Math 1201
Unit 1: Measurement

Read *Building On*, *Big Ideas* and *New Vocabulary*, p. 2 text.

Ch. 1 Notes

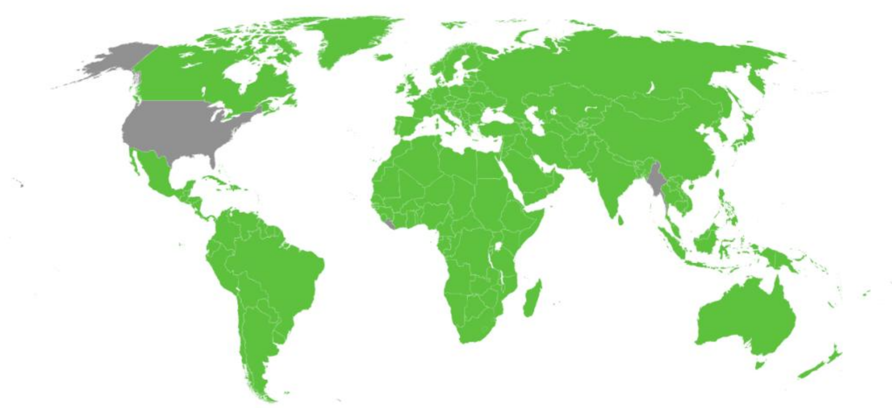
§1.1 Imperial Measures of Length (2 classes)

Read *Lesson Focus* p. 4 text.

Outcomes

1. Define the **Imperial System**. See notes
2. Define and give examples, and non-examples, of **imperial units**. pp. 4, 6, 537
3. State the imperial units for length. p. 6
4. Recognize and use the symbols for the imperial units of length. p. 6 and notes
5. Define and give an example of a **referent**. pp. 6, 539
6. Give referents for various imperial units. See notes
7. Convert from one imperial length unit to another. p. 7
8. Solve problems where conversion of imperial units is required. pp. 8-9
9. Solve problems using scale diagrams. p. 10

Much of the world (including Canada) uses Le Système International d'Unités or SI (metric) units (see the figure below – green color).



Some quantities and their SI units of measure are shown in the Table below.

Quantity	Unit of Measure
***Distance/Length	***millimeters, centimeters, meters, kilometers
Speed	meters per second, kilometers per hour
Mass	grams, kilograms
Weight	Newtons

However, 3 countries (most notably the US) use the Imperial System and imperial units of measure (see the figure on the previous page – gray color).

Def$^{\underline{n}}$: The **Imperial System** is a system of weights and measures originally developed in England.

Def$^{\underline{n}}$: **Imperial units** are the units used in the Imperial System.

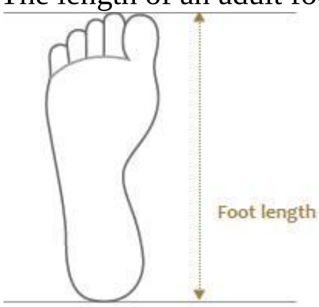
Some quantities and their imperial units of measure are shown in the Table below.

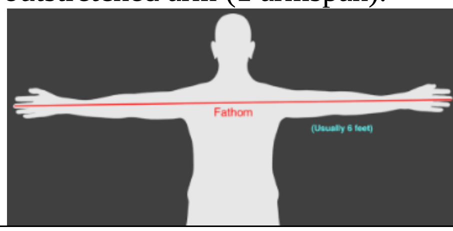
Quantity	Unit of Measure
***Distance/Length	***inch (in), foot (ft), yard (yd), mile (mi), fathom (ftm)
Speed	feet per second (fps), miles per hour (mph)
Mass	slug
Weight	ounces (oz), pounds (lbs)

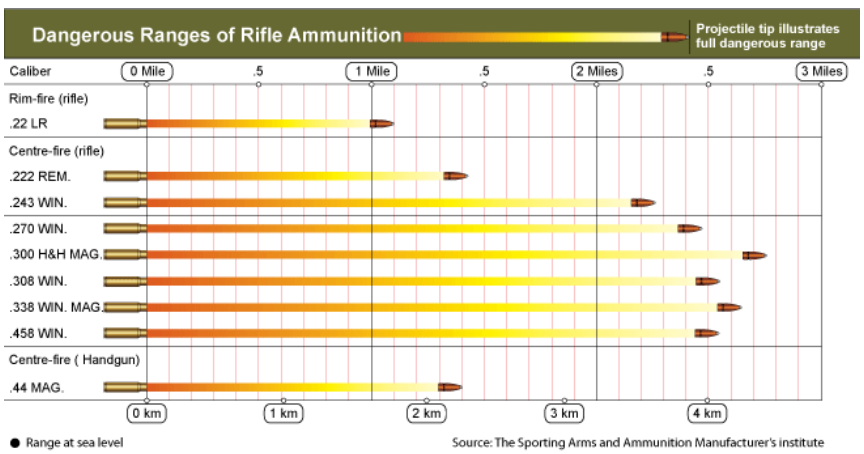
To give you some idea about the size of imperial units of distance/length, we are going to use referents.

Def$^{\underline{n}}$: A **referent** is an object that can be used to estimate a measure (a length/distance in this chapter). Referents do NOT have to be exact.

Some imperial units and possible referents are shown in the Table below.

Imperial Unit	Symbol for Unit	Referent
inch	in.	Tip of your thumb to the first knuckle (see p. 5 text or left pic below).   Thickness of a hockey puck (see right pic above).
foot	ft.	The length of an adult foot. 

yard	yd.	The distance from one footprint to another when you take a step. 
fathom	ftm.	The distance from the tip of one outstretched arm to the tip of the other outstretched arm (1 armspan). 
mile	mi.	The distance a 22 rifle bullet can travel and still be dangerous (see figure below).



Sample Exam Questions

Which referent would best represent 1 mile?

(A) The length of the school gymnasium.

(B) The height of the church in Musgravetown.

(C) The road distance between Heritage Collegiate and Pye's Service Station.

(D) The length of the Trans Canada Highway.

Juan was told to plant trees two armspans apart. Which of the following estimates is closest to "two armspans apart"?

A. 6 ft

B. 3.5 m

C. 60 cm

D. 30 in

Converting From One Imperial Unit to Another

Sometimes you have to convert from one imperial length unit to another. This can be done using the table below.

Imperial Unit & Symbol	Relationship between Units
Inch (in.)	
Foot (ft.)	1 ft = 12 in
Yard (yd.)	1 yd = 3 ft = 36 in
Fathom (ftm.)	1 ftm = 2 yd = 6 ft = 72 in
Mile (mi.)	1 mi = 880 ftm = 1760 yd = 5280 ft

Problem: Convert 4 ft to inches. (Like 7 a), p. 11)		
Solution 1	Solution 2	Solution 3
Since 1 ft = 12 in (see table above) then, 4 ft = 4(12) = 48 inches.	Let x be the number of inches in 4 ft. Since, 1 ft = 12 in (see table above) we can write the **proportion** $\frac{12\text{ in}}{1\text{ ft}} = \frac{x\text{ in}}{4\text{ ft}}$ Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get $12(4) = x(1)$ $48 = x$	Let x be the number of inches in 4 ft. 1 ft = 12 in (see table above) 4 ft = x in So, $\frac{1}{4} = \frac{12}{x}$ Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get $1(x) = 4(12)$ $x = 48$
Statement	There are 48 inches in 4 feet.	

<table>
<tr><th colspan="3">Problem: Convert 16 yd to feet. (Like 7 b), p. 11)</th></tr>
<tr><th>Solution 1</th><th>Solution 2</th><th>Solution 3</th></tr>
<tr>
<td>Since, 1 yd = 3 ft (see table p. 4) then,

16 yd = 3(16) = 48 ft.</td>
<td>Let x be the number of feet in 16 yd.

Since, 1 yd = 3 ft (see table p. 4), we can write the proportion

$\frac{3\text{ ft}}{1\text{ yd}} = \frac{x\text{ ft}}{16\text{ yd}}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$3(16) = x(1)$
$48 = x$</td>
<td>Let x be the number of feet in 16 yd.

1 yd = 3ft (see table p. 4)
16 yd = x ft

So we can write the proportion

$\frac{1}{16} = \frac{3}{x}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$1(x) = 16(3)$
$x = 48$</td>
</tr>
<tr><td>Statement</td><td colspan="2">There are 48 feet in 16 yards.</td></tr>
</table>

<table>
<tr><th colspan="3">Problem: Convert 72 in to feet. (Like 7 c), p. 11)</th></tr>
<tr><th>Solution 1</th><th>Solution 2</th><th>Solution 3</th></tr>
<tr>
<td>Since, 1 ft = 12 in (see table p. 4) then,

72 in = 72/12 = 6 ft.</td>
<td>Let x be the number of feet in 72 in.

Since, 1 ft = 12 in (see table p. 4) we can write the proportion

$\frac{1\text{ ft}}{12\text{ in}} = \frac{x\text{ ft}}{72\text{ in}}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$1(72) = x(12)$
$72 = 12x$
$\frac{72}{12} = \frac{\cancel{12}x}{\cancel{12}}$
$6 = x$</td>
<td>Let x be the number of feet in 72 in.

1 ft = 12 in (see table p. 4)
x ft = 72 in

So we can write the proportion

$\frac{1}{x} = \frac{12}{72}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$1(72) = 12(x)$
$72 = 12x$
$\frac{72}{12} = \frac{\cancel{12}x}{\cancel{12}}$
$6 = x$</td>
</tr>
<tr><td>Statement</td><td colspan="2">There are 6 feet in 72 inches.</td></tr>
</table>

Do # 7, p. 11 text in your homework booklet.

Solving Problems Involving Converting From One Imperial Unit to Another

E.g.: A carpenter would like to place trim around a rectangular window measuring 41 in. by 27 in. If the trim is 3in wide and costs $1.92/ft., what is the approximate cost of the trim for the window before taxes? After taxes?

The perimeter of the window is 41in + 27in + 41in + 27in = 136in.

The outside perimeter of the window with 3inch trim is 136 + 8(3) = 160in.

Since the cost of the trim is per foot, we will change 160in to feet. We will use the method in Solution 3 above.

12in = 1ft

160in = *x* ft

$$\frac{12}{160} = \frac{1}{x}$$
$$12x = 1(160)$$
$$12x = 160$$
$$\frac{12x}{12} = \frac{160}{12}$$
$$x = 13.\bar{3}\text{ft} \approx 14\text{ft}$$

So the cost of the trim is $14 \times \$1.92 = \$26.88$ before taxes.

