+12]$ from the second group. Note that the binomials inside both parentheses must be the same $(y+5)$.	$y(y+5)+12(y+5)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$(y+5)(y+12)$
Step 6: Verify your factoring by expanding	$(y+5)(y+12)=y^2+12y+5y+60=y^2+17y+60$

Do # 11 a, c, e, g, p. 166 text in your homework booklet.

E.g.: Factor $g^2-6g-16$ completely.

Step 1: Find two numbers with a sum of -6 and a product of $1\times -16=-16$.	The numbers are -8 and 2.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$g^2+2g-8g-16$
Step 3: Group the first two terms and the last two terms using parentheses.	$(g^2+2g)(-8g-16)$
Step 4: remove a common factor $[g]$ from the first group and a sign and a common factor $[-8]$ from the second group. Note that the binomials inside both parentheses must be the same $(g+2)$.	$g(g+2)-8(g+2)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$(g+2)(g-8)$
Step 6: Verify your factoring by expanding	$(g+2)(g-8)=g^2-8g+2g-16=g^2-6g-16$

Do #'s 14 a, c, e, g, 17, p. 167 text in your homework booklet.

E.g.: Find the value(s) that can replace $\square$ so that $x^2 + \square x + 10$ is factorable.

The sum of the factors of $1 \times 10 = 10$ will make $x^2 + \square x + 10$ factorable.

Factors of 10	Sum of Factors
1 and 10	11
2 and 5	7
-1 and -10	-11
-2 and -5	-7

So if $\square$ is replaced by $\pm 7, \pm 11$, $x^2 + \square x + 10$ is factorable.

Do # 19 b, e, f, p. 167 text in your homework booklet.

E.g.: Find one value that can replace $\square$ so that $g^2 - 3g + \square$ is factorable.

The product of any two numbers with a sum of -3 will make $g^2 - 3g + \square$ factorable. Some are listed below. There are many others.

Sum of -3	Product
1 and -4	-4
2 and -5	-10
3 and -6	-18
4 and -7	-28

So if $\square$ is replaced by $-4, -10, -18, -28 \cdots$, $g^2 - 3g + \square$ is factorable.

Do # 20 b, d, e, p. 167 text in your homework booklet.

Combining Methods of Factoring: GCF and Decomposition

You will also be asked to factor trinomials in which the terms have a common factor and the trinomial that remains after the common factor is removed can also be expressed as the product of two binomials.

****** Whenever you factor, if possible, remove a common factor first. ******

E.g.: Factor $-3m^2-18m-24$ completely.

Step 1: Determine the GCF of the terms and rewrite the trinomial as a product of the GCF and another trinomial.	GCF = -3 $-3m^2-18m-24=-3\left(m^2+6m+8\right)$
Step 2: Find two numbers with a sum of 6 and a product of 8.	The numbers are 2 and 4.
Step 3: Rewrite the middle term of the trinomial using the two numbers found in Step 2.	$-3\left(m^2+6m+8\right)=-3\left(m^2+2m+4m+8\right)$
Step 4: Group the first two terms and the last two terms using parentheses.	$-3\left[\left(m^2+2m\right)\left(+4m+8\right)\right]$
Step 5: remove a common factor $[m]$ from the first group and a sign and a common factor $[+4]$ from the second group. Note that the binomials inside both parentheses must be the same $(m+2)$.	$-3\left[m(m+2)+4(m+2)\right]$
Step 6: The common binomial from each group becomes one factor and the remaining terms form another factor.	$-3\left[(m+2)(m+4)\right]$
Step 7: Verify your factoring by expanding	$-3\left[(m+2)(m+4)\right]=-3\left[m^2+4m+2m+8\right]$ $=-3\left(m^2+6m+8\right)=-3m^2-18m-24$

Do # 21 a, c, e, p. 167 text in your homework booklet.

Section 3.7 Multiplying Polynomials (2 classes)

Read *Lesson Focus* p. 182

Outcomes:

1. Multiply polynomials using the distributive property. p. 183
2. Multiply polynomials in more than one variable using the distributive property. p. 184
3. Simplify the sum and difference of polynomial products. p. 185
4. Identify and explain errors in a polynomial multiplication. p. 186

Multiplying a Binomial in One Variable by a Trinomial in One Variable

E.g.: Simplify $(3k+4)(k^2-2k-7)$.

