CHAPTER 4 Rational Expressions and Equations

Section 4.1: Equivalent Rational Expressions

Rational Expression:

A **rational expression** is any expression that can be written as the quotient of two polynomials, in the form $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$.

Rational Expressions include:

- A fraction that contains a polynomial in the numerator or denominator or both.
- A fraction that contains a variable.
- NOTE: All rational expressions are algebraic fractions, but not all algebraic fractions are rational expressions.

$\frac{x}{1} = x$

For example,

$$\frac{1}{x}, \quad \frac{m}{m+1}, \quad \frac{y^2-1}{y^2+2y+1}$$

$\frac{x^2-1}{1}$ is a rational expression with a denominator of 1.

Example 1:

Which of the following are rational expressions?

$$\frac{2x}{y}, \cancel{\frac{4}{5}}, \frac{x^2-4}{x+1}, \cancel{2\pi}, \frac{x^2}{4}, \cancel{\sqrt{5}}, \frac{\sqrt{x}}{2y}$$

→

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Non - Permissible Values (NPVs):

Values that make the denominator of a rational expression equal zero.

To determine the NPVs:

1) Set the denominator equal to zero.
2) Solve for the variable.

You may need to FACTOR to solve the equation.

All NPVs must be stated as **restrictions** on the variable in order to ensure the expression is defined.

Example 2:

Determine the non permissible values for the expression $\frac{x}{x+2}$ and then state the restrictions.

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$x+2=0$

$\boxed{x=-2}$

$\frac{x}{(-2)+2}=\frac{x}{0}$

Ask yourself, for what values of x will $x + 2 = 0$?

Noooo!!!...

The non permissible value(s) for the expression are: $x=-2$

and the restrictions are: $\frac{x}{x+2}$, $x \neq -2$

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Example 3:

Difference of Squares

What are the non - permissible values of $\frac{x}{x^2-9}$?

$x^2 - 9 = 0$
$x^2 = 9$
$x = \sqrt{9}$
$x = \pm 3$

$\sqrt{x^2} = x$
$\sqrt{9} = 3$

$x^2 - 9 = 0$
$(x+3)(x-3) = 0$
$x+3=0 \quad x-3=0$
$x=-3 \quad x=3$

Example 4:

Determine the non-permissible values for: $\frac{x-1}{3x^2-12x}$

$3x^2 - 12x = 0$
$3x(x-4) = 0$
$3x=0 \quad x-4=0$
$\boxed{x=0 \quad x=4}$

Your Turn: (ex. 3, p. 219)

Determine the NPVs for each rational expression and then state all the restrictions.

a) $\frac{4x^3}{6-2x}$

$6-2x=0$
$\frac{-2x}{-2} = \frac{-6}{-2}$
$\boxed{x=3}$

$\frac{4x^3}{6-2x}, x \neq 3$

b) $\frac{-15}{x^3-4x}$

$x^3 - 4x = 0$
$x(x^2-4) = 0$
$x(x+2)(x-2) = 0$
$x=0 \quad x=-2 \quad x=2$

$\frac{-15}{x^3-4x}, x \neq 0, -2, 2$

→

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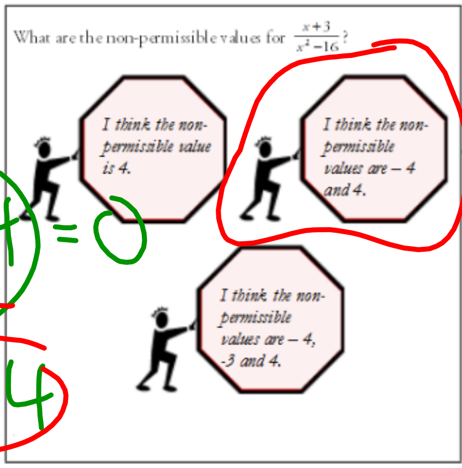
Example 5:



$$\frac{x+3}{x^2-16}$$

$$x^2-16=0$$

$$(x+4)(x-4)=0$$

$$x=-4 \qquad x=4$$

Who is correct? Justify your answer.

NOTE:

Non - permissible values and *inadmissable values* are not the same.

Non - permissible values are values that make the denominator of a rational expression zero.

Inadmissible values are values that do not make sense in a given context. For example, you cannot have a negative length.

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Equivalent Rational Expressions:

Recall:

If we have a rational number and multiply/divide the numerator and denominator by a number (that is, if we multiply/divide the fraction by 1), it does not change the number.

We say the two fractions are ***equivalent rational numbers***.

For example, consider the fraction $\frac{3}{2}$.

$$\frac{3}{2} = \frac{3}{2}\times 1 = \frac{3}{2}\times\frac{4}{4} = \frac{12}{8}$$

We say $\frac{3}{2}$ and $\frac{12}{8}$ are equivalent fractions.