The trim would cost $\$26.88 \times 1.13 = \$30.37$ after taxes.

Do #'s 10b, 12, 15 a, pp. 11-12 text in your homework booklet.

Solving Problems Involving Two Imperial Unit Convertions

E.g.: Thick round wire is wound to make coil springs for the suspension of a pickup. Each spring requires 30in of wire. How many springs can be made from 10yd of wire?

First convert 10yd to feet.

1yd = 3ft

10yd = *x* ft

$$\frac{1}{10} = \frac{3}{x}$$
$$1x = 10(3)$$
$$x = 30$$

So 10yd = 30ft.

Next convert 30ft to inches.

1ft = 12in

30ft = y in

$$\frac{1}{30}=\frac{12}{y}$$
$$1y=30(12)$$
$$y=360$$

So 30ft = 360in.

Therefore 10yd = 360in.

So $\frac{360}{30}=12$ springs.

So 12 springs can be made from 10yd of wire.

Do #11 a, p. 11 text in your homework booklet. (See Example 3, p. 9 text)

Solving Problems Involving Scale Diagrams

E.g.: The scale for a diecast car is 1:14. If the diecast car is $11\frac{3}{4}$ in, what is the length (in feet) of the actual car?

A scale of 1:14 means that 1 inch on the diecast car equals 14 inches on the actual car.

1 in = 14 in

$11\frac{3}{4}$ in = x in

So

$$\frac{1}{11.75}=\frac{14}{x}$$
$$1x=11.75(14)$$
$$x=164.5\text{in}$$

and

$$\frac{164.5}{12} = 13.708\bar{3}\text{ft}$$

So the actual car is just under 14ft long.

E.g.:

Convert $13.708\bar{3}\text{ft}$ to feet and inches.

We know we have more than 13 ft but less than 14 feet, so let's change the decimal part to inches.

1ft = 12in

$0.708\bar{3}$ft = z in

$$\frac{1}{0.708\bar{3}} = \frac{12}{z}$$
$$1z = 12\left(0.708\bar{3}\right)$$
$$x = 8.5\text{in}$$

So, $13.708\bar{3}\text{ft} = 13\text{ft } 8.5\text{in OR } 13\text{ft } 8\tfrac{1}{2}\text{in OR } 13' \, 8\tfrac{1}{2}''$ and the actual car is $13' \, 8\tfrac{1}{2}''$ long.

Sample Exam Questions

Change 73 in into feet and inches.

A) 12 feet
B) 6 feet .08 in
C) 6 feet 9 in
D) 6 feet 1 in

A map of Orlando, Florida has a scale 1in = 4.8 mi. If the straight-line map distance from Orlando International Airport to Winter Park is $4\tfrac{1}{8}''$, what is the actual straight-line distance between the airport and the park? Show all workings for full credit. $\left(19\text{mi, } 4224\text{ft}\right)$

Do #'s 16-17, p. 12 text in your homework booklet. (See Example 4, p. 10 text)

§1.2 Measuring Length and Distance (1 class)

Read *Lesson Focus* p. 13 text.

Outcomes

1. Give referents for various SI (metric) units. See notes
2. Find length/distance of various objects using referents. See notes

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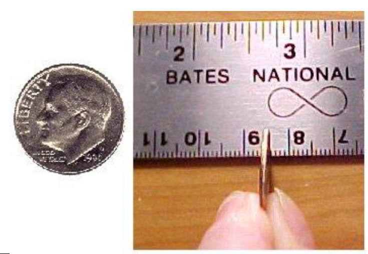
Some quantities and their SI units of measure are shown in the Table below.

Quantity	Unit of Measure
***Distance/Length	***millimeters, centimeters, meters, kilometers
Speed	meters per second, kilometers per hour
Mass	grams, kilograms
Weight	Newtons

We can also use referents to get an idea of the size of these units. (See table below)

SI (metric) Unit	Symbol for Unit	Referent
millimeter	mm	Thickness of a dime. (See pic below). 
centimeter	cm	Ten dimes stacked on top of one another. 
meter	m	The length of a running stride.  The height of a doorknob off the floor or the height from the trunk cover of a car to the ground.
kilometer	km	Distance from ___________________ to ______________________ .

Sample Exam Question

As an estimation strategy, which could be used to best approximate one centimetre?

A. the length of your foot
B. the width of your hand
C. the width of your finger
D. the width of a pencil lead

Complete the table below by finding the indicated measures using referents and then find the actual measure in Imperial or SI units. You select the last three objects.

Object	Measure	Referent	Length/Distance using Referent	Length/Distance with Imperial/SI Units
Classroom door	Height			
Classroom door	Width	G.C.	4.8	35.5cm
Filing cabinet	Height			
Filing cabinet	Width			
Can	Height			
Can	Diameter			
Can	Radius			
Can	Circumference			
Hallway	Length			

Converting From One SI (metric) Unit to Another

Sometimes you have to convert from one SI length unit to another. This can be done using the table below.

SI Unit & Symbol	Relationship between Units
millimeter (mm)	
centimeter (cm)	1 cm = 10 mm
meter (m)	1 m = 100 cm
kilometer (km)	1 km = 1000 m

<table>
<tr><th colspan="3">Problem: Convert 95 cm to mm.</th></tr>
<tr><th>Solution 1</th><th>Solution 2</th><th>Solution 3</th></tr>
<tr>
<td>Since 1 cm = 10 mm (see table p. 10) then,

95 cm = 95(10mm)
= 950mm</td>
<td>Let x be the number of mm in 95cm.

Since, 1 cm = 10 mm (see table p. 10) we can write the proportion

$\frac{10\text{ mm}}{1\text{ cm}} = \frac{x\text{ mm}}{95\text{ cm}}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$10(95) = x(1)$
$950 = x$</td>
<td>Let x be the number of mm in 95cm.

1 cm = 10 mm (see table p. 10)
95 cm = x mm

So, we can write the proportion

$\frac{1}{95} = \frac{10}{x}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$1(x) = 95(10)$
$x = 950$</td>
</tr>
<tr><td>Statement</td><td colspan="2">There are 950mm in 95cm.</td></tr>
</table>

<table>
<tr><th colspan="3">Problem: Convert 950 m to km.</th></tr>
<tr><th>Solution 1</th><th>Solution 2</th><th>Solution 3</th></tr>
<tr>
<td>Since 1 km = 1000 m (see table p. 10) then,

$\frac{950}{1000} = 0.950\text{km}$</td>
<td>Let x be the number of km in 950m.

Since, 1 km = 1000 m (see table p. 10) we can write the proportion

$\frac{1\text{ km}}{1000\text{ m}} = \frac{x\text{ km}}{950\text{ m}}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$1(950) = x(1000)$
$950 = 1000x$
$\frac{950}{1000} = \frac{\cancel{1000}x}{\cancel{1000}}$
$0.950 = x$</td>
<td>Let x be the number of km in 950m.

1 km = 1000 m (see table p. 10)
x km = 950 m

So, we can write the proportion

$\frac{1}{x} = \frac{1000}{950}$

Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get

$1(950) = x(1000)$
$950 = 1000x$
$\frac{950}{1000} = \frac{1000x}{1000}$
$0.950 = x$</td>
</tr>
<tr><td>Statement</td><td colspan="2">There are 0.950km in 950m.</td></tr>
</table>

Sample Test Question

Which is equivalent to 2500 m?

A) 2.5 mm B) 2.5 km C) 250cm D) 250hm

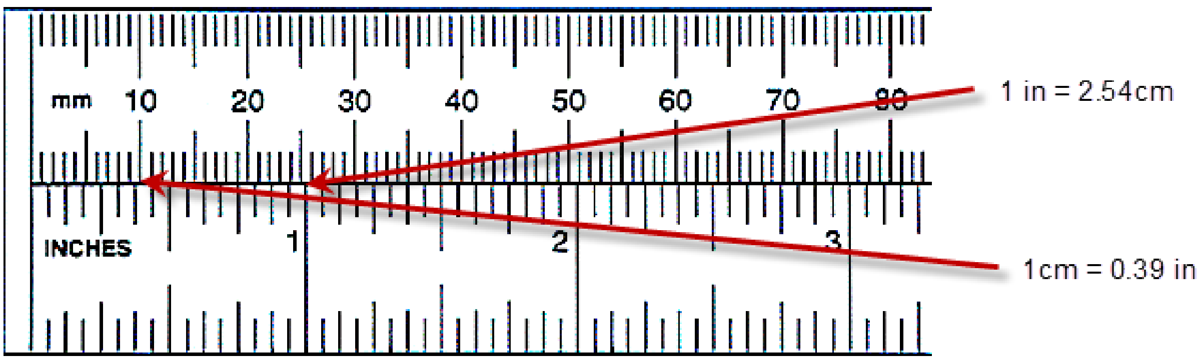
§1.3 Relating SI and Imperial Units (1 class)

Read *Lesson Focus* p. 16 text.

Outcomes

1. Convert between SI units and imperial units. p. 16-19
2. Solve problems involving conversions between SI and imperial units. pp. 20-21

If you place a ruler with inches next to a ruler with millimeters, you should see that 1 inch is just a tiny bit above 2.5cm.



In fact 1 inch is about 2.54cm.

Likewise, you should also see that 1cm is just a tiny bit above $\frac{6}{16} = \frac{3}{8} = 0.375$

In fact 1 cm is about 0.39in.

So $1 \text{ in} = 2.54 \text{ cm}$ and $1 \text{ cm} = 0.39 \text{ in}$ allows us to convert between SI and imperial units.

Complete the table below.