Step 1: Use the distributive property to multiply each term in the binomial by the trinomial.	$(3k+4)(k^2-2k-7)$ $=3k(k^2-2k-7)+4(k^2-2k-7)$
Step 2: Use the distributive property again to multiply each monomial by each term of the trinomial.	$3k(k^2-2k-7)+4(k^2-2k-7)$ $=3k(k^2)+3k(-2k)+3k(-7)+4(k^2)+4(-2k)+4(-7)$
Step 3: Find each product using the laws of exponents.	$3k(k^2)+3k(-2k)+3k(-7)+4(k^2)+4(-2k)+4(-7)$ $=3k^3-6k^2-21k+4k^2-8k-28$
Step 4: Combine the like terms.	$3k^3-6k^2-21k+4k^2-8k-28$ $=3k^3-2k^2-29k-28$

E.g.: Simplify $(5x-3)(2x^2+x-4)$.

Step 1: Use the distributive property to multiply each term in the binomial by the trinomial.	$(5x-3)(2x^2+x-4)$ $=5x(2x^2+x-4)-3(2x^2+x-4)$
Step 2: Use the distributive property again to multiply each monomial by each term of the trinomial.	$5x(2x^2+x-4)-3(2x^2+x-4)$ $=5x(2x^2)+5x(x)+5x(-4)+-3(2x^2)+-3(x)+-3(-4)$
Step 3: Find each product using the laws of exponents.	$5x(2x^2)+5x(x)+5x(-4)+-3(2x^2)+-3(x)+-3(-4)$ $=10x^3+5x^2-20x-6x^2-3x+12$
Step 4: Combine the like terms.	$10x^3+5x^2-20x-6x^2-3x+12$ $=10x^3-x^2-23x+12$

Do #'s 4 a, c, 8 a, b, p. 186 text in your homework booklet.

Multiplying a Trinomial in One Variable by a Trinomial in One Variable

E.g.: Simplify $\left(-2t^2+4t-3\right)\left(5t^2-2t+1\right)$.

Step 1: Use the distributive property to multiply each term in the trinomial by the trinomial.	$\left(-2t^2+4t-3\right)\left(5t^2-2t+1\right)$ $=-2t^2\left(5t^2-2t+1\right)+4t\left(5t^2-2t+1\right)-3\left(5t^2-2t+1\right)$
Step 2: Use the distributive property again to multiply each monomial by each term of the trinomial.	$-2t^2\left(5t^2-2t+1\right)+4t\left(5t^2-2t+1\right)-3\left(5t^2-2t+1\right)$ $=-2t^2\left(5t^2\right)-2t^2\left(-2t\right)-2t^2\left(+1\right)+4t\left(5t^2\right)+4t\left(-2t\right)+4t\left(+1\right)-3\left(5t^2\right)-3\left(-2t\right)-3\left(+1\right)$
Step 3: Find each product using the laws of exponents.	$=-2t^2\left(5t^2\right)-2t^2\left(-2t\right)-2t^2\left(+1\right)+4t\left(5t^2\right)+4t\left(-2t\right)+4t\left(+1\right)-3\left(5t^2\right)-3\left(-2t\right)-3\left(+1\right)$ $=-10t^4+4t^3-2t^2+20t^3-8t^2+4t-15t^2+6t-3$
Step 4: Combine the like terms.	$-10t^4+4t^3-2t^2+20t^3-8t^2+4t-15t^2+6t-3$ $=-10t^4+24t^3-25t^2+10t-3$

Do #'s 13 a, c, 14, p. 186 text in your homework booklet.

Multiplying a Binomial in Two Variables by a Binomial in Two Variables

E.g.: Simplify $\left(2z+y\right)\left(3z+4y\right)$.

Step 1: Use the distributive property to multiply each term in the first binomial by the second binomial.	$\left(2z+y\right)\left(3z+4y\right)=2z\left(3z+4y\right)+y\left(3z+4y\right)$
Step 2: Use the distributive property again to multiply each monomial by each term of the binomial.	$2z\left(3z+4y\right)+y\left(3z+4y\right)$ $=2z\left(3z\right)+2z\left(4y\right)+y\left(3z\right)+y\left(4y\right)$
Step 3: Find each product using the laws of exponents.	$2z\left(3z\right)+2z\left(4y\right)+y\left(3z\right)+y\left(4y\right)$ $=6z^2+8yz+3yz+4y^2$
Step 4: Combine the like terms.	$6z^2+8yz+3yz+4y^2=6z^2+11yz+4y^2$

E.g.: Simplify $(5z-2y)^2$.

