CHAPTER 3 Probability

Section 3.1: Exploring Probability / Section 3.2: Probability and Odds

Statements of probability and odds are referenced often in various media in many fields including political polls, sports and social statistics.

Have you noticed?!

But do you know the difference between probability and odds?

Review of Probability

Event: collection of outcomes that satisfy a specific condition.

Ex: When throwing a standard die, the event "throwing an odd #" is the collection of outcomes 1, 3, 5 or 3 ways.

Probability of an event: the ratio of favourable outcomes to the total possible outcomes. (part:whole)

$$\boxed{P(event) = \frac{favourable}{total} = \frac{part}{whole}}$$

For example, the probability of throwing an odd number on a standard die is:

$$P(odd) = \frac{favourable}{total} = \frac{3}{6} = \frac{1}{2}$$

Probability can be expressed as a fraction, a ratio, a decimal, or a percent.

$P(odd) = \frac{1}{2}$ or $P(odd) = 1:2$ or $P(odd) = 0.5$ or $P(odd) = 50\%$

NOTE:

The probability of an event can range from 0 (impossible) to 1 (certain) or 0 to 100%.

Experimental Probability VS. Theoretical Probability

Experimental probability of event A: $P(A) = \frac{n(A)}{n(T)}$

where $n(A)$ is the number of times event A occurred

and $n(T)$ is the total number of trials, T, in the experiment

Theoretical probability of event A: $P(A) = \frac{n(A)}{n(S)} = \frac{fav}{total} = \frac{part}{whole}$

where $n(A)$ is the number of favourable outcomes for event A

and $n(S)$ is the total number of outcomes in the sample space, S, where all outcomes are equally likely

Probability VS. Odds

Consider choosing a heart from a deck of cards.

Prob → P(heart) = $\frac{13}{52} = \frac{1}{4}$ or $1:4$

When discussing odds, we need to be specific!

It is not enough to ask, "what are the odds?" We need to ask:

"What are the *odds in favour*?" or "What are the *odds against*?"

Odds in favour: the ratio of favourable outcomes to unfavourable outcomes. (fav:unfav) (part:part)

Odds in favour of choosing a heart from a deck of cards is:

$\frac{13}{39} = \frac{1}{3}$ or $1:3$

Odds against: the ratio of unfavourable outcomes to favourable outcomes. (unfav:fav) (part:part)

Odds against choosing a heart from a deck of cards is:

$\frac{39}{13} = \frac{3}{1}$ or $3:1$

NOTE:

The formula for the odds against is the reciprocal of the formula for finding odds in favour of an event.

Example 1:

Identify the following as odds or probability.

a) The chances of rolling a 1 on a fair six-sided die is $\frac{1}{6}$

Prob

b) The chances of drawing a 4 from a standard 52-card deck is 1:12.

A, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

Odds

Example 2:

a) The odds of winning a contest are 5:9. What is the probability of winning the contest?

$\frac{5}{9}$ $P = \frac{fav}{total}$ fav unfav

$= \frac{5}{5+9} = \frac{5}{14}$ or 5:14

b) The probability of you passing the next math test is 75%. What are the odds of you passing?

$P = \frac{3}{4}$ $\frac{3}{4}$

Odds in favor: $\frac{3}{1}$ or 3:1

c) A jar contains 3 red marbles and some green marbles. The odds are 3:1 that a randomly chosen marble is green. How many green marbles are in the jar?

$$\text{Odds}(g) = \frac{3}{1} \times \frac{3}{3} = \frac{9}{3}$$

← green

↑ red

9 green marbles

NOTE:

All odds and probability calculations begin with 2 of 3 values:

total possibilities, favourable outcomes and non-favourable outcomes,

and we have to determine the third.

In general,

- If the odds in favour of an event is a:b, then the total number of possibilities is a+b.
- Therefore, the probability of an event occurring is a:a+b.
- The odds against an event is b:a.
- The probability of an event not happening is b:a+b. (complement)

Total: 13

Example 3: (ex.1, p. 143)

Bailey holds all the hearts from a standard deck of 52 playing cards. He asks Morgan to choose a single card without looking.

Determine the odds in favour of Morgan choosing a face card.