Imperial Units to SI Units	SI Units to Imperial Units
$1 \text{ in} \approx 2.54 \text{ cm}$	$1 \text{ cm} \approx 0.39 \text{ in}$
1 ft = 30.48 cm	1 m = ________ in 1 m = ________ ft = ________ in 1 m = ________ yd = ________ ft = ________ in
1 yd = ________ cm 1 yd = ________ m	1 km = ________ yd 1 km = ________ mi
1 mi = ________ m = ________ km	

Converting between SI and Imperial Units

Problem: Convert 3500 m to miles.		
Solution 1	Solution 2	Solution 3
Since 3500m = 3.5km and 1km ≈ 0.6 mi (See p. 13) then, $3.5\times 0.6=2.1$	**Change 3500m to km** Let x be the number of km in 3500m. 1000 m = 1 km 3500 m = x km $\frac{1000}{3500}=\frac{1}{x}$ $1000x=3500$ $\frac{1000x}{1000}=\frac{3500}{1000}$ $x=3.5$ **Change 3.5 km to miles.** Let y be the number of mi in 3.5km. Since, 1 km ≈ 0.6 mi (see table p. 13) we can write the **proportion** $\frac{1\text{ km}}{0.6\text{ mi}}=\frac{3.5\text{ km}}{y\text{ m}}$ Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get $1(y)=0.6(3.5)$ $y\approx 2.1$	**Change 3500m to km** Let x be the number of km in 3500m. 1 km = 1000 m x km = 3500 m $\frac{1}{x}=\frac{1000}{3500}$ Since there is only 1 fraction on each side of the equal sign, we can cross multiply to get $1(3500)=x(1000)$ $3500=1000x$ $\frac{3500}{1000}=\frac{1000x}{1000}$ $3.5=x$ **Change 3.5 km to miles.** Let y be the number of mi in 3.5km. 1 km = 0.6 mi 3.5 km = y mi $\frac{1}{3.5}=\frac{0.6}{y}$ $y=0.6(3.5)$ $y\approx 2.1$
Statement	There are about 2.1 mi in 3500m.	

Sample Exam Questions

If you have a driver's license that has a stated height of 170 cm, what height does this represent in inches (rounded to the nearest inch)?

(A) 432 in. (B) 85 in. (C) 67 in. (D) 66 in.

You are vacationing in the United States to watch the World Series of baseball. For game seven of the series you have to fly from Texas to St. Louis. Just as you are departing the pilot announces that it is 539 miles. Which represents the distanceto the ?

(A) 323 km (B) 337 km (C) 862 km (D) 863 km

Do #'s 4 a,b,e; 5 b,d; 6 a; p. 22 text in your homework booklet.

Solving Problems Involving Converting Between Imperial and SI Units

E.g.: During winter 2013-2014, New York City received 57.4 inches of snow. How many centimeters was this?

Abundant Snowfall:
Totals for Winter 2013-2014

Location	Total	Normal	Percent of Normal	Location	Total	Normal	Percent of Normal
Boston	58.6"	40.6"	144%	Indianapolis	55.3"	25.4"	218%
Chicago	80.0"	34.6"	231%	New York City	57.4"	24.0"	239%
Charlotte	9.3"	4.2"	221%	Norfolk	15.4"	5.8"	266%
Cincinnati	47.1"	21.2"	222%	Philadelphia	67.6"	21.5"	314%
Dayton	50.5"	22.9"	221%	Pittsburgh	62.3"	39.3"	159%
Detroit	90.7"	39.8"	228%	Washington, D.C. (Dulles)	49.0"	21.3"	230%

* as of March 24, 2014 AccuWeather.com

Let *x* be the number of centimeters in 57.4 in.

1 cm = 0.39 in

x cm = 57.4 in

We write the proportion

$$\frac{1}{x} = \frac{0.39}{57.4}$$

We cross multiply to get

$$1(57.4) = x(0.39)$$
$$57.4 = 0.39x$$
$$\frac{57.4}{0.39} = \frac{\cancel{0.39}x}{\cancel{0.39}}$$
$$147.2 \approx x$$

So New York City had 147.2cm or 1.472m of snow.

E.g.: The distance from Earth to the sun is called an astronomical unit (AU). This distance is about 149,597,871km. How many miles is this?

From the last table we know that 1 mi = 1.6km

Let *x* be the number of miles in 149,597,871km

1 mi = 1.6km

x mi = 149,597,871 km

$$\frac{1}{x} = \frac{1.6}{149,597,871}$$
$$1(149,597,871) = 1.6x$$
$$149,597,871 = 1.6x$$
$$\frac{149,597,871}{1.6} = \frac{\cancel{1.6}x}{\cancel{1.6}}$$
$$93,498,669.38 \approx x$$

The sun is about 93,498, 669 mi from Earth.

Sample Exam Question

A road sign warns of an intersection in 1000 ft. Approximately how far is this distance in kilometres? Give your answer to the nearest tenth of a kilometre. (0.3)

Do #'s 9, 11, 13, 15 pp. 22-23 text in your homework booklet.
Do #'s 3, 4, 7, 8 p. 25 text in your homework booklet.

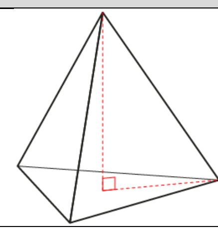
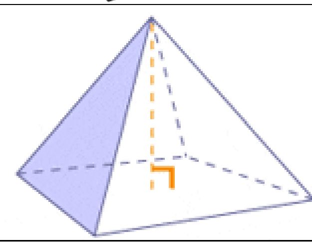
§1.4 Surface Area of Right Pyramids and Right Cones (3 classes)

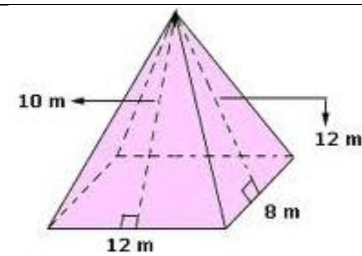
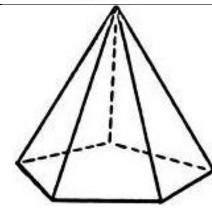
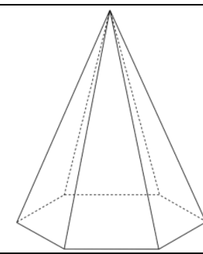
Read *Lesson Focus* p. 26 text.

Outcomes

1. Define a **right pyramid**. p. 27
2. Draw a right pyramid. p. 27
3. Identify the **apex**, **base**, **faces**, **height (altitude)**, and **slant height** of a right pyramid. p. 27
4. Draw and recognize a right pyramid with various bases. (E.g.: triangular prism, square pyramid, rectangular pyramid, pentagonal pyramid) p. 27
5. Given a right pyramid, recognize its corresponding net. p. 28
6. Define the term **lateral surface area** or **lateral area**. p. 30
7. Find the lateral surface area of a right pyramid. p. 30
8. Determine the surface area of a right pyramid, using an actual object or its labelled diagram. pp. 28-31
9. Determine an unknown dimension of a right pyramid, given the object's surface area and the remaining dimensions. p. 31
10. Draw a **right cone**. p. 31
11. Identify the **apex**, **base**, **base radius**, **base diameter**, **lateral surface**, **height (altitude)**, and **slant height** of a right cone. p. 31
12. Find the lateral surface area of a right cone. p. 31
13. Determine the surface area of a right cone, using an object or its labelled diagram. p. 32
14. Determine an unknown dimension of a right pyramid, given the object's surface area and the remaining dimensions. # 16, p. 35
15. Determine an unknown dimension of a right cone, given the object's surface area and the remaining dimensions. p. 33
16. Solve problems that involve surface area, using an actual 3-D object or its labelled diagram.

Def$^{\underline{n}}$: A **right pyramid** is a 3-D object that has sides that are triangles and a base that is a polygon. The shape of the base polygon determies the name of the pyramid.

Base Shape	Pyramid Name	Illustration
Triangle	(Triangular Pyramid)	
Square	Right Square Pyramid (Square Pyramid)	

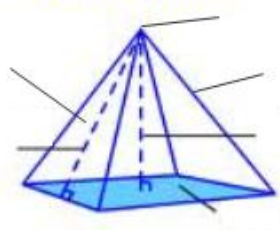
Rectangle	Right Rectangular Pyramid (Rectangular Pyramid)	
Pentagon	Right Pentagonal Pyramid (Pentagonal Pyramid)	
Hexagon	Right Hexagonal Pyramid (Hexagonal Pyramid)	

You need to know the parts of a right pyramid.

Term/Part	Definition/Description
Apex (Vertex)	The point where the triangular faces meet.
Base	The bottom surface of the pyramid.
Edge	The line segment where two faces meet (intersect) or where a base and a face meet.
Face (lateral face)	Any one of the triangular sides of the pyramid (Not the base).
Height (altitude)	The perpendicular distance from the apex to the centre of the base.
Slant Height	The height of a triangular face. It extends from the apex to the midpoint of the base of the triangular face.

Use the table above to label the diagrams below.

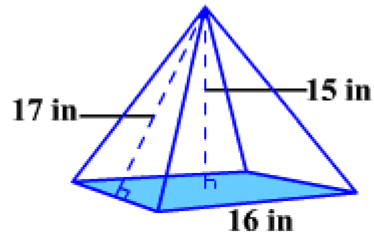
Square Pyramid



Hexagonal Pyramid

E.g.: Give the length of the altitude and of the slant height for the right pyramid shown below.

a. Height (altitude) ______
b. Slant height ______



Finding the Surface Area of a Right Pyramid Using Nets.

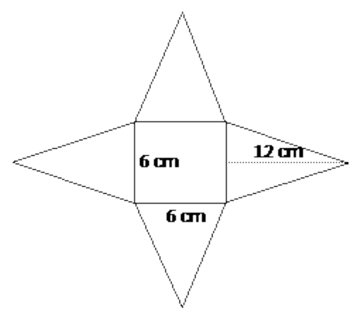
Go to http://www.learner.org/interactives/geometry/3d_pyramids.html to see some right pyramids and their nets.

Recall the common area formulas below.

Shape	Area Formula
Square	$A = \text{side}^2 = s^2$
Rectangle	$A = \text{length} \times \text{width} = l \times w$
Triangle	$A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh = \frac{bh}{2}$
Circle	$A = \pi \times \text{radius}^2 = \pi r^2$

To find the surface area (SA) of a right pyramid, we need to find the area of the base and the area of each triangular face (lateral face) and find the sum (add).

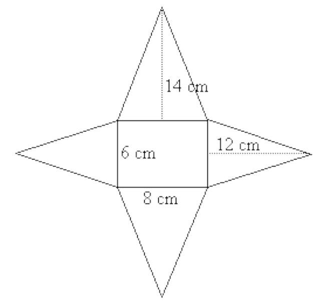
E.g.: Find the area of the right **square** pyramid whose net is given below.



For the right square pyramid to the right, we have to find the area of the square base plus the area of the 4 triangles.

So $\text{SA} = s^2 + 4\left(\frac{1}{2}bh\right) = 6^2 + 4\left(\frac{1}{2} \times 6 \times 12\right) = 36 + 144 = 180\text{cm}^2$

E.g.: Find the area of the right **rectangular** pyramid whose net is given below.