Step 1: Use the distributive property to multiply each term in the first binomial by the second binomial.	$(5z-2y)^2=(5z-2y)(5z-2y)=5z(5z-2y)-2y(5z-2y)$
Step 2: Use the distributive property again to multiply each monomial by each term of the binomial.	$5z(5z-2y)-2y(5z-2y)$ $=5z(5z)-5z(2y)-2y(5z)-2y(-2y)$
Step 3: Find each product using the laws of exponents.	$5z(5z)-5z(2y)-2y(5z)-2y(-2y)$ $=25z^2-10yz-10yz+4y^2$
Step 4: Combine the like terms.	$25z^2-10yz-10yz+4y^2=25z^2-20yz+4y^2$

E.g.: Simplify $(2x+3y)(2x-3y)$.

Step 1: Use the distributive property to multiply each term in the first binomial by the second binomial.	$(2x+3y)(2x-3y)=2x(2x-3y)+3y(2x-3y)$
Step 2: Use the distributive property again to multiply each monomial by each term of the binomial.	$2x(2x-3y)+3y(2x-3y)$ $=2x(2x)+2x(-3y)+3y(2x)+3y(-3y)$
Step 3: Find each product using the laws of exponents.	$2x(2x)+2x(-3y)+3y(2x)+3y(-3y)$ $=4x^2-6xy+6xy-9y^2$
Step 4: Combine the like terms.	$4x^2-6xy+6xy-9y^2=4x^2-9y^2$

Do #'s 5 a, c, e, f, 7 a(ii), (iii), (iv) , p. 186 & # 13, p. 167 text in your homework booklet.

Multiplying a Binomial in Two Variables by a Trinomial in Two or more Variables

E.g.: Simplify $(2v-5w)(3v+2w-7)$.

Step 1: Use the distributive property to multiply each term in the binomial by the trinomial.	$(2v-5w)(3v+2w-7)$ $=2v(3v+2w-7)-5w(3v+2w-7)$
Step 2: Use the distributive property again to multiply each monomial by each term of the trinomial.	$2v(3v+2w-7)-5w(3v+2w-7)$ $=2v(3v)+2v(2w)+2v(-7)-5w(3v)-5w(2w)-5w(-7)$
Step 3: Find each product using the laws of exponents.	$2v(3v)+2v(2w)+2v(-7)-5w(3v)-5w(2w)-5w(-7)$ $=6v^2+4vw-14v-15vw-10w^2+35w$
Step 4: Combine the like terms.	$6v^2+4vw-14v-15vw-10w^2+35w$ $=6v^2-11vw-14v-10w^2+35w$

E.g.: Simplify $(4x-5y)(3x-y+2z)$.

Step 1: Use the distributive property to multiply each term in the binomial by the trinomial.	$(4x-5y)(3x-y+2z)$ $=4x(3x-y+2z)-5y(3x-y+2z)$
Step 2: Use the distributive property again to multiply each monomial by each term of the trinomial.	$4x(3x-y+2z)-5y(3x-y+2z)$ $=4x(3x)+4x(-y)+4x(2z)-5y(3x)-5y(-y)-5y(2z)$
Step 3: Find each product using the laws of exponents.	$4x(3x)+4x(-y)+4x(2z)-5y(3x)-5y(-y)-5y(2z)$ $=12x^2-4xy+8xz-15xy-5y^2-10yz$
Step 4: Combine the like terms.	$12x^2-4xy+8xz-15xy-5y^2-10yz$ $=12x^2-19xy+8xz-5y^2-10yz$

Do #'s 9 b, d, 10 b, d, 11, p. 186 text in your homework booklet.

Simplifying the Sum or Difference of Polynomial Products

E.g.: Simplify $(4m+1)(3m-2)+2(2m-1)(-3m+4)$

$(4m+1)(3m-2)+2(2m-1)(-3m+4)=(12m^2-8m+3m-2)+2(-6m^2+8m+3m-4)$
$=(12m^2-5m-2)+2(-6m^2+11m-4)=12m^2-5m-2-12m^2+22m-8$
$=17m-10$