Does this apply to rational expressions as well?

Yes, it is similar but there are some restrictions!

Equivalent Rational Expressions:

Two rational expressions are equivalent only if they have the **same restrictions**.

This is accomplished by:

1. Multiplying or dividing the numerator and denominator by a number.
2. Multiplying the numerator and denominator by a factor that appears in the denominator.

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Example 6:

a) Are $\frac{7x}{x-2}$ and $\frac{14x}{2(x-2)}$ equivalent rational expressions?

$x-2=0$
$x=2$

$\frac{7x}{x-2} \cdot \frac{2}{2}$
$= \frac{14x}{2x-4}$

$2(x-2)=0$
$2x-4=0$
$2x=4$
$x=2$

$\frac{14x}{2(x-2)}$
$= \frac{14x}{2x-4}$

$\frac{7x}{x-2} = \frac{14x}{2(x-2)}, x \neq 2$

b) Are $\frac{7x}{x-2}$ and $\frac{7x^2}{x(x-2)}$ equivalent rational expressions?

$x-2=0$
$x=2$

$x(x-2)=0$
$x=0$
$x-2=0$
$x=2$

NPVs are not the same therefore, not equivalent

Example 7: (ex. 2, p. 218)

a) Write a rational number that is equivalent to $\frac{8}{12}$.

$\frac{8}{12} = \frac{\cancel{2}(4)}{\cancel{2}(6)} = \frac{4}{6}$

b) Write a rational expression that is equivalent to $\frac{4x^2+8x}{4x}$, $x \neq 0$

$\frac{4x^2+8x}{4x} = \frac{\cancel{4x}(x+2)}{\cancel{4x}(1)}$

$= \frac{x+2}{1}$

$= x+2, x \neq 0$

$4x=0$
$x=0$

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Example 8: (ex. 4, p. 220)

For each of the following, determine if the rational expressions are equivalent.

a) $\frac{9}{3x-1}$ *and* $\frac{-18}{2-6x}$ b) $\frac{2-2x}{4x}$ *and* $\frac{x-1}{2x}$

$3x-1=0$
$3x=1$
$x=\frac{1}{3}$

$2-6x=0$
$-6x=-2$
$x=\frac{-2}{-6}$
$x=\frac{1}{3}$

$\frac{9}{3x-1}\cdot\frac{-2}{-2}$

$=\frac{-18}{-6x+2}$

$=\frac{-18}{2-6x}, x\neq\frac{1}{3}$

$\frac{-18}{2-6x}, x\neq\frac{1}{3}$

$4x=0$, $\frac{4x}{4}=\frac{0}{4}$
$x=0$

$2x=0$, $\frac{2x}{2}=\frac{0}{2}$
$x=0$

Use substitution to check
let $x=2$

$\frac{2-2(2)}{4(2)}$
$=\frac{2-4}{8}$
$=\frac{-2}{8}=-\frac{1}{4}$

$\frac{(2)-1}{2(2)}$
$=\frac{1}{4}$

Practice Questions:
Worksheet 4.1,
p. 222 - 223, #9a(i, ii, iii), 11a(i, ii, iii), 16abcd

$\frac{9}{3x-1}=\frac{-18}{2-6x}, x\neq\frac{1}{3}$

$\frac{2-2x}{4x}\neq\frac{x-1}{2x}, x\neq 0$

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Section 4.2: Simplifying Rational Expressions

Simplifying Rational Expressions

The common factors in rational expressions can be reduced in the numerator and denominator to create equivalent rational expressions.

Remember that the simplified expression MUST retain the non-permissible values of the original for both to be equivalent.

Review of Factoring:

The two methods of factoring we will need in this section are:
1) Remove a common factor
2) Completing the square ← Diff of squares

1. $2x^2+4x$

$2x(x+2)$

$2x=0$ $\quad x+2=0$

$x=0$ $\quad x=-2$

2. x^2-16

$(x+4)(x-4)$

$x=-4$ $\quad x=4$

3. $9x^4-15x^3$

$3x^3(3x-5)$

$x=0$

$3x-5=0$

$3x=5$

$x=\frac{5}{3}$

4. $4x^2-36$

$(2x+6)(2x-6)$ $\quad \sqrt{4x^2}=2x$ $\quad \sqrt{36}=6$

$2x+6=0$ $\quad 2x-6=0$

$2x=-6$ $\quad 2x=6$

$x=\frac{-6}{2}$ $\quad \cancel{x}=\frac{6}{2}$

$x=-3$ $\quad x=3$

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M3201 - Section 4.2

Example 1:

Simplify each of the following and state the restrictions.