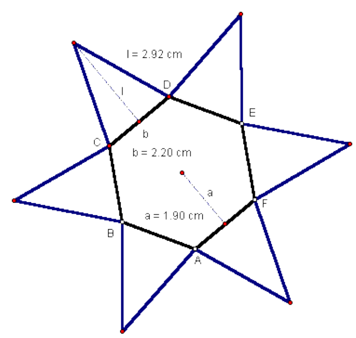
For the right rectangular pyramid in the picture, we have to find the area of the rectangular base plus the area of the two triangles with base 6cm and height 12cm plus the area of the two triangles with base 8cm and height 14cm.

So

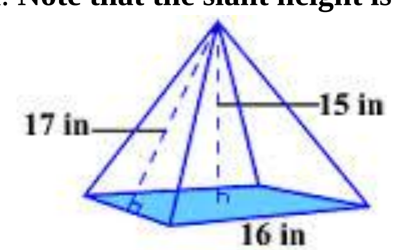
$$\begin{aligned}\text{SA} &= (l\times w)+2\left(\frac{1}{2}bh\right)+2\left(\frac{1}{2}bh\right)\\ &=(8\times 6)+2\left(\frac{1}{2}\times 6\times 12\right)+2\left(\frac{1}{2}\times 8\times 14\right)\\ &=48+72+112\\ &=232\text{cm}^2\end{aligned}$$

Explain how you would find the surface area of a right **triangular** prism given its net.

Think about how you might find the surface area of a right **hexagonal** pyramid with a net like this.



E.g.: Find the lateral surface area of the right **square** pyramid in the diagram. **Note that the slant height is given**.



To find the lateral surface area, we only need to find the areas of the four triangular faces and add them.

$$\text{SA}=4\left(\frac{1}{2}bh\right)=4\left(\frac{1}{2}\times 16\times 17\right)=544\text{in}^2$$

E.g.: Find the total surface area of the pyramid in the last example.

We only need to find the area of the base and add it to the lateral surface area.

$$A_{base} = 16^2 = 256\text{in}^2$$

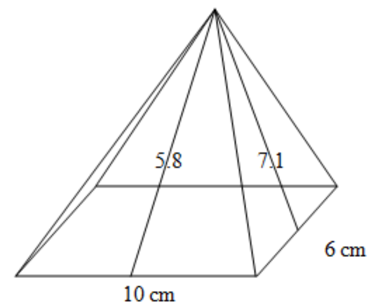
and

$$SA = 256 + 544 = 800\text{in}^2$$

Do #'s 4 a, 5 (using 4 a) only), 8 a, p. 34 text in your homework booklet.

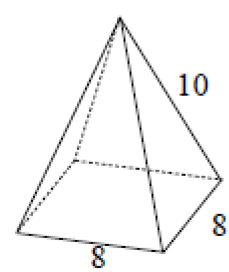
Sample Test Question

Calculate the surface area of the right **rectangular** pyramid with a base with side lengths 10cm and 6cm and sides with slant heights of 5.8cm and 7.1cm. $\left(160.6\text{cm}^2\right)$

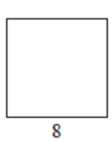
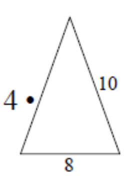


Do # 13 a, p. 34 text in your homework booklet.

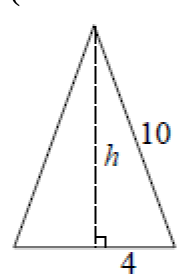
E.g.: Find the surface area of the right **square** pyramid in the diagram. **Note that the slant height is NOT given**.



To find the surface area, we only need to find the area of the base and the areas of the four triangular faces and add them.

SA =  + 4 • 

To find the area of the triangles, we need to find the height of one of the triangles (i.e. we need to find the slant height of the pyramid). We do this using the Pythagorean Theorem.



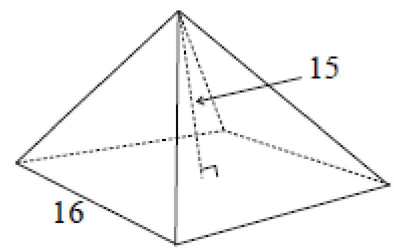
$$h^2 + 4^2 = 10^2$$
$$h^2 + 16 = 100$$
$$h^2 = 84$$
$$h = \pm\sqrt{84}$$
$$h \approx \pm 9.2$$
$$h \approx 9.2$$

So,

$$\text{SA} = s^2 + 4\left(\frac{1}{2}bh\right) = 8^2 + 4\left(\frac{1}{2}\times 8\times 9.2\right) = 64 + 147.2 = 211.2\text{units}^2$$

The surface area of the square pyramid is 211.2units^2.

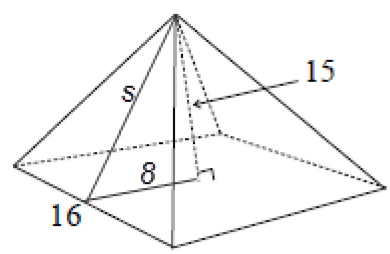
E.g.: Find the surface area of the right **square** pyramid in the diagram. **Note that the slant height is NOT given**.



To find the surface area, we only need to find the area of the base and the areas of the four triangular faces and add them.

To find the area of the triangular faces, we need the slant height (s) of the pyramid.

To find the slant height we can use the Pythagorean Theorem



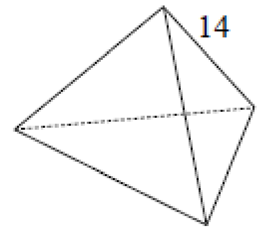
$$s^2 = 8^2 + 15^2$$
$$s^2 = 64 + 225$$
$$s^2 = 289$$
$$s = \pm\sqrt{289}$$
$$s = \pm 17$$
$$s = 17$$

So

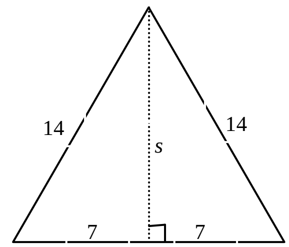
$$\text{SA} = s^2 + 4\left(\frac{1}{2}bs\right) = 16^2 + 4\left(\frac{1}{2}\times 16\times 17\right) = 256 + 544 = 800\text{units}^2$$

Do #'s 13 b, 18, pp. 34-35 text in your homework booklet.

E.g.: Find the surface area of the regular tetrahedron (right **triangular** pyramid) in the diagram. **Note that the slant height is NOT given**.



In this pyramid, all the edges have length 14. So we need to find the area of one triangle and multiply by 4 to get the total surface area. We need to find the slant height first.



$$s^2 + 7^2 = 14^2$$
$$s^2 + 49 = 196$$
$$s^2 + 49 - 49 = 196 - 49$$
$$s^2 = 147$$
$$s = \pm\sqrt{147}$$
$$s = \pm 12.1$$
$$s = 12.1$$

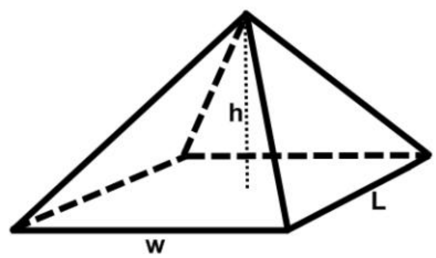
So

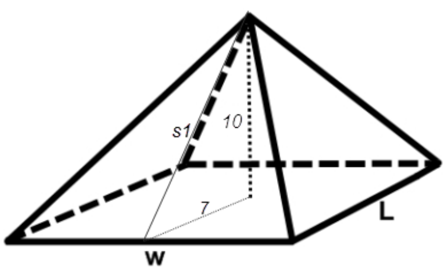
$$\text{SA} = 4\left(\frac{1}{2}bs\right) = 4\left(\frac{1}{2}\times 14\times 12.1\right) = 338.8\text{units}^2$$

Do #'s 4 b and 5 (using 4 b) only) p. 34 text in your homework booklet.

E.g.: Find the surface area of the right **rectangular** pyramid below if $h = 10\text{ft}, w = 8\text{ft},$ and $L = 14\text{ft}$.

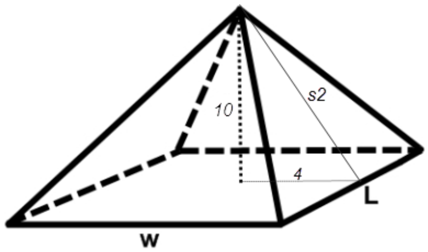
Since the base is a rectangle, we need to find two slant heights, one for the front and rear lateral faces (s1), and one for the left and right lateral faces (s2).





$$(s1)^2 = 7^2 + 10^2$$
$$(s1)^2 = 49 + 100$$
$$(s1)^2 = 149$$
$$s1 = \pm\sqrt{149}$$
$$s1 = \pm 12.2$$
$$s1 = 12.2$$

So the area of the front and back lateral faces is $SA = 2\left(\frac{1}{2}w \times s1\right) = 2\left(\frac{1}{2} \times 8 \times 12.2\right) = 97.6\text{ft}^2$



$$
\begin{aligned}
(s2)^2 &= 4^2 + 10^2 \\
(s2)^2 &= 16 + 100 \\
(s2)^2 &= 116 \\
s2 &= \pm\sqrt{116} \\
s2 &= \pm 10.8 \\
s2 &= 10.8
\end{aligned}
$$

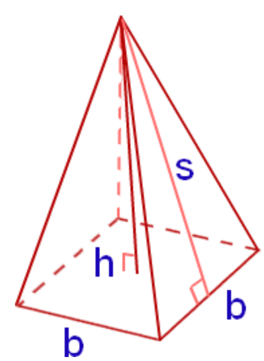
So the area of the left and right lateral faces is $SA = 2\left(\frac{1}{2}L \times s2\right) = 2\left(\frac{1}{2} \times 14 \times 10.8\right) = 151.2\text{ft}^2$

The area of the base is $14 \times 8 = 112\text{ft}^2$

So the total surface area is $97.6 + 151.2 + 112 = 360.8\text{ft}^2$

Now instead of finding the surface area given some dimensions of a right pyramid, let's work backwards and find a missing dimension given the surface area.

E.g.: The surface area of the right square pyramid below is 256yd^2. If the base has length of 8 yd, find the slant height.



Since this is a right **square** pyramid, the surface area is

$$SA = b^2 + 4\left(\frac{1}{2}bs\right)$$

Substituting numbers for the variables gives

$$256 = 8^2 + 4\left(\frac{1}{2}\times 8\times s\right)$$
$$256 = 64 + 16s$$
$$256 - 64 = 64 - 64 + 16s$$
$$192 = 16s$$
$$\frac{192}{16} = \frac{\cancel{16}s}{\cancel{16}}$$
$$12 = s$$

So the slant height is 12 yd.