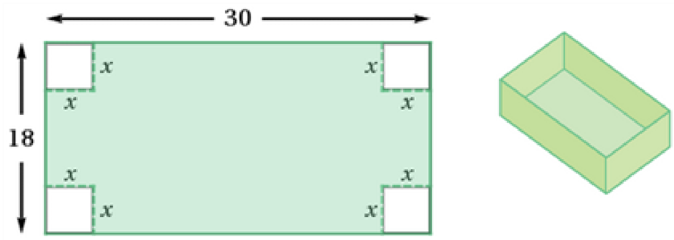
E.g.: Simplify $(3x-2)^2-(2x+6)(3x-1)$

$(3x-2)^2-(2x+6)(3x-1)=(3x-2)(3x-2)-(2x+6)(3x-1)$
$=(9x^2-6x-6x+4)-(6x^2-2x+18x-6)$
$=(9x^2-12x+4)-(6x^2+16x-6)$
$=(9x^2-12x+4)+(-6x^2-16x+6)$
$=9x^2-12x+4-6x^2-16x+6$
$=3x^2-28x+10$

Do #'s 15 a, b, d, f, p. 186 text in your homework booklet.

Problem Solving with Polynomials

E.g.: A box with no top is made from a piece of cardboard 30cm by 18 cm. Equal squares are cut from each corner and the sides are folded up.



Let x represent the side length of the square that is cut from each corner. Write a polynomial to represent each measurement below. Simplify the polynomial.

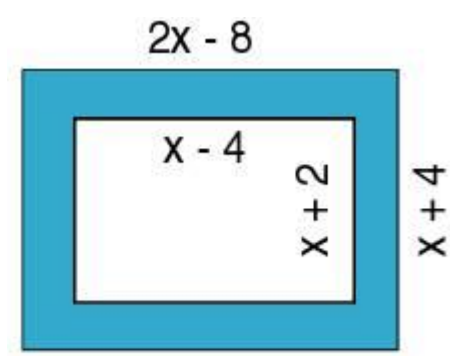
a) The length of the box.
b) The width of the box
c) The area of the base of the box.
d) The volume of the box.

a) The length of the box is $30-x-x=30-2x$
b) The width of the box is $18-x-x=18-2x$
c) The area of the base of the box is $(30-2x)(18-2x)=540-60x-36x+4x^2=540-96x+4x^2$
d) The volume of the box is $\left(540-96x+4x^2\right)x=540x-96x^2+4x^3$

Do # 16, p. 187 text in your homework booklet.

E.g.: Each shape below is a rectangle. Write a polynomial to represent the area of the shaded region. Simplify the polynomial.

The area of the shaded region can be found by subtracting the area of the smaller rectangle from the area of the larger rectangle. As a formula,



$$A_{\text{shaded}} = A_{\text{large rectangle}} - A_{\text{small rectangle}}$$

$$A_{\text{shaded}} = (2x-8)(x+4)-(x-4)(x+2)$$
$$A_{\text{shaded}} = \left(2x^2+8x-8x-32\right)-\left(x^2+2x-4x-8\right)$$
$$A_{\text{shaded}} = \left(2x^2-32\right)-\left(x^2-2x-8\right)$$
$$A_{\text{shaded}} = \left(2x^2-32\right)+\left(-x^2+2x+8\right)$$
$$A_{\text{shaded}} = 2x^2-32-x^2+2x+8$$
$$A_{\text{shaded}} = x^2+2x-24$$

Do # 17 b, p. 187 text in your homework booklet.

Section 3.8 Factoring Special Polynomials (2 classes)

Read *Lesson Focus* p. 188

Outcomes:

1. Factor perfect square trinomials. p. 190
2. Factor trinomials in two variables. p. 191
3. Factor the difference of squares. pp. 192-193

Recall from section 3.4 that a **perfect square trinomial** is any trinomial that can be written in the form $\left(\text{term 1}+\text{term 2}\right)^2$

E.g.: $4n^2+4n+1$ is a perfect square trinomial because $4n^2+4n+1=\left(2n+1\right)^2$. Each side of the square is $2n+1$.

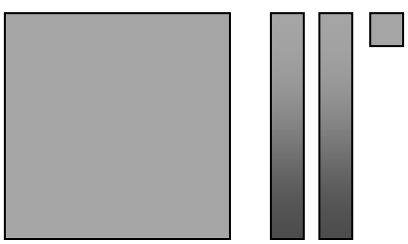
E.g.: $9r^2+12r+4$ is a perfect square trinomial because $9r^2+12r+4=\left(3r+2\right)^2$. Each side of the square is $3r+2$.