$F = \{J, Q, K\}$ $n(F) = 3$

$F' = \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $n(F') = 10$

Odds in favor: $\frac{n(F)}{n(F')} = \frac{3}{10}$ or 3:10

Example 4: (ex. 2, p. 144)

Research shows that the probability of an expectant mother, selected at random, have twins is $\frac{1}{32}$.

a) What are the odds in favour of an expectant mother having twins?

$P(\text{twins}) = \frac{1}{32}$

$P(\text{not twins}) = \frac{31}{32}$

Odds in favor $\frac{1}{31}$ or $1:31$

b) What are the odds against an expectant mother have twins?

Odds against $\frac{31}{1}$ or $31:1$

Example 5: (ex. 3, p. 144)

A computer randomly selects a university student's name from the university database to award a $100 gift certificate for the bookstore. The odds against the selected student being male are 57:43. Determine the probability that the randomly selected university student will be male.

Total $57 + 43 = 100$

$P(\text{male}) = \frac{43}{100}$ or 43%

57 : 43

female male

Example 6: (ex. 4, p. 145)

A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Ellen or Brittany should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the players' shootout records. Who should go first?

Odds in favor

Player	Attempts	Goals Scored
Ellen	13	8
Brittany	17	10

$\frac{8}{13} = 62\%$

8:5

10:7

$P(E) = \frac{10}{17} = 59\%$

Ellen should go first

Example 7: (ex. 5, p. 146)

A group of Grade 12 students are holding a charity carnival to support a local animal shelter. The students have created a dice game that they call Bim and a card game that they call Zap. The odds against winning Bim are 5:2, and the odds against winning Zap are 7:3. Which game should Madison play?

Bim

Total $5+2=7$

$P(win)\ \frac{2}{7} = 0.285$

Zap

Total $7+3=10$

$P(win) = \frac{3}{10} = 0.3$

Practice Questions:

p. 148-150, # 1, 2, 3, 5, 6, 7, 9, 10,12, 14, 17

Section 3.3: Probabilities Using Counting Methods

In the previous unit we solved problems using the Fundamental Counting Principle, permutations and combinations.

Now we will use these counting techniques to solve probability problems.

Remember: Probability $= \dfrac{favourable}{total}$

Example 1:

A survey was conducted of 500 adults who wore Halloween costumes to a party. Each person was asked how he/she acquired the costume:

- 360 adults created their costumes
- 60 adults rented their costumes
- 60 adults bought their costumes
- 20 adults borrowed costumes

What is the probability that the first four people who were polled all created their costumes?

What are the total number of possible outcomes?

$_{500}C_4 = \cancel{73\,515\,175}\ 25\,7303\,1125$

What are the number of favourable outcomes?

$_{360}C_4 = \cancel{19\,663\,866}\ 688\,235\,310$

Probability $= \dfrac{fav}{tot} = \dfrac{_{360}C_4}{_{500}C_4} = 0.267 \rightarrow$

or

26.7%

Example 2:

There are ~~7~~ 10 teachers and 3 administrators at a conference. Find the probability of 3 different door prizes being awarded to teachers only.

Probability = $\frac{fav}{tot} = \frac{_7C_3}{_{10}C_3} = \frac{35}{120} = 0.292$ or 29.2%

Example 3:

A 4-digit number is generated at random from the digits 2, 3, 5 and 7 (without repetition of the digit), what is the probability that it will be even?

□ □ □ □

Total # of outcomes: $_4P_4 = 24$

Total # of favourable outcomes: $_4P_1 = 4$

Probability = $\frac{4}{24} = \frac{1}{6}$ or 0.167 or $\boxed{16.7\%}$

Example 4:

If a 4-digit PIN number can begin with any digit, except zero, and the remaining digits have no restriction. If repeated digits are allowed, determine the probability of the PIN code beginning with a number greater than 7 and ending with a 3.

$\underline{9} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$

Total # of outcomes: $= 9000$

Total # of favourable outcomes: $\underline{2} \cdot \underline{10} \cdot \underline{10} \cdot \underline{1}$ $= 200$

Probability = $\frac{200}{9000} = \frac{1}{45}$

Example 5:

Mark, Abby and 5 other students are standing in a line.

a) Determine the probability Mark and Abby are standing together.

Fav: $\frac{{}_6P_6}{{}_2P_2} = \frac{6!}{2!} = 360$ Prob: $\frac{\text{fav}}{\text{tot}} = \frac{360}{5040}$

Total: ${}_7P_7 = 7! = 5040$

$= 0.071$

$\simeq 7\%$

b) Determine the probability Mark and Abby are not standing together.