Do # 16 b, p. 35 text in your homework booklet.

Def $^{\underline{n}}$: A **right cone** is a 3-D object that has a curved lateral surface and a base that is a circle.

Now let's apply the things we did with right pyramids to a right cone. Near the middle of page 31 of the text, note that if we start with a right square pyramid and increase the number of sides in the base to infinity, we get a right cone.

Note the general formula for the surface area of a right pyramid with a regular polygon base on the top of page 31 of the text. This formula states that to find the area of a right pyramid with a regular polygon base, we need to find the area of the base and add the area of each triangular face. If the base has *n* sides, then the formula for the total surface area would be

$$\text{SA} = \text{ Area of Base} + n\left(\frac{1}{2}\times \text{base}\times \text{slant height}\right)$$

OR

$$\text{SA} = \text{ Area of Base} + n\times\frac{1}{2}\times \text{base}\times \text{slant height}$$

Since multiplication is commutative (we can multiply in any order) we can rewrite the last formula as

$$\text{SA} = \text{ Area of Base} + \frac{1}{2}\times \text{n}\times \text{base}\times \text{slant height}$$

OR

$$\text{SA} = \text{ Area of Base} + \frac{1}{2}\times(\text{n}\times \text{base})\times \text{slant height}$$

But $\text{n}\times\text{base}$ is just the perimeter of the base so we could write

$$\text{SA} = \text{ Area of Base} + \frac{1}{2}\times(\text{Perimeter of the base})\times \text{slant height}$$

If we are using a cone then the perimeter of the base is just the circumference of the circular base and the area of the base is just the area of the circle.

So for a right cone,

$$\text{SA} = \text{ Area of Base} + \frac{1}{2}\times(\text{Circumference of the base})\times \text{slant height}$$

Recall that the circumference of a circle is given by the expression $2\pi r$ and the area is given by πr^2, where r is the radius of the circle. If we use *s* for the slant height, then the formula for the area of a right cone is

$$SA = \pi r^2 + \frac{1}{2} \times (2\pi r) \times s$$

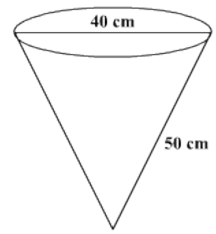
Which simplifies to

$$SA = \pi r^2 + \frac{\cancel{1}}{\cancel{2}} \times \cancel{2}\pi r \times s$$

********$SA = \pi r^2 + \pi rs$********

Note that πr^2 is the base area and πrs is the area of the lateral surface.

E.g.: Find the surface area of the cone to the right.



$s = 50\text{cm}, \ r = \frac{40}{2} = 20\text{cm}$

$SA = \pi r^2 + \pi rs$

$= \pi(20)^2 + \pi(20)(50)$

$= 400\pi + 1000\pi$

$= 1400\pi \approx 4398.2\text{cm}^2$

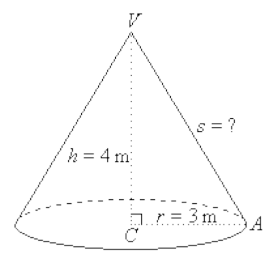
So the surface area of the cone is 4398.2cm^2.

Do #'s 6 a, and 7 (use 6 a) only); 6 b, and 7 (use 6 b) only); 11 p. 34 text in your homework booklet.

E.g.: Find the surface area of the cone below.

$h = 4\text{m}, \ r = 3\text{m}$

Since we need the slant height (s) to find the surface area, we use the Pythagorean Theorem.



$s^2 = 4^2 + 3^2$

$s^2 = 16 + 9$

$s^2 = 25$

$s = \pm\sqrt{25}$

$s = \pm 5$

$s = 5$

$SA = \pi r^2 + \pi rs$

$= \pi(3)^2 + \pi(3)(5)$

$= 9\pi + 15\pi$

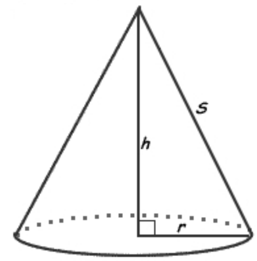
$= 24\pi \approx 75.4$

The surface area is 75.4m^2.

Do #'s 8 b, 20 pp. 34-35 text in your homework booklet.

Now let's work backwards again and find the slant height or the height (altitude) of a cone given its surface area and radius.

E.g.: The surface area of the right cone below is $282.74\overline{3}\ \text{ft}^2$. If the radius is 5 feet, find the slant height of the cone.



$\text{SA} = 282.74\overline{3},\quad r = 5,\quad s = ?$

Substituting into $\text{SA} = \pi r^2 + \pi r s$ gives

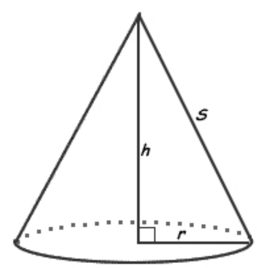
$$282.74\overline{3} = \pi(5)^2 + \pi(5)s$$
$$282.74\overline{3} = 25\pi + 5\pi s$$
$$282.74\overline{3} - 25\pi = 25\pi - 25\pi + 5\pi s$$
$$204.20 = 5\pi s$$
$$\frac{204.20}{5\pi} = \frac{\cancel{5\pi} s}{\cancel{5\pi}}$$
$$13 = s$$

The slant height is 13 ft.

Do # 16 a, p. 35 text in your homework booklet.

E.g.: The surface area of the right cone below is $628.32\ \text{in}^2$. If the radius is 8 inches, find the height (altitude) of the cone.

$\text{SA} = 628.32,\quad r = 8,\quad \text{h} = ?$



Before we can find *h*, we must find the value of *s*.

Substituting into $\text{SA} = \pi r^2 + \pi r s$ gives

$$628.32 = \pi(8)^2 + \pi(8)s$$
$$628.32 = 64\pi + 8\pi s$$
$$628.32 - 64\pi = 64\pi - 64\pi + 8\pi s$$
$$427.26 = 8\pi s$$
$$\frac{427.26}{8\pi} = \frac{\cancel{8\pi} s}{\cancel{8\pi}}$$
$$17 = s$$

If $s = 17$ then,

$17^2 = 8^2 + h^2$

$289 = 64 + h^2$

$289 - 64 = 64 - 64 + h^2$

$225 = h^2$

$\pm\sqrt{225} = h^2$

$\pm 15 = h$

$15 = h$

So the height of the cone is 15 inches.

Do # 21 p. 35 text in your homework booklet.

§1.5 Volumes of Right Pyramids and Right Cones (2 classes)

Read *Lesson Focus* p. 36 text.

Outcomes

1. Define the **volume** of an object. p. 36
2. Define the **capacity** of an object. p. 36
3. Distinguish between volume and capacity. p. 36
4. Give the correct units for volume and capacity. p. 36
5. Describe the relationship between the volumes of:
   • right cones and right cylinders with the same base and height. p. 39
   • right pyramids and right prisms with the same base and height. p. 37
6. Determine the volume of a right cone, right cylinder, right prism, or a right pyramid using an object or its labelled diagram. pp. 38-40
7. Determine an unknown dimension of a right cone, right cylinder, right prism, or right pyramid, given the object's volume and the remaining dimensions. p. 41

Def$^{\underline{n}}$: The **volume** of an object is the amount of space it occupies. It is measured in cubic units (e.g.: $\text{in}^3,\ \text{ft}^3,\ \text{yd}^3,\ \text{m}^3,\ \text{cm}^3$)

Def$^{\underline{n}}$: The **capacity** of an object is how much it can hold. The capacity of an object is normally less than its volume. It is measures in cubic units (e.g.: $\text{in}^3,\ \text{ft}^3,\ \text{yd}^3,\ \text{m}^3,\ \text{cm}^3$) but also in capacity units (e.g.: mL, L).

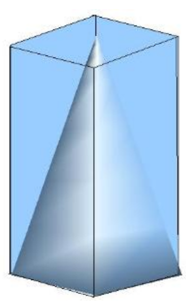
Recall that the volume of a **prism** is the area of its base multiplied by its height.

$$V_{\text{prism}} = \text{Area of Base} \times \text{height}$$

If a **pyramid** has the same base area and height as a prism, we can find the volume of the pyramid using the volume of the prism.

The Relationship Between the Volume of a Right Prism and the Volume of the Corresponding Pyramid

Corresponding means that the prism and the pyramid have the same base area and height.



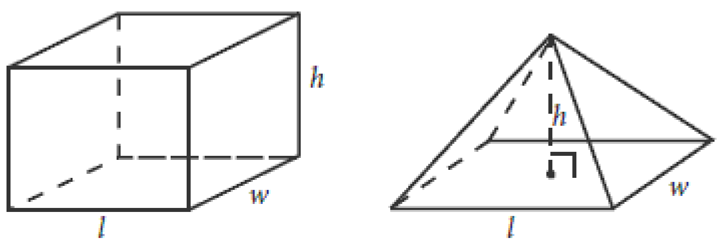
Watch http://www.youtube.com/watch?v=Fqj8svaBwc8 (0-2:42) to see the relationship between the volume of a right prism and the volume of the corresponding pyramid.

Watch https://www.k12pl.nl.ca/curr/10-12/math/math1201/classroomclips/measurement/volume3-dobjects.html (1:15-4:08) to see the relationship between the volume of a right prism and the volume of the corresponding pyramid.

What is the relationship between the volume of a pyramid and the volume of the corresponding prism? Write a formula that shows this relationship. (See p. 37, text)

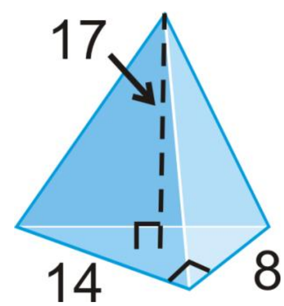
So, to find the volume of a right pyramid, all I need to do is find the area of the corresponding prism and divide by 3 $\left(\text{or multiply by } \frac{1}{3}\right)$.

E.g.:



$V = lwh$ or $V = Bh$ $\qquad$ $V = \frac{1}{3}lwh$ or $V = \frac{1}{3}Bh$

E.g.: Find the volume of the triangular pyramid to the right.



First I will find the volume of the corresponding prism.

I need the base area. The base is a triangle with one side of length 14 (base) and a height of 8.