Factoring Perfect Square Trinomials

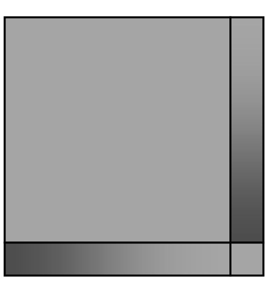
Perfect square trinomials can be factored using algebra tiles (if the terms are positive and not too large) or by decomposition.

E.g.: Factor n^2+2n+1 completely.

We can represent n^2 using one large square tile, each with side length *n*. We can represent $2n$ using two rectangular rods/strips, each of which has a length of *n* and a width of 1. We can represent 1 using one square unit tile, each with side length 1 (see below).



To factor n^2+2n+1 using algebra tiles, we must arrange these tiles to form a rectangle with area n^2+2n+1 (see below).



Note that our rectangle is actually a square. The width of this square is $n+1$ and the length of the square is $n+1$. Therefore we can factor n^2+2n+1 and write

$$n^2+2n+1=(n+1)(n+1)=(n+1)^2$$

If the coefficients of the terms are large, decomposition is a better method.

E.g.: Factor $16-56y+49y^2$ completely.

Step 1: Find two numbers with a sum of -56 and a product of $16\times 49=784$.	<table><tr><th>Factors of 784</th><th>Sum of Factors</th></tr><tr><td>-1 and -784</td><td>-785</td></tr><tr><td>-2 and -392</td><td>-394</td></tr><tr><td>-4 and -196</td><td>-200</td></tr><tr><td>-7 and -112</td><td>-119</td></tr><tr><td>-14 and -56</td><td>-70</td></tr><tr><td>-16 and -49</td><td>-65</td></tr><tr><td>-28 and -28</td><td>-56</td></tr></table> The numbers are -28 and -28.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$16-28y-28y+49y^2$
Step 3: Group the first two terms and the last two terms using parentheses.	$(16-28y)(-28y+49y^2)$
Step 4: remove a common factor $[4]$ from the first group and a sign and a common factor $[-7y]$ from the second group. Note that the binomials inside both parentheses must be the same $(4-7y)$.	$4(4-7y)-7y(4-7y)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$(4-7y)(4-7y)=(4-7y)^2$
Step 6: Verify your factoring by expanding	$(4-7y)(4-7y)=16-28y-28y+49y^2=16-56y+49y^2$

Do # 8, p. 194 text in your homework booklet.

Factoring Trinomials in Two Variables

E.g.: Factor $5c^2 - 13cd + 6d^2$ completely.

Step 1: Find two numbers with a sum of -13 and a product of $5 \times 6 = 30$.	The numbers are -10 and -3.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$5c^2 - 10cd - 3cd + 6d^2$
Step 3: Group the first two terms and the last two terms using parentheses.	$\left(5c^2 - 10cd\right)\left(-3cd + 6d^2\right)$
Step 4: remove a common factor $[5c]$ from the first group and a sign and a common factor $[-3d]$ from the second group. Note that the binomials inside both parentheses must be the same $(c - 2d)$.	$5c(c - 2d) - 3d(c - 2d)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$(c - 2d)(5c - 3d)$
Step 6: Verify your factoring by expanding	$(c - 2d)(5c - 3d) = 5c^2 - 3cd - 10cd + 6d^2 = 5c^2 - 13cd + 6d^2$

E.g.: Factor $3p^2 - 5pq - 2q^2$ completely.

Step 1: Find two numbers with a sum of -5 and a product of $3 \times -2 = -6$.	The numbers are -6 and 1.
Step 2: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$3p^2 - 6pq + 1pq - 2q^2$
Step 3: Group the first two terms and the last two terms using parentheses.	$\left(3p^2 - 6pq\right)\left(+1pq - 2q^2\right)$
Step 4: remove a common factor $[3p]$ from the first group and a sign and a common factor $[q]$ from the second group. Note that the binomials inside both parentheses must be the same $(p - 2q)$.	$3p(p - 2q) + q(p - 2q)$
Step 5: The common binomial from each group becomes one factor and the remaining terms form another factor.	$(p - 2q)(3p + q)$
Step 6: Verify your factoring by expanding	$(p - 2q)(3p + q) = 3p^2 + pq - 6pq - 2q^2 = 3p^2 - 5pq - 2q^2$

Do # 11, p.195 text in your homework booklet.