a) $\frac{x+3}{2x+6}$

$2x+6=0$
$2x=-6$
$x=-3$

$\frac{x+3}{2x+6} = \frac{\cancel{x+3}}{2(\cancel{x+3})}$

$\frac{1}{2}, x \neq -3$

b) $\frac{x-4}{x^2-16}$

$x^2-16=0$
$(x+4)(x-4)=0$
$x=4 \quad x=-4$

$\frac{\cancel{x-4}}{(x+4)(\cancel{x-4})}$

$= \frac{1}{x+4}, x \neq 4, -4$

Example 2: (ex.1, p. 226)

Simplify each of the following and state the restrictions.

a) $\frac{-24a^2}{18a^3}$ $a \neq 0$

$\frac{-24a^2}{18a^3} = \frac{\cancel{6a^2}(-4)}{\cancel{6a^2}(3a)}$

$= -\frac{4}{3a}, a \neq 0$

b) $\frac{12x^3y^2}{9x^4y}$ $x \neq 0, y \neq 0$

$\frac{12x^3y^2}{9x^4y} = \frac{\cancel{3x^3y}(4y)}{\cancel{3x^3y}(3x)}$

$= \frac{4y}{3x}, x \neq 0, y \neq 0$

Example 3: (ex.2, p. 227)

Simplify each of the following and state the restrictions.

a) $\frac{15x^3-5x}{15x^3}$ $x \neq 0$

$\frac{15x^3-5x}{15x^3} = \frac{\cancel{5x}(3x^2-1)}{\cancel{5x}(3x^2)}$

$= \frac{3x^2-1}{3x^2} \quad x \neq 0$

b) $\frac{3x^2-12}{6x+12}$

$6x+12=0$
$6x=-12$
$\boxed{x=-2}$

2

$\frac{3x^2-12}{6x+12}$

$= \frac{\cancel{3}(x^2-4)}{\cancel{3}(2x+4)}, x \neq -2$

$= \frac{x^2-4}{2x+4}, x \neq -2$

$= \frac{(\cancel{x+2})(x-2)}{2(\cancel{x+2})}$

$= \frac{x-2}{2}, x \neq -2$

Example 4:

Identify and correct the errors in the following examples.

a) $\frac{8x-12}{6x^2-4x}, x \neq 0, \frac{2}{3}$

$= \frac{4(2x-3)}{2x(3x-2)}$

$= \frac{4}{2x}(1)$

$= 2x, x \neq 0, \frac{2}{3}$

$\frac{\cancel{2}(4x-6)}{\cancel{2}(3x^2-2x)}$

$= \frac{4x-6}{3x^2-2x}\ x \neq 0, 2/3$

$3x^2-2x=0$

$x(3x-2)=0$

$x=0 \quad x=\frac{2}{3}$

b) $\frac{x^2+1}{x^2-1}$

$= \frac{\cancel{x^2}+1}{\cancel{x^2}-1}$

$= \frac{1}{-1}$

$= -1, x \neq \pm 1$

Can't be simplified

$\frac{x^2+1}{x^2-1}, x \neq -1, 1$

$x^2-1=0$

$(x+1)(x-1)=0$

$x=-1 \quad x=1$

c) $\frac{3}{6x}$

$= \frac{\cancel{3}}{\cancel{6}x}$

$= 2x, x \neq 0$

$\frac{\cancel{3}}{\cancel{3}(2x)}$

$= \frac{1}{2x}, x \neq 0$

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NOTE: $x+5=5+x$

however,
$x-5 \neq 5-x$

$5-x=-x+5$
$5-x=-1(x-5)$

$(x-5)(-1)$
$=(-x+5)(-1)$
$=(5-x)(-1)$
$5-x$

Example 5:

Simplify and state the restrictions.

a) $\dfrac{2x-10}{5x-x^2}$

$5x-x^2=0$
$x(5-x)=0$
$x=0 \quad 5-x=0$
$x=5$

$\dfrac{2x-10}{x(5-x)}$
$=\dfrac{2(x-5)}{-x(-5+x)}$
$=\dfrac{2\cancel{(x-5)}}{-x\cancel{(x-5)}}$
$=-\dfrac{2}{x}$ $x \neq 0,5$

b) $\dfrac{2x^2-18}{12x-4x^2}$

$12x-4x^2=0$
$4x(3-x)=0$
$x=0 \quad 3-x=0$
$x=3$

$=\dfrac{2(x^2-9)}{4x(3-x)}$
$=\dfrac{2(x+3)\cancel{(x-3)}}{-4x\cancel{(x-3)}}$
$=-\dfrac{x+3}{2x}$ $x \neq 3, 0$

Practice Questions:

p. 229 - 231, #2cd, 3abcd, 4ad, 5cd, 7, 13ab

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Section 4.3: Multiplying and Dividing Rational Expressions

Multiplying Rational Expressions

Multiplying rational expressions follows the same procedure as multiplying rational numbers , however you have to determine the non - permissible values for the variables.