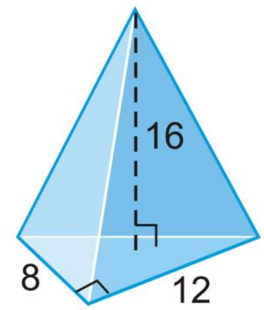
$$\text{Area}_{\text{base}} = \frac{1}{2}(14)(8) = 56\text{units}^2$$

So the volume of the prism is:

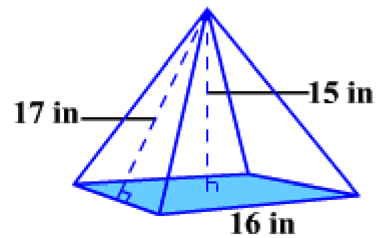
$$V_{prism} = 56 \times 17 = 952\text{units}^3$$

This means that the volume of the pyramid is $\frac{952}{3} = 317.\bar{3}$ units^3.

Your Turn: Find the volume of the triangular pyramid to the right. $\left(256 \text{ units}^3\right)$



E.g.: Find the volume of the square pyramid to the right.



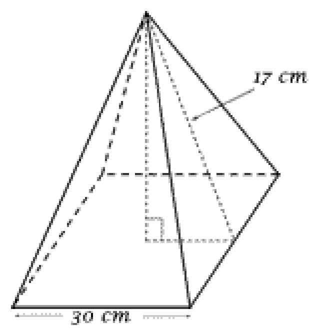
First we will find the volume of the corresponding prism.

$$V_{prism} = \text{ Base Area} \times \text{ height}$$
$$= 16^2 \times 15$$
$$= 3840 \text{ in}^3$$

This means that the volume of the pyramid is $\dfrac{3840}{3} = 1280 \text{ in}^3$.

Do #'s 8, 13 p. 42 text in your homework booklet.

E.g.: Find the volume of the square pyramid to the right.



To find the volume of the corresponding prism, I first need to find the height.

$$17^2 = 15^2 + h^2$$
$$289 = 225 + h^2$$
$$289 - 225 = 225 - 225 + h^2$$
$$64 = h^2$$
$$\pm\sqrt{64} = h^2$$
$$\pm 8 = h$$
$$8 = h$$

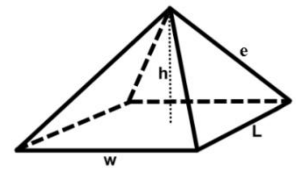
So,

$$V_{prism} = \text{ Base Area} \times \text{ height}$$
$$= 30^2 \times 8$$
$$= 7200 \text{ cm}^3$$

This means that the volume of the pyramid is $\dfrac{7200}{3} = 2400 \text{ cm}^3$.

Do # 12 p. 42 text in your homework booklet.

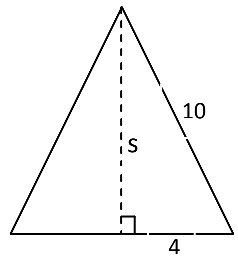
E.g.: Find the volume of the rectangular pyramid if $L = 8$ ft, $w = 2$ yd, and $e = 10$ ft.



Before we can find the volume, we need to find the value of *h*.

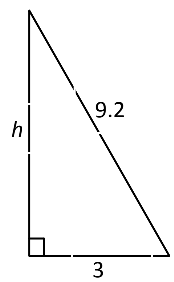
Before we can find *h*, we need to find the slant height (*s*).

$$10^2 = 4^2 + s^2$$
$$100 = 16 + s^2$$
$$100 - 16 = 16 - 16 + s^2$$
$$84 = s^2$$
$$\pm\sqrt{84} = s^2$$
$$\pm 9.2 \approx s$$
$$9.2 \approx s$$



Now let's use the value of *s* to find the value of *h*.

$$9.2^2 = 3^2 + h^2$$
$$84.64 = 9 + h^2$$
$$84.64 - 9 = 9 - 9 + h^2$$
$$75.64 = h^2$$
$$\pm\sqrt{75.64} = \sqrt{h^2}$$
$$\pm 8.7 \approx h$$
$$8.7 \approx h$$



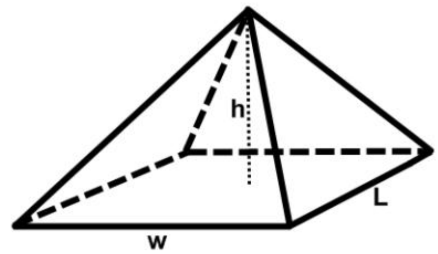
So the volume of the pyramid is

$$V = \frac{1}{3}(6)(8)(8.7)$$
$$V = 139.2\text{ft}^3$$

Do # 16 p. 43 text in your homework booklet.

Now let's work backwards and find the missing dimension(s) of a prism or pyramid given its volume.

E.g.: The volume of the right pyramid is 40yd^3. If $\text{L} = 6\text{ yd}$ and $h = 5\text{ yd}$, what is the value of *w*?



The formula for the volume of the right pyramid is $V = \frac{1}{3}wLh$.

Substituting values for *V*, *L*, and *h* gives

$$40 = \frac{1}{3}w(6)(5)$$
$$40 = 10w$$
$$\frac{40}{10} = \frac{10w}{10}$$
$$4 = w$$

So the width of the pyramid is 4 yd.

Do # 18 a, b p. 43 text in your homework booklet.

Recall that the volume of a **cylinder** is the area of its base multiplied by its height.

$$\text{V}_{\text{cylinder}} = \text{Area of Base} \times \text{height}$$

If a **cone** has the same base area and height as a cylinder, we can find the volume of the cone using the volume of the cylinder.

The Relationship Between the Volume of a Right Cylinder and the Volume of the Corresponding Cone

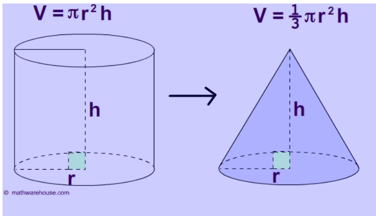
Corresponding means that the prism and the pyramid have the same base area and height.

Watch http://www.youtube.com/watch?v=xwPiA0COi8k to see the relationship between the volume of a right cylinder and the volume of the corresponding cone.

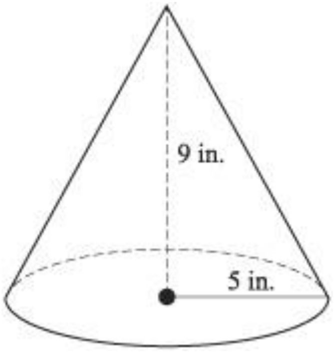
Watch http://www.youtube.com/watch?v=0ZACAU4SGyM to see the relationship between the volume of a right cylinder and the volume of the corresponding cone.

What is the relationship between the volume of a cylinder and the volume of the corresponding cone? Write a formula that shows this relationship. (See p. 40, text)

So, to find the volume of a right cone, all I need to do is find the area of the corresponding cylinder and divide by 3 $\left(\text{or multiply by } \frac{1}{3}\right)$.



E.g.: Find the volume of the cone to the right.



First we will find the volume of the corresponding cylinder.

We need the base area. The base is a circle with radius 5in.

$\text{Area}_{\text{base}} = \pi(5)^2 = 78.5 \text{ in}^2$

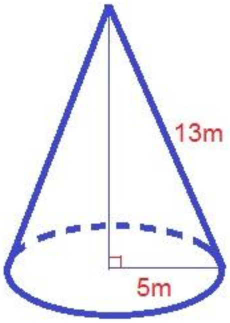
So the volume of the cylinder is:

$V_{cylinder} = 78.5 \times 9 = 706.5 \text{ in}^3$

This means that the volume of the pyramid is $\frac{706.5}{3} = 235.62 \text{ in}^3$.

Do #'s 9, 14 a, b, pp. 42-43 text in your homework booklet.

E.g.: Find the volume of the cone to the right.



We need the base radius and the height to find the volume, so I need to find the height (altitude) of the cone.

Using the Pythagorean Theorem,

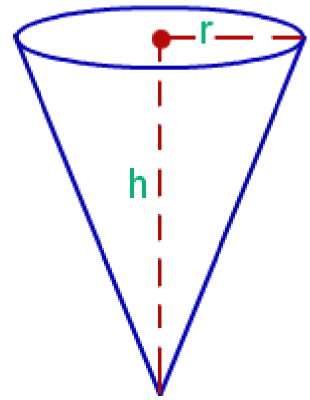
$13^2 = 5^2 + h^2$
$169 = 25 + h^2$
$169 - 25 = 25 - 25 + h^2$
$144 = h^2$
$\pm\sqrt{144} = \sqrt{h^2}$
$\pm 12 = h$
$12 = h$

So the volume of the cone is $\frac{\pi(5)^2(12)}{3} \approx 314.16 \text{ m}^3$

Do # 11 p. 42 text in your homework booklet.

Now let's work backwards and find a missing dimension given the volume and the remaining dimensions.

E.g.: The volume of the cone to the right is 678.6 ft^3. If the radius of the base of the come is 2 yd, what is the height of the cone?



Substituting into $V = \frac{1}{3}\pi r^2 h$ gives

$$678.6 = \frac{1}{3}\pi(6)^2 h$$

$$678.6 = 12\pi h$$

$$\frac{678.6}{12\pi} = \frac{12\pi h}{12\pi}$$

$$18 = h$$

The height of the cone is 18ft or 6yd.

The volume of the cone to the right is 45.8 cm^3. Find the value of r.

Substituting into $V = \frac{1}{3}\pi r^2 h$ gives

$$45.8 = \frac{1}{3}\pi r^2(7)$$

$$45.8 = \frac{7}{3}\pi r^2$$

$$\frac{45.8}{\frac{7}{3}\pi} = \frac{\frac{7}{3}\pi r^2}{\frac{7}{3}\pi}$$

$$6.25 \approx r^2$$

$$\sqrt{6.25} \approx \sqrt{r^2}$$

$$2.5 \approx r$$

The radius of the cone is 2.5cm.

Do # 18 c, d, p. 43 text in your homework booklet.

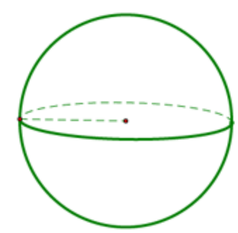
§1.6 Surface Area and Volume of a Sphere (2 classes)

Read *Lesson Focus* p. 45 text.