Def$^{\underline{n}}$: Any polynomial that can be written as $(\text{term } 1)^2 - (\text{term } 2)^2$ is the **difference of squares**.

E.g.: $x^2 - 36 = (x)^2 - (6)^2$; $x^2 - y^2 = (x)^2 - (y)^2$; $49 - r^2 = (7)^2 - (r)^2$; $\frac{x^2}{4} - \frac{1}{9} = \left(\frac{x}{2}\right)^2 - \left(\frac{1}{3}\right)^2$

Factoring the Difference of Squares

Decomposition can be used to factor the difference of squares.

E.g.: Factor $81m^2 - 49$ completely.

This is the difference of squares because $81m^2 - 49$ can be written as $(9m)^2 - (7)^2$.

Step 1: Rewrite the binomial as a trinomial by adding $0m$ as the middle term.	$81m^2 + 0m - 49$
Step 2: Find two numbers with a sum of 0 and a product of $81 \times -49 = -3969$.	The numbers are 63 and -63.
Step 3: Rewrite the middle term of the trinomial using the two numbers found in Step 1.	$81m^2 + 63m - 63m - 49$
Step 4: Group the first two terms and the last two terms using parentheses.	$(81m^2 + 63m)(-63m - 49)$
Step 5: Remove a common factor $[9m]$ from the first group and a sign and a common factor $[-7]$ from the second group. Note that the binomials inside both parentheses must be the same $(9m + 7)$.	$9m(9m + 7) - 7(9m + 7)$
Step 6: The common binomial from each group becomes one factor and the remaining terms form another factor.	$(9m + 7)(9m - 7)$
Step 7: Verify your factoring by expanding	$(9m + 7)(9m - 7) = 81m^2 - 63m + 63m - 49 = 81m^2 - 49$

There is a **shortcut** to factoring the difference of squares if you want to use it.

E.g.: Factor $81m^2 - 49$ completely.

Step 1: Rewrite the binomial as $(\text{term } 1)^2 - (\text{term } 2)^2$	$(9m)^2 - (7)^2$
Step 2: Write 2 sets of parentheses that indicate multiplication.	$(\qquad)(\qquad)$
Step 3: Place term 1 first in each set of parentheses.	$(9m \qquad)(9m \qquad)$
Step 4: Place term 2 last in each set of parentheses.	$(9m \quad 7)(9m \quad 7)$
Step 5: Place a $"+"$ sign between terms 1 and 2 in the first set of parentheses.	$(9m+7)(9m \quad 7)$
Step 6: Place a $"-"$ sign between terms 1 and 2 in the second set of parentheses.	$(9m+7)(9m-7)$
Step 7: Verify your factoring by expanding	$(9m+7)(9m-7) = 81m^2 - 63m + 63m - 49 = 81m^2 - 49$

E.g.: Factor $36f^2 - 81g^2$ completely.

Step 1: Rewrite the binomial as $(\text{term } 1)^2 - (\text{term } 2)^2$	$(6f)^2 - (9g)^2$
Step 2: Write 2 sets of parentheses that indicate multiplication.	$(\qquad)(\qquad)$
Step 3: Place term 1 first in each set of parentheses.	$(6f \qquad)(6f \qquad)$
Step 4: Place term 2 last in each set of parentheses.	$(6f \quad 9g)(6f \quad 9g)$
Step 5: Place a $"+"$ sign between terms 1 and 2 in the first set of parentheses.	$(6f+9g)(6f \quad 9g)$
Step 6: Place a $"-"$ sign between terms 1 and 2 in the second set of parentheses.	$(6f+9g)(6f-9g)$
Step 7: Verify your factoring by expanding	$(6f+9g)(6f-9g) = 36f^2 - 54fg + 54fg - 81g^2 = 36f^2 - 81g^2$

E.g.: Factor $25r^2 - \frac{1}{16}t^2$ completely.