Recall: $\frac{3}{4} \times \frac{6}{9} = \frac{18}{36} = \frac{1}{2}$

You can also **reduce first** before you multiply.

When Multiplying Rational Expressions, you should:

1. Factor the numerators and denominators of both expressions, if possible.
2. Identify the non - permissible values.
3. Reduce like factors.
4. Write the product and state the restrictions.

Example 1: (ex. 1, p. 233)

$x=0$ $\quad$ $x-6=0$

$x=6$

Simplify: $\frac{2x^2-12x}{15x} \cdot \frac{5x}{x-6}$

$$= \frac{2x\cancel{(x-6)}}{3\cancel{(5x)}} \cdot \frac{\cancel{5x}}{\cancel{x-6}}$$

$$= \frac{2x}{3}, x \neq 0, 6$$

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Your Turn: Simplify each of the following:

a) $\cancel{\dfrac{40x^2-20x}{18x}\cdot\dfrac{30x}{x-5}}$

b) $\dfrac{18x^3-12x}{5x-15x^2}\cdot\dfrac{1-9x^2}{24x^2}$

$\sqrt{1}=1$ $\sqrt{9x^2}=3x$

$1-3x=0$
$-3x=-1$
$x=1/3$

$$=\frac{\cancel{6x}(3x^2-2)}{5x\cancel{(1-3x)}}\cdot\frac{(1+3x)\cancel{(1-3x)}}{\cancel{6x}(4x)}$$

$$=\frac{3x^2-2}{5x(4x)}\cdot(1+3x)$$

$$=\frac{(3x^2-2)(1+3x)}{5x(4x)}$$

$x\neq 0, -1/3$

Dividing Rational Expressions

The rule for dividing rational expressions is the same as dividing rational numbers,

Multiply by the Reciprocal

Recall: $\frac{3}{4}\div\frac{1}{2}=\frac{3}{4}\times\frac{2}{1}=\frac{6}{4}=\frac{3}{2}$

When Dividing Rational Expressions, you should:

1. Factor the numerators and denominators of both expressions, if possible.
2. Identify the non - permissible values.

   Remember to consider both the numerator and denominator of the second rational expression (divisor) when identifying NPVs.

3. Multiply by the reciprocal.
4. Reduce like factors.
5. Write the quotient and state the restrictions.

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M3201 - Section 4.3

$6+w=0$, $w=-6$ $\quad$ $w-1=0$, $w=1$

Example 2: (ex. 2, p. 234)

Simplify each quotient and state the restrictions.

a) $\dfrac{x-5}{3x^2-9x} \div \dfrac{5}{6x-18}$, $x \neq 0, 3$

$= \dfrac{x-5}{3x(x-3)} \div \dfrac{5}{6(x-3)}$

$= \dfrac{x-5}{3x\cancel{(x-3)}} \times \dfrac{6\cancel{(x-3)}}{5}$

$= \dfrac{6(x-5)}{15x}$, $x \neq 0, 3$

b) $\dfrac{2w}{24w+4w^2} \div \dfrac{6w^2-6w}{9w^3+54w^2}$, $w \neq 0, -6, 1$

$= \dfrac{2w}{4w(6+w)} \div \dfrac{6w(w-1)}{9w^2(w+6)}$

$= \dfrac{2\cancel{w}}{4\cancel{w}\cancel{(6+w)}} \cdot \dfrac{9w^2\cancel{(w+6)}}{{}^3\cancel{6w}(w-1)}$

$= \dfrac{9w}{4(3)(w-1)} = \dfrac{3w}{4(w-1)}$, $w \neq 0, -6, 1$

Your Turn: Simplify each of the following:

$2x+1=0$, $2x=-1$, $x=-\frac{1}{2}$

a) $\dfrac{30x^2+15x}{x-3} \div \dfrac{2x^3+x^2}{x^2-3x}$, $x \neq 0, 3, -\frac{1}{2}$

$= \dfrac{15x(2x+1)}{x-3} \div \dfrac{x^2(2x+1)}{x(x-3)}$

$= \dfrac{15x\cancel{(2x+1)}}{\cancel{x-3}} \cdot \dfrac{\cancel{x-3}}{x\cancel{(2x+1)}}$