Outcomes

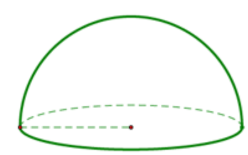
1. Define a **sphere**. pp. 45, 541
2. Define the radius of a sphere. p. 45
3. Define the diameter of a sphere. p. 45
4. Determine the surface area of a sphere, using an object or its labelled diagram. p.46
5. Determine an unknown dimension of a sphere, given the object's surface area and the remaining dimensions. p. 47
6. Determine the volume of a sphere, using an object or its labelled diagram. p. 49
7. Define a **hemisphere**. pp. 49, 537
8. Determine the volume of a hemisphere, using an object or its labelled diagram. p. 50

Def$^{\underline{n}}$: A **sphere** is a set of points that are equidistant (the same distance) from a fixed point called the centre of the sphere.



Sphere

Def$^{\underline{n}}$: A **hemisphere** is half a sphere.



Hemisphere

Finding the Surface Area of a Sphere

The surface area of a sphere is related to the area of a circle with the same radius.

Watch http://www.youtube.com/watch?v=cAxHYFRx1Fs to see the relationship between the surface area of a sphere and the area of a circle with the same radius.

Watch http://www.youtube.com/watch?v=Bbf3agEH_3M to see the relationship between the surface area of a sphere and the area of a circle with the same radius.

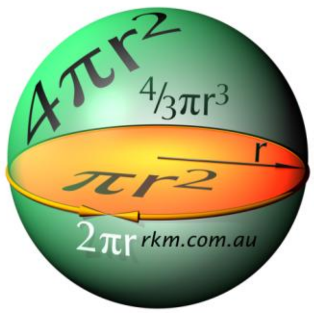
Watch http://www.youtube.com/watch?v=Fyvq-jIQKr8 to see the relationship between the surface area of a sphere and the area of a circle with the same radius.

What is the relationship between the relationship between the surface area of a sphere and the area of a circle with the same radius? Write a formula that shows this relationship. (See p. 46, text) The pictures below may help.

So, to find the surface area of a sphere, all I need to do is find the area of the circle with the same radius and multiply by 4.

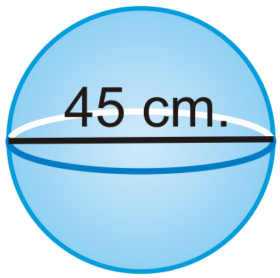
Your Turn

If the area of a circle with radius r is given by the expression πr^2, which is the expression for the surface area of the sphere with the same radius?



a) r
b) $2\pi r$
c) $4\pi r^2$
d) $\frac{4}{3}\pi r^3$

E.g.: Find the surface area of the sphere to the right. Give your answer to the nearest hundredth.



Since the diameter is 45cm then $r = \frac{45}{2} = 22.5\text{cm}$. So the surface area of the sphere is

$4\pi(22.5)^2 \approx 6361.73\text{cm}^2$

E.g.: If the diameter of the beach-ball to the right is 24 in. how much material is needed to make the ball? Give your answer to the nearest tenth.

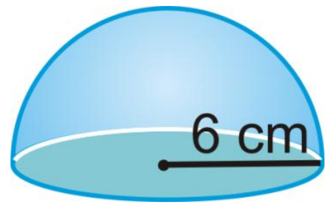
Since the diameter is 24 in then $r = \frac{24}{2} = 12$ in. So the surface area of the sphere is

$4\pi(12)^2 \approx 1809.6 \text{ in}^2$.

This means that 1809.6 in^2 of material would be needed to make the ball.

Do # 3 a, c, p. 51 text in your homework booklet.

E.g.: Find the surface area of the hemisphere to the right (no bottom).



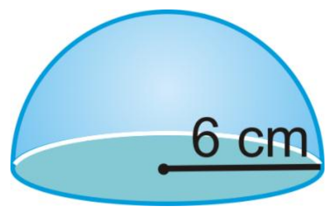
Since a hemisphere is half a sphere, we will find the surface area of the corresponding sphere and divide by 2.

$$\text{SA}_{\text{sphere}} = 4\pi(6)^2 = 144\pi$$

So $\text{SA}_{\text{hemisphere}} = \frac{144\pi}{2} \approx 226.2$

The surface area of the hemisphere is 226.2cm^2.

E.g.: Find the surface area of the hemisphere to the right including the bottom.



In the last example, we found the surface area of the lateral (curved) surface, so we need to add the area of the circular base.

$$\text{A}_{\text{cirle}} = \pi(6)^2 = 36\pi \approx 113.1\text{cm}^2.$$

So the total surface area, including the bottom, is $226.2 + 113.1 = 339.3\text{cm}^2$.

Notice that the area of the lateral surface (226.2) is 2 times the area of the circle (113.1). Why is this?

Do #'s 5 b (SA only, top included), 11 a, p. 51 text in your homework booklet.

Now let's work backwards and find the radius (or diameter) of a sphere given its surface area.

E.g.: If it takes about 168cm^2 of leather to cover a baseball, what is the diameter of the baseball? Give your answer to the nearest hundredth.

Substituting into $\text{SA}_{\text{sphere}} = 4\pi r^2$ gives

$$168 = 4\pi r^2$$
$$\frac{168}{4\pi} = \frac{\cancel{4\pi} r^2}{\cancel{4\pi}}$$
$$r^2 = \frac{168}{4\pi}$$
$$\sqrt{r^2} = \pm\sqrt{\frac{168}{4\pi}}$$
$$r \approx \pm 3.66$$
$$r \approx 3.66$$

So the diameter of the baseball is about $2(3.66) = 7.32\text{cm}$

Do # 8, p. 51 text in your homework booklet.

Finding the Volume Area of a Sphere

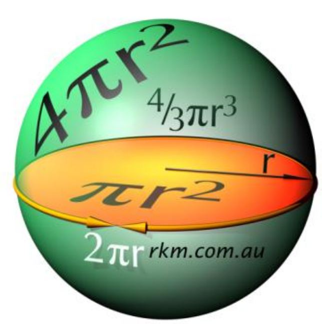
Watch https://www.youtube.com/watch?v=xuPl_8o_j7k to see the how the formula for the volume of a sphere is derived. (See p. 48)

Watch https://learnzillion.com/lessons/1361-develop-and-apply-the-formula-for-volume-of-a-sphere to see the how the formula for the volume of a sphere is derived. (See p. 48, text)

What is the formula for the volume of a sphere?

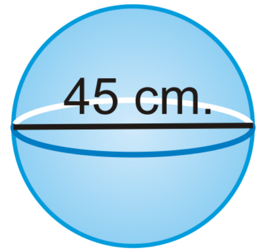
Your Turn

Which is the expression for the volume of the sphere with radius *r*?



a) r
b) $2\pi r$
c) $4\pi r^2$
d) $\frac{4}{3}\pi r^3$

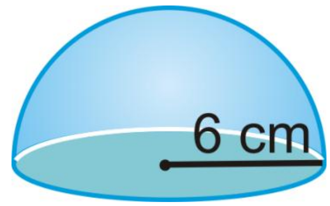
E.g.: Find the volume of the sphere to the right. Give your answer to the nearest hundredth. Recall that volume is given in cubic units.



Since the diameter is 45cm then $r=\frac{45}{2}=22.5\text{cm}$. So the surface area of the sphere is $\frac{4}{3}\pi(22.5)^3\approx 47712.9\text{cm}^3$

Do #'s 19, 24 p. 52 text in your homework booklet.

E.g.: Find the volume of the hemisphere to the right.



Since a hemisphere is half a sphere, we will find the volume of the corresponding sphere and divide by 2.

$$\text{V}_{\text{sphere}}=\frac{4}{3}\pi(6)^3=288\pi$$

So $\text{V}_{\text{hemisphere}}=\frac{288\pi}{2}\approx 452.4$

The surface area of the hemisphere is 452.4cm^3.

Do #'s 5 b (V only), 11 b, p. 51 text in your homework booklet.

§1.7 Solving Problems Involving Objects (2 classes)

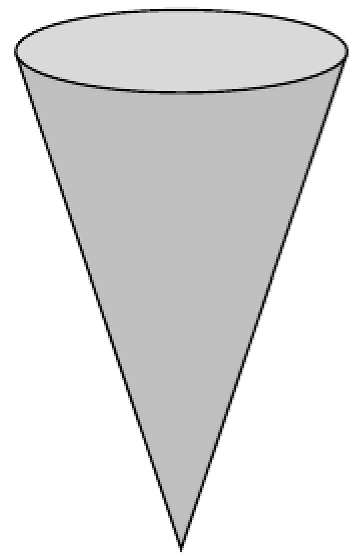
Read *Lesson Focus* p. 55 text.

Outcomes

1. Define a **composite object**. pp. 56, 535
2. Solve a problem that involves surface area or volume, using an object or its labelled diagram of a composite 3-D object. pp. 55-58

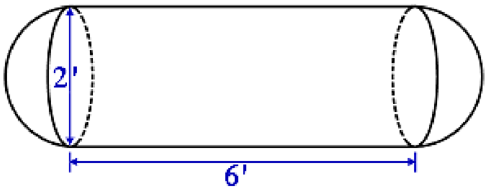
Now you have to put everything together and solve problems involving surface area and volume.

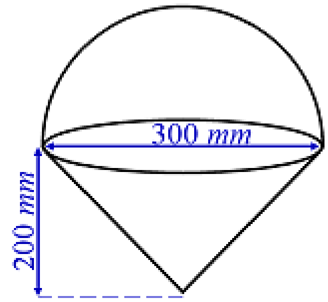
E.g.: Fraser Greening has feed bin that is in the shape of a cone. He wants to paint the outside surface of the bin. Explain how he could determine how many gallons of paint he needs, assuming no wastage and only one coat of paint. One gallon of paints covers about $200\ \text{ft}^2$.

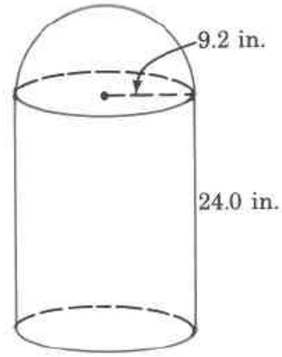


E.g.: Now that Fraser has his feed bin painted, he wants to know how much feed he can put in the bin. Explain how he could determine the capacity of his bin.

Def$^{\underline{n}}$: A **composite object** is an object made up of 2 or more 3-D objects.