Step 1: Rewrite the binomial as $(\text{term 1})^2 - (\text{term 2})^2$	$(5r)^2 - \left(\frac{1}{4}t\right)^2$
Step 2: Write 2 sets of parentheses that indicate multiplication.	$(\qquad)(\qquad)$
Step 3: Place term 1 first in each set of parentheses.	$(5r \qquad)(5r \qquad)$
Step 4: Place term 2 last in each set of parentheses.	$\left(5r \quad \frac{1}{4}t\right)\left(5r \quad \frac{1}{4}t\right)$
Step 5: Place a $"+"$ sign between terms 1 and 2 in the first set of parentheses.	$\left(5r + \frac{1}{4}t\right)\left(5r \quad \frac{1}{4}t\right)$
Step 6: Place a $"-"$ sign between terms 1 and 2 in the second set of parentheses.	$\left(5r + \frac{1}{4}t\right)\left(5r - \frac{1}{4}t\right)$
Step 7: Verify your factoring by expanding	$\left(5r + \frac{1}{4}t\right)\left(5r - \frac{1}{4}t\right) = 25r^2 - \frac{5}{4}rt + \frac{5}{4}rt - \frac{1}{16}r^2 = 25r^2 - \frac{1}{16}r^2$

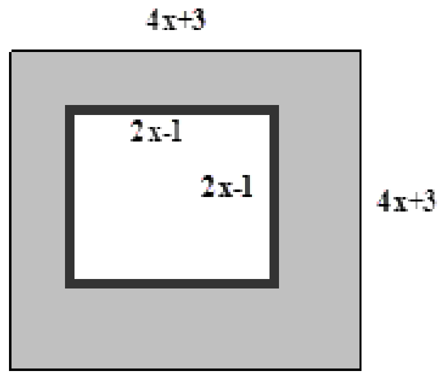
E.g.: Factor $\frac{r^2}{49} - \frac{t^2}{144}$ completely.

Step 1: Rewrite the binomial as $(\text{term 1})^2 - (\text{term 2})^2$	$\left(\frac{r}{7}\right)^2 - \left(\frac{t}{12}\right)^2$
Step 2: Write 2 sets of parentheses that indicate multiplication.	$(\qquad)(\qquad)$
Step 3: Place term 1 first in each set of parentheses.	$\left(\frac{r}{7} \qquad\right)\left(\frac{r}{7} \qquad\right)$
Step 4: Place term 2 last in each set of parentheses.	$\left(\frac{r}{7} \quad \frac{t}{12}\right)\left(\frac{r}{7} \quad \frac{t}{12}\right)$
Step 5: Place a $"+"$ sign between terms 1 and 2 in the first set of parentheses.	$\left(\frac{r}{7} + \frac{t}{12}\right)\left(\frac{r}{7} \quad \frac{t}{12}\right)$
Step 6: Place a $"-"$ sign between terms 1 and 2 in the second set of parentheses.	$\left(\frac{r}{7} + \frac{t}{12}\right)\left(\frac{r}{7} - \frac{t}{12}\right)$
Step 7: Verify your factoring by expanding	$\left(\frac{r}{7} + \frac{t}{12}\right)\left(\frac{r}{7} - \frac{t}{12}\right) = \frac{r^2}{49} - \frac{rt}{84} + \frac{rt}{84} - \frac{t^2}{144} = \frac{r^2}{49} - \frac{t^2}{144}$

Do #’s 10, b, d, f, h, 21 b, d, f, p.195 text in your homework booklet.

Problem Solving with Polynomials

E.g.: Each shape below is a square. Write a polynomial to represent the area of the shaded region. Simplify the polynomial.



The area of the shaded region can be found by subtracting the area of the smaller square from the area of the larger square. As a formula,

$$A_{\text{shaded}} = A_{\text{large square}} - A_{\text{small square}}$$

$$A_{\text{shaded}} = (4x+3)(4x+3) - (2x-1)(2x-1)$$
$$A_{\text{shaded}} = (16x^2 + 12x + 12x + 9) - (4x^2 - 2x - 2x + 1)$$
$$A_{\text{shaded}} = (16x^2 + 24x + 9) - (4x^2 - 4x + 1)$$
$$A_{\text{shaded}} = (16x^2 + 24x + 9) + (-4x^2 + 4x - 1)$$
$$A_{\text{shaded}} = 16x^2 + 24x + 9 - 4x^2 + 4x - 1$$
$$A_{\text{shaded}} = 12x^2 + 28x + 8$$

Do # 18, p.195 text in your homework booklet.
Do #'s 2 b, 3 a, 6 b, 7 b, 8, d, e, f, 9, 10, 12 e, f, 14, 15 a, c, 18 a, c, e, g, 19 a, c, e, 21, 22 b, 23 a) (i), (ii), (iii), 24 a, c, e, 25 a, c, e, 26 a, b, c, 27 a, c, 28 b, d, 29 b, 30, 32 a, c, 33 b, d, f, 34 b, d, 35, pp. 198-200 text in your homework booklet.