$= \dfrac{15x}{x}$, $x \neq 0, 3, -\frac{1}{2}$

$= 15$, $x \neq 0, 3, -\frac{1}{2}$

b) $\dfrac{25-x^2}{3x^2+6x} \div \dfrac{7x-35}{x^2-4}$, $x \neq 0, -2, 2, 5$

$= \dfrac{(5-x)(5+x)}{3x(x+2)} \div \dfrac{7(x-5)}{(x+2)(x-2)}$

$= \dfrac{\cancel{(5-x)}(5+x)}{3x\cancel{(x+2)}} \cdot \dfrac{\cancel{(x+2)}(x-2)}{-7\cancel{(5-x)}}$

$= \dfrac{(5+x)(x-2)}{-21x}$, $x \neq 0, 2, -2, 5$

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M3201 - Section 4.3

Example 3:

Simplify: $\dfrac{\dfrac{x^2-16}{2x^2-10x}}{\dfrac{4x^3+16x^2}{x^2-5x}}$ $, x \neq 0, 5, -4$

$x = 0$

$x = 5$

$x = -4$

$$= \frac{x^2-16}{2x^2-10x} \div \frac{4x^3+16x^2}{x^2-5x}$$

$$= \frac{(x+4)(x-4)}{2x(x-5)} \div \frac{4x^2(x+4)}{x(x-5)}$$

$$= \frac{\cancel{(x+4)}(x-4)}{2x\cancel{(x-5)}} \cdot \frac{x\cancel{(x-5)}}{4x^2\cancel{(x+4)}}$$

$$= \frac{x(x-4)}{8x^3}, x \neq 0, 5, -4$$

Practice Questions:

p. 238 - 239, #1ab, 2bc, 3cd, 4ad, 5b, 6bd, 7bd

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M3201 - Section 4.4

Section 4.4: Adding and Subtracting Rational Expressions

Adding or Subtracting Rational Expressions

Recall: $\frac{1}{3}+\frac{1}{4}$ 12

When adding or subtracting rational numbers, you must get a **lowest common denominator.**

$\frac{1}{3}\cdot\frac{4}{4}+\frac{1}{4}\cdot\frac{3}{3}$

$=\frac{4}{12}+\frac{3}{12}$

$=7/12$

Adding or Subtracting Rational Expressions:

1. Factor the numerators and denominators of both expressions, if possible.
2. Determine the lowest common denominator (LCD).
3. Rewrite each rational expression as an equivalent expression with the LCD as the denominator.
4. Add or subtract the numerators of the equivalent expressions while keeping the denominator the same.
5. Simplify the rational expression and restate the restrictions.

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M3201 - Section 4.4

Example 1: Simplify and state the restrictions.

$$\frac{x-4}{x-2}-\frac{x-10}{x-2}$$

NOTE:
Same Denominator!

$$=\frac{x-4-(x-10)}{x-2}$$

$x-2=0$
$x=2$

$$=\frac{\cancel{x}-4-\cancel{x}+10}{x-2}$$

$$=\frac{6}{x-2},\ x\neq 2$$

YOUR TURN:

Simplify and state the restrictions.

$$\frac{x^2}{x+1}-\frac{1}{x+1}\qquad x\neq -1$$

$$=\frac{x^2-1}{x+1}$$

$\sqrt{x^2}=x$

$\sqrt{1}=1$

$$=\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}}$$

$$=x-1,\ x\neq -1$$

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M3201 - Section 4.4

Example 2: (ex. 2, p. 246)

Simplify and state the restrictions.

$$\frac{3}{8x^2}+\frac{1}{4x}$$ $x \neq 0$

NOTE:
One denominator is a multiply of another.

$$= \frac{3}{8x^2} + \frac{1}{4x} \cdot \frac{2x}{2x}$$

$$= \frac{3}{8x^2} + \frac{2x}{8x^2}$$

$$= \frac{3+2x}{8x^2}, x \neq 0$$

YOUR TURN:

$x+5 \cdot \_\_ = 4x+20$

Simplify and state the restrictions.

$$\frac{3}{x+5}-\frac{1}{4x+20}$$ $x \neq -5$

$x+5=0$

$x=-5$

$$= \frac{3}{x+5} \cdot \frac{4}{4} - \frac{1}{4x+20}$$

$$= \frac{12}{4x+20} - \frac{1}{4x+20}$$

→

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$$= \frac{12-1}{4x+20}$$

$$= \frac{11}{4x+20}, x \neq -5$$

M3201 - Section 4.4

Example 3: (ex. 3, p. 246)

Simplify: $\frac{3n}{2n+1}-\frac{4}{n-3}$ $n \neq -\frac{1}{2}$ or 3

$n-3=0$
$n=3$

$2n+1=0$
$n=-\frac{1}{2}$

NOTE:
Denominators do not have any common factors.

$$= \frac{3n}{2n+1}\cdot\frac{n-3}{n-3} - \frac{4}{n-3}\cdot\frac{2n+1}{2n+1}$$

$$= \frac{3n^2-9n}{(2n+1)(n-3)} - \frac{8n+4}{(n-3)(2n+1)}$$

$$= \frac{3n^2-9n-8n-4}{(2n+1)(n-3)} \longrightarrow = \frac{3n^2-17n-4}{(2n+1)(n-3)},\ n \neq -\frac{1}{2}, 3$$

YOUR TURN:

Simplify and state the restrictions.