$5040 - 360 = 4680$

Prob: $\frac{4680}{5040} = 0.928 \simeq 93\%$

Example 6:

A bookcase contains 6 different math books and 12 different biology books. If a student randomly selects two of these books, determine the probability they are both math or both biology books.

$\frac{\text{fav}}{\text{total}} \quad \frac{{}_6C_2 + {}_{12}C_2}{{}_{18}C_2} = \frac{15+66}{153} = \frac{81}{153} \simeq 53\%$

Example 7:

A jar contains 5 red, 7 blue and 5 purple candies. 3 green If the total number of candies is 20, determine the probability that a handful of four candies contains one of each colour.

$\frac{\text{Fav}}{\text{Total}} = \frac{{}_5C_1 \cdot {}_7C_1 \cdot {}_5C_1 \cdot {}_3C_1}{{}_{20}C_4} = \frac{5 \cdot 7 \cdot 5 \cdot 3}{4845} \rightarrow$

$= 0.108$

$\simeq 11\%$

Example 8: (ex. 1, p. 152)

Jamaal, Ethan and Alberto are competing with seven other boys to be on their school's cross-country team. All the boys have an equal chance of winning the trial race. Determine the probability that Jamaal, Ethan and Alberto will place first, second, and third, in any order.

Total # of outcomes: ${}_{10}P_3 = 720$

Total # of favourable outcomes: ${}_3P_3 = 6$

Probability = $\frac{6}{720} = \frac{1}{120} = 0.83\%$

Example 9: (ex. 2, p. 154)

About 20 years after they graduated from high school, Blake, Mario and Simon met in a mall. Blake had two daughters with him, and he said he had three other children at home. Determine the probability that at least one of Blake's children is a boy.

Total $= \underline{2} \times \underline{2} \times \underline{2}$
$= 8$

Girls $= \underline{1} \times \underline{1} \times \underline{1}$
$= 1$

$P(\text{all girls}) = \frac{1}{8}$

$P(\text{at least 1 boy}) = 1 - \frac{1}{8}$
$\boxed{= \frac{7}{8}}$

Example 10: (ex. 3, p. 156)

Bob hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up.

He asks Sally to arrange the tiles in a row and turn them face down. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

12 letters, 2 S's, 3 A's

Total: $\dfrac{12!}{2!\,3!} = 39\,916\,800$

S's → 2!   3! ← A's

$P(\text{winning}) = \dfrac{\text{fav}}{\text{tot}} = \dfrac{1}{39\,916\,800}$

Practice Questions:

p. 159 - 161, # 1, 2, 3, 5, 10, 11, 15, 16

Section 3.4: Mutually Exclusive Events

Recall from Unit 1 we classified events as:

mutually exclusive (disjoint sets) and non-mutually exclusive.

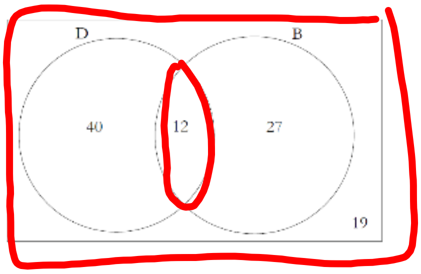
Mutually exclusive sets did not intersect.

(i.e, $n(A \cap B) = 0$)

Recall also that the outcomes of an event and the outcomes of the complement make up the entire sample space.

We will now solve probability problems involving mutually exclusive and non-mutually exclusive events.

Let's look at an example of a Venn diagram where D represents students on the debate team, and B represents students on the Basketball team.



In order to determine the probability of events, we have to think about the following questions:

- **Are the two sets intersecting or disjoint?**
- **How many elements are in each set?**
- **How many elements are in the universal set *S*?**

Using the Principle of Inclusion and Exclusion, we can develop the probability formula for non-mutually exclusive events:

$$n(D \cup B) = n(D) + n(B) - n(D \cap B)$$

$$P(D \cup B) = \frac{n(D) + n(B) - n(D \cap B)}{n(S)}$$

$$P(D \cup B) = \frac{n(D)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(D \cap B)}{n(S)}$$

$$\boxed{P(D \cup B) = P(D) + P(B) - P(D \cap B)}$$

Recall if events are mutually exclusive, the sets are disjoint, so $n(D \cap B) = 0$.