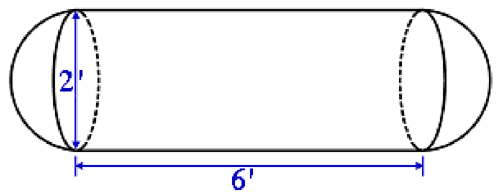


The left composite object is made up of a cylinder and two hemispheres, the middle composite object is made up of a cone and a hemisphere, and the right composite object is made up of a cylinder and a hemisphere.

Finding the Volume of a Composite Object

To find the volume of a composite object, find the volume of each object separately and add the volumes.

E.g.: Find the volume of milk tank to the right.



The ends of the tank are identical hemispheres so placed together they would make a sphere. So we will find the volume of the sphere and the volume of the cylinder and add them.

$$V = V_{sphere} + V_{cylinder}$$

$$V = \frac{4}{3}\pi r^3 + \pi r^2 h$$

$$V = \frac{4}{3}\pi (1)^3 + \pi (1)^2 (6)$$

$$V = \frac{4}{3}\pi + 6\pi$$

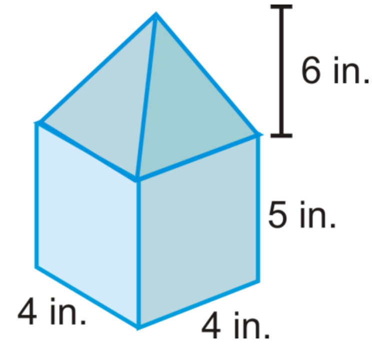
$$V = \frac{4}{3}\pi + \frac{18}{3}\pi$$

$$V = \frac{4\pi}{3} + \frac{18\pi}{3}$$

$$V = \frac{22}{3}\pi \approx 23.0 \text{ ft}^3$$

So the milk tank can hold about 23.0 ft^3.

E.g.: Find the volume of composite object to the right.



The object consists of a square prism and a square pyramid.

$$V = V_{prism} + V_{pyramid}$$

$$V = lwh_1 + \frac{1}{3}lwh_2$$

$$V = (4)(4)(5) + \frac{1}{3}(4)(4)(6)$$

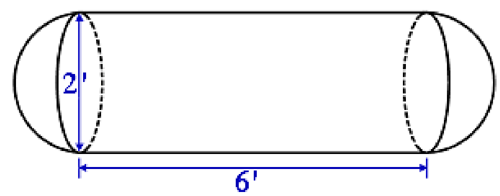
$$V = 80 + 32 = 112 \text{ in}^3$$

Do # 10, p. 60 text in your homework booklet.

Finding the Surface Area of a Composite Object

To find the surface area of a composite object, find the surface area of each object separately and add the surface areas.

Find the surface area of the object to the right.



The object consists of a sphere and a cylinder with no ends.

$$SA = SA_{sphere} + SA_{cylinder \text{ (no ends)}}$$

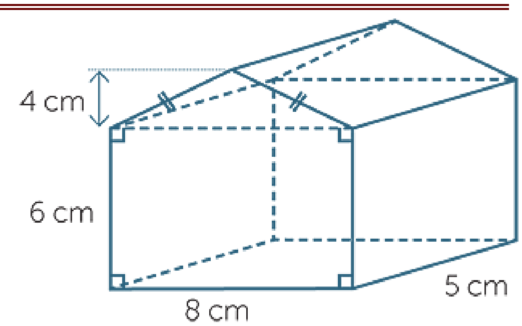
$$SA = 4\pi r^2 + 2\pi rh$$

$$SA = 4\pi (1)^2 + 2\pi (1)(6)$$

$$SA = 4\pi + 12\pi$$

$$SA = 16\pi = 50.27 \text{ ft}^2$$

Find the exterior surface area of the object to the right.



The object consists of a rectangular prism for the base and a rectangular pyramid for the top.

Let's find the surface area of the base first.

Since the front and back rectangles are the same and the left and right rectangles are the same, then

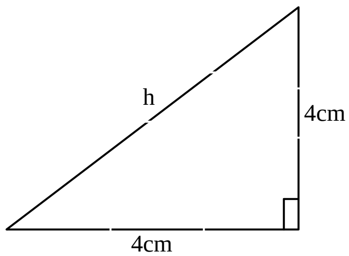
$$SA_{\text{front \& back}} + SA_{\text{left \& right}} = 2(6 \times 8) + 2(5 \times 6) = 156\text{cm}^2 \ldots\ldots 1$$

Now let's find the surface area of the top.

For the triangles on the front and the back we have the base length and the height so we can find those areas using the formula for the area of a triangle. Since both triangles are identical,

$SA_{\text{front \& back }\Delta\text{'s}} = 2\left(\frac{1}{2}\times 8\times 4\right) = 32\text{cm}^2$

Now let's find the surface area of the roof. We know that one dimension of the roof is 5cm but we don't know the length of the sloped part of the roof. We can find this length using the right triangle below and the Pythagorean Theorem.



$(h)^2 = 4^2 + 4^2$

$(h)^2 = 16 + 16$

$(h)^2 = 32$

$\sqrt{(h)^2} = \pm\sqrt{32}$

$h \approx \pm 5.7\text{cm}$

$h \approx 5.7\text{cm}$

Since the left and right slopes of the roof are identical, the surface area of both slopes is

$SA_{\text{front \& back slopes}} = 2(5\times 5.7) \approx 57\text{cm}^2$

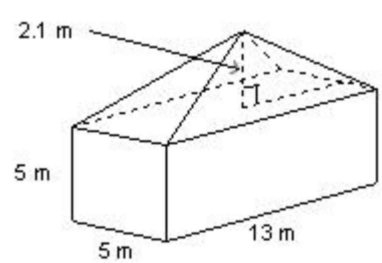
So the total surface area of the roof is $32 + 57 = 89\text{cm}^2$ ……..2

This means that the total surface area is $156 + 89 = 245\text{cm}^2$.

Do #'s 3 c, d; 5, 6 b; pp. 59-60 text in your homework booklet.

Problem Solving with Volume and Surface Area

E.g.: The entire structure to the right is to be painted on the outside. If the paint costs $45/gallon and one gallon of paint covers 40m^2, how much will it cost to buy the paint?



Let's do the easy part first. To find the surface area of the base (rectangular prism), we need to find the area of each side and add these areas.

Since the front and back sides are the same size,

$SA_{\text{front \& back}} = 2(5\times 5) = 50\text{m}^2$

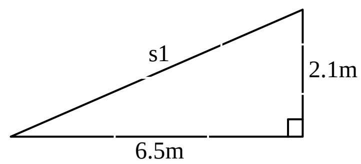
Similarly, the left and right sides are the same size so,

$$SA_{\text{left \& right}} = 2(13\times 5) = 130\text{m}^2$$

So the total surface area of the base is $50+130=180\text{m}^2$ ………1

Now let's do the hard part. There are two different triangles that make up the roof (rectangular pyramid). We need the slant height of each before we can find their areas.

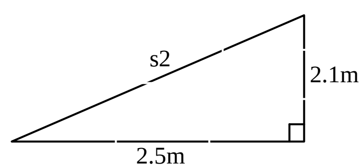
To find the slant height (s1) of for the front and back we will use the right triangle below.



Using the Pythagorean Theorem,

$$(s1)^2 = 6.5^2 + 2.1^2$$
$$(s1)^2 = 42.25 + 4.41$$
$$(s1)^2 = 46.66$$
$$\sqrt{(s1)^2} = \pm\sqrt{46.66}$$
$$s1 \approx 6.8\text{m}$$

To find the slant height (s2) of for the left and right we will use the right triangle below.



Using the Pythagorean Theorem,

$$(s2)^2 = 2.5^2 + 2.1^2$$
$$(s2)^2 = 6.25 + 4.41$$
$$(s2)^2 = 10.66$$
$$\sqrt{(s2)^2} = \pm\sqrt{10.66}$$
$$s2 \approx 3.3\text{m}$$

Now we will find the surface area of the front and back triangles using s1. These triangles are the same size so,

$$SA_{\text{front \& back}} = 2\left(\frac{1}{2}\times 5\times 6.8\right) = 34\text{m}^2$$

Similarly, we will find the surface area of the left and right triangles using s2. These triangles are the same size so,

$$SA_{\text{left \& right}} = 2\left(\frac{1}{2}\times 13\times 3.3\right) = 42.9\text{m}^2$$

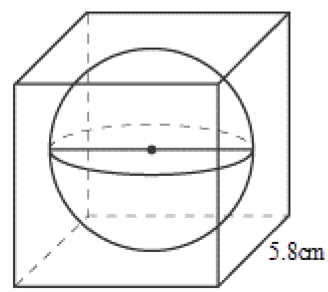
So the total surface area of the roof is $34 + 42.9 = 76.9\text{m}^2 \ldots\ldots\ldots 2$

This means that the total surface area to be painted is $180 + 76.9 = 256.9\text{m}^2$.

We will need $\frac{256.9}{40} \approx 6.4 = 7$ gallons of paint .

So the cost will be $\$45\times 7 = \$315$ before taxes and $\$315\times 1.13 = \$355.95$ after taxes.

E.g.: A solid sphere just fits inside a cube. If the edge of the cube is 5.8cm, what is the volume of air inside the cube to the nearest cubic centimeter?



To find the volume of air inside the cube, we need to find the volume of the cube and subtract the volume of the sphere.

If the edge of the cube is 5.8cm, then the radius of the sphere is $\frac{5.8}{2} = 2.9\text{cm}$.

The volume of air inside the cube is

$$V_{air} = V_{cube} - V_{sphere}$$
$$V_{air} = s^3 - \frac{4}{3}\pi r^3$$
$$V_{air} = (5.8)^3 - \frac{4}{3}\pi (2.9)^3$$
$$V_{air} \approx 195.112 - 102.1604043$$
$$V_{air} \approx 93$$

So the volume of air left inside the cube is 93cm^3.

Do #'s 7, 9, p. 60 text in your homework booklet.
Do #'s 3, 4, 6 a,c, 7 b, 9, 14-16, 19, 20 a, 22, 25, 27; pp. 64-66 text in your homework booklet.

FORMULAE

Surface Area

Cylinder	Cone	Sphere
$2\pi r^2 + 2\pi rh$	$\pi r^2 + \pi rs$	$4\pi r^2$

Volume

Pyramid	Cone	Sphere
$\frac{1}{3}Ah$	$\frac{1}{3}\pi r^2 h$	$\frac{4}{3}\pi r^3$

Conversions

1 foot = 12 inches	1 yard = 3 feet	1 mile = 1760 yards
1 inch $\approx$ 2.54 centimetres $\approx$ 2.5 centimetres		1 mile $\approx$ 1.6 kilometres