$\frac{3}{2x}-\frac{4}{x-1}$, $x \neq 0, 1$

$2x=0$
$x=0$

$x-1=0$
$x=1$

$$= \frac{3}{2x}\cdot\frac{x-1}{x-1} - \frac{4}{x-1}\cdot\frac{2x}{2x}$$

$$= \frac{3x-3}{2x(x-1)} - \frac{8x}{(x-1)(2x)}$$

$$= \frac{3x-8x-3}{2x(x-1)}$$

$$= \frac{-5x-3}{2x(x-1)},\ x \neq 0, 1$$

$$= \frac{-5x-3}{2x^2-2x},\ x \neq 0, 1$$

$\longrightarrow$

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M3201 - Section 4.4

$x-4=0$, $x=4$ $x+4=0$, $x=-4$

Example 4: (ex. 4, p. 247)

Simplify: $\frac{32}{x^2-16}+\frac{4}{x+4}$, $x\neq 4,-4$

NOTE: The denominators have a common factor.

$$=\frac{32}{(x+4)(x-4)}+\frac{4}{x+4}$$

$$=\frac{32}{(x+4)(x-4)}+\frac{4}{x+4}\cdot\frac{x-4}{x-4}$$

$$=\frac{32+4x-16}{(x+4)(x-4)}$$

$$=\frac{4x+16}{(x+4)(x-4)}$$

$$=\frac{4\cancel{(x+4)}}{\cancel{(x+4)}(x-4)}$$

$$=\frac{4}{x-4},\ x\neq 4,-4$$

YOUR TURN:

Simplify and state the restrictions.

$\frac{7}{x^2-9}-\frac{1}{4x+12}$, $x\neq -3,3$

$$=\frac{7}{(x+3)(x-3)}-\frac{1}{4(x+3)}$$

$$=\frac{7}{(x+3)(x-3)}\cdot\frac{4}{4}-\frac{1}{4(x+3)}\cdot\frac{x-3}{x-3}$$

~~Practice Questions:~~

~~p. 249 - 250, #1bd, 3bc, 4ab, 5bc, 6ac, 7b, 8a, 9~~

$$=\frac{28}{4(x+3)(x-3)}-\frac{x-3}{4(x+3)(x-3)}$$

$$=\frac{28-x+3}{4(x+3)(x-3)}$$

$$=\frac{31-x}{4(x+3)(x-3)},\ x\neq -3,3$$

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M3201 - Section 4.5

Section 4.5 - Solving Rational Equations

Rational Equation

- an equation that contains at least one rational expression.

For example: $x = \frac{x-3}{x+1}$ and $\frac{x}{4} - \frac{7}{x} = 3$

To Solve a Rational Equation:

1. **Factor each denominator**
2. Identify the non - permissible values
3. ***Method 1:*** Multiply both sides of the equation by the LCD
   OR
   Method 2: Add/Subtract fractions by obtaining LCD to get a single fraction on both sides of the equation and then equate numerators.
4. Solve the resulting linear or quadratic equation
5. Check your answers for **extraneous** roots

Example 1:

$4 \cdot x = 4x$

Solve: $\frac{x}{4} - \frac{7}{x} = 3$, $x \neq 0$

Note:

Non - permissible values are identified from the original equation.

$\frac{x}{\cancel{4}} \cdot \cancel{4}x - \frac{7}{\cancel{x}} \cdot 4\cancel{x} = 3 \cdot 4x$

$x^2 - 28 = 12x$

$x^2 - 12x - 28 = 0$

$(x-14)(x+2) = 0$

$x - 14 = 0$ $\quad$ $x + 2 = 0$

$x = 14$ $\quad$ $x = -2$

$\underline{2} + (\underline{-14}) = -12$

$\underline{2} \cdot \underline{-14} = -28$

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Example 2: (ex. 2, p. 254)

Solve for x: $\frac{18}{x^2-3x} = \frac{6}{x-3} - \frac{5}{x}$

$x \neq 0, 3$

Extraneous Root, No solution

$$\frac{18}{x(x-3)} = \frac{6}{x-3} - \frac{5}{x}$$

$$\frac{18}{\cancel{x(x-3)}} \cdot \cancel{x(x-3)} = \frac{6}{\cancel{x-3}} \cdot x\cancel{(x-3)} - \frac{5}{\cancel{x}} \cdot \cancel{x}(x-3)$$

$18 = 6x - (5x - 15)$

$18 = 6x - 5x + 15$

$18 - 15 = x$

$3 = x$

$x = 3$

Example 4:

Solve: $\frac{3x-5}{x^2+4x+3} + \frac{2x+2}{x+3} = \frac{x-3}{x+1}$

$$\frac{3x-5}{(x+3)(x+1)} + \frac{2x+2}{x+3} = \frac{x-3}{x+1}$$

$$\frac{3x-5}{\cancel{(x+3)(x+1)}} \cdot \cancel{(x+3)(x+1)} + \frac{2x+2}{\cancel{x+3}} \cdot \cancel{(x+3)}(x+1) = \frac{x-3}{\cancel{x+1}} \cdot (x+3)\cancel{(x+1)}$$

$3x - 5 + (2x+2)(x+1) = (x-3)(x+3)$

$3x - 5 + 2x^2 + 2x + 2x + 2 = x^2 + 3x - 3x - 9$

2

$2x^2 + 7x - 3 = x^2 - 9$

$2x^2 - x^2 + 7x - 3 + 9 = 0$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$\boxed{x = -6}$ $\boxed{x = -1}$

$\underline{6} + \underline{1} = 7$

$\underline{6} \cdot \underline{1} = 6$

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Besides factoring, we may have to use the Quadratic Formula to solve for the variable in a trinomial.

Recall:

Quadratic Formula:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example 3:

A B C

Solve for x: $3x^2 + 4x - 6 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-6)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{16 - (-72)}}{6}$$

$$= \frac{-4 \pm \sqrt{16 + 72}}{6}$$

$$= \frac{-4 \pm \sqrt{88}}{6}$$

3

$$\frac{-4 + \sqrt{88}}{6} \qquad\qquad \frac{-4 - \sqrt{88}}{6}$$

$$\boxed{x = 0.897} \qquad\qquad \boxed{x = -2.23}$$

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Your Turn:

1. Solve for x: $\frac{2}{a+2}-\frac{a^2+4}{a^2-4}=\frac{a}{2-a}$ $a \neq -2, 2$

$$\frac{2}{a+2}-\frac{a^2+4}{(a+2)(a-2)}=\frac{a}{-1(a-2)}$$

$$\frac{2}{\cancel{a+2}}\cdot\cancel{(a+2)}(a-2)-\frac{a^2+4}{\cancel{(a+2)}\cancel{(a-2)}}\cancel{(a+2)}\cancel{(a-2)}=\frac{-a}{\cancel{a-2}}\cdot(a+2)\cancel{(a-2)}$$

$$2(a-2)-a^2-4=-a(a+2)$$

$$2a-4-a^2\ 4=-a^2-2a$$

$$2a+2a-4-a^2+\cancel{a^2}-4=0$$

Practice Questions:

p. 258, Section 4.5 Worksheet

$$4a=-8$$

$$\boxed{a=-2}$$ ← Extraneous Root!

No solution

4

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Part 2: Word Problems

Example 1:

The sum of a number and its reciprocal is $\frac{5}{2}$. Determine the number.

$\frac{1}{2} \Rightarrow \frac{2}{1}$

$+ \quad x \quad \frac{1}{x}$

$x + \frac{1}{x} = \frac{5}{2}, x \neq 0$

$x \cdot 2x + \frac{1}{\cancel{x}} \cdot 2\cancel{x} = \frac{5}{\cancel{2}} \cdot \cancel{2}x$

$2x^2 + 2 = 5x$

$2x^2 - 5x + 2 = 0$

$(2x-1)(x-2) = 0$

factors

$2x - 1 = 0$ $\quad x - 2 = 0$

$x = \frac{1}{2}$ $\quad \boxed{x = 2}$

Example 2:

One positive integer is 5 more than the other. When the reciprocal of the larger number is subtracted from the reciprocal of the smaller the result is $\frac{5}{14}$. Find the two integers.

$x \quad x+5$

$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14}$

$\frac{1}{\cancel{x}} \cdot \cancel{x}(x+5)(14) - \frac{1}{\cancel{x+5}} \cdot x\cancel{(x+5)}(14) = \frac{5}{\cancel{14}} \cdot x(x+5)\cancel{14}$

5

$\cancel{14x} + 70 - \cancel{14x} = 5x(x+5)$

$70 = 5x^2 + 25x$

$\frac{0}{5} = \frac{5x^2}{5} + \frac{25x}{5} - \frac{70}{5}$ $\quad (-2) + 7 = 5$

$0 = x^2 + 5x - 14$ $\quad (-2) \cdot 7 = -14$

$0 = (x-2)(x+7)$

$\boxed{x = 2}$ $\quad \cancel{x = -7}$ ← Must be positive

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Example 3:

Sherry can mow a lawn in 5 hours. Terry can mow the same lawn in 4 hours. Determine how long it would take to mow the lawn if Sherry and Terry worked together.

	Time to mow lawn	Fraction of lawn mowed in 1 hour
Sherry	5	$1/5$
Terry	4	$1/4$
Together	x	$1/x$

$$\frac{1}{5} + \frac{1}{4} = \frac{1}{x}$$

$$\frac{1}{\cancel{5}} \cdot \cancel{5}(4)x + \frac{1}{\cancel{4}} \cdot 5\cancel{(4)}(x) = \frac{1}{\cancel{x}} \cdot 5(4)\cancel{x}$$

$$4x + 5x = 20$$

$$\frac{\cancel{9}x}{\cancel{9}} = \frac{20}{9}$$

$$x = \frac{20}{9} = \boxed{2.22 \text{ hours}}$$

→

6

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Example 4:

Gerard takes 5 hours longer than Hubert to assemble a play set. If Gerard and Hubert worked together, they could assemble the play set in 6 hours. Determine how long it takes each person to assemble the play set if they worked alone.

completed

	Time	Fraction ~~of time~~ in 1 hour
Gerard	$x+5$	$\frac{1}{x+5}$
Hubert	x	$\frac{1}{x}$
Together	6	$\frac{1}{6}$

$$\frac{1}{x+5}+\frac{1}{x}=\frac{1}{6}$$

$$\frac{1}{\cancel{x+5}}\cdot x\cancel{(x+5)}6+\frac{1}{\cancel{x}}\cdot \cancel{x}(x+5)6=\frac{1}{\cancel{6}}\cdot x(x+5)\cancel{6}$$

$$6x+6x+30=x^2+5x$$

7

$$0=x^2+5x-12x-30$$

$$0=x^2-7x-30 \qquad \underline{-10}+\underline{3}=-7$$

$$0=(x-10)(x+3) \qquad \underline{-10}\cdot\underline{3}=-30$$

$\boxed{x=10}$ $\quad \cancel{x=-3}$

Hubert : 10 hours

Gerard : $x+5$

$=10+5$

$=15$ hours

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Example 5:

A skiing club is going on a skiing trip that costs $1500 total for bussing. If 10 non-members are allowed to go, the price per person drops by $5. If x represents the number of members and the situation is modelled by

$$\frac{1500}{x} - \frac{1500}{x+10} = 5 \quad , x \neq 0, -10$$

algebraically determine how many members there are.

$$\frac{1500}{\cancel{x}} \cdot \cancel{x}(x+10) - \frac{1500}{\cancel{x+10}} \cdot x\cancel{(x+10)} = 5 \cdot (x)(x+10)$$

$$\cancel{1500x} + 15000 - \cancel{1500x} = 5(x^2+10x)$$

$$15000 = 5x^2 + 50x$$

$$\frac{0}{5} = \frac{\cancel{5}x^2}{\cancel{5}} + \frac{50x}{5} - \frac{15000}{5}$$

$$0 = x^2 + 10x - 3000$$

$$0 = (x-50)(x+60)$$

$$\boxed{x = 50} \quad \cancel{x = -60}$$

Inadmissable

$\underline{60} + (\underline{-50}) = 10$

$\underline{60}(\underline{-50}) = -3000$

8

50 members!

Example 6:

Priddle Inc. is having a Christmas party for all of its employees. Initially, all employees agree to attend. The cost of the catering is $1800, which is to be divided amongst all people who attend the party. At the last minute, 30 people decide not to come, increasing the cost per person by $2. If x represents the number of employees and the situation is modelled by

$$\frac{1800}{x-30}-\frac{1800}{x}=2$$

algebraically determine the number of people who are employed at Priddle Inc.

$$\frac{1800}{\cancel{x-30}}\cdot x\cancel{(x-30)} - \frac{1800}{\cancel{x}}\cdot \cancel{x}(x-30) = 2(x)(x-30)$$

$$180\cancel{0}x - 18\cancel{00}x - 54\,000 = 2x^2-60x$$

$$0 = 2x^2-60x-54000$$

$$0 = x^2-30x-27\,000$$

Practice Questions:

p. 259, #10,11,12 + Worksheet

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{30 \pm \sqrt{900-4(1)(27\,000)}}{2(1)}$$

$$= \frac{30 \pm \sqrt{108\,900}}{2}$$

$$= \frac{30 \pm 330}{2}$$

$$= \frac{30+330}{2} \qquad \frac{30-330}{2}$$

$$x = \frac{360}{2} = \boxed{180} \qquad = \frac{-300}{2} = -\cancel{150}$$