Therefore, $P(D \cup B) = P(D) + P(B)$

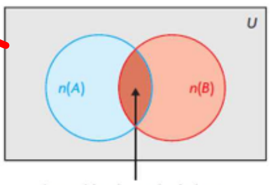
To summarize:

If A and B are non-mutually exclusive events,

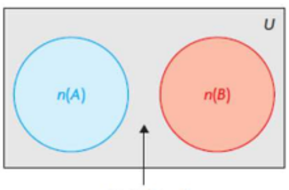
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events,

$$P(A \cup B) = P(A) + P(B)$$



$n(A \cap B)$ has been shaded twice



$n(A \cap B) = 0$
(no common elements)

Example 1:

Classify the events in each experiment as either mutually exclusive or non-mutually exclusive:

a) The experiment is rolling a die. The first event is rolling an even number and the second event is rolling a prime number.

Mutually exclusive

b) The experiment is playing a game of hockey. The first event is that your team scores a goal, and the second event is that your team wins the game.

non ME

c) The experiment is selecting a gift. The first event is that the gift is edible and the second event is that the gift is an iPhone.

ME

NOTE:
The sum of the probability of an event and its complement must equal 1.

$$\boxed{P(A) + P(A') = 1}$$

Rearranging the formula would give us:

$P(A') = 1 - P(A)$ and $P(A) = 1 - P(A')$

For example,
If the probability that a student picks the ace of diamonds from a standard deck of cards is: $\frac{1}{52}$

then the probability that he/she will not pick the ace of diamonds is: $\frac{51}{52}$ $\left(\frac{1}{52} + \frac{51}{52} = 1\right)$

Example 2:

Determine if the events are mutually exclusive or non-mutually exclusive and display the information in a Venn diagram.

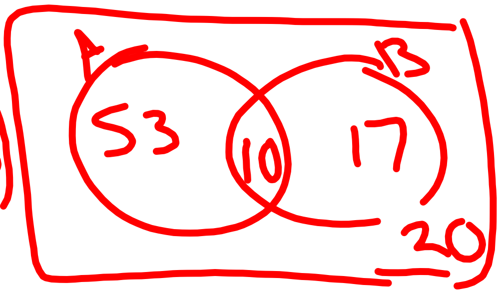
$P(A)$ →
$P(B)$ →

Class Survey
63% of students play sports
27% of the students play a musical instrument
20% play neither sports nor a musical instrument

$63 + 27 + 20 = 110$

$P(A \cup B) = 100 - 20 = 80\%$

$P(A) + P(B) + P(A \cap B) = 53 + 17 + 10 = 80\%$



Example 3: *(ex. 2, p. 168)*

Jack and Ellen are playing a board game. If a player rolls a sum that is greater than 8 or a multiple of 5 when using 2 standard dice, the player gets a bonus of 100 points. Determine the probability that Ellen will receive a bonus of 100 points on her next roll.

Possible Sums When a Pair of Dice are Rolled						
Die 1/ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Greater than 8

Multiple of 5

$P(A) = \frac{10}{36}$

5

$P(B) = \frac{7}{36}$

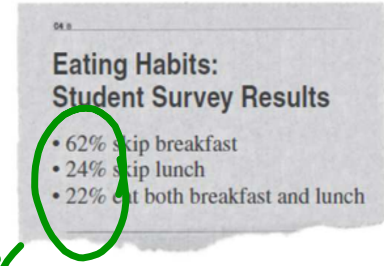
$P(A \cup B) = \frac{14}{36}$

$= 0.388 \rightarrow$

$\approx 39\%$

Example 4: (ex. 3, p. 170)

A school newspaper published the results of a recent survey.



Eating Habits:
Student Survey Results

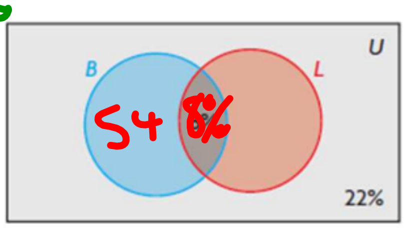
- 62% skip breakfast
- 24% skip lunch
- 22% eat both breakfast and lunch

a) Are skipping breakfast and skipping lunch mutually exclusive events?

$62 + 24 + 22 = 108\%$

8% counted twice

NOT mutually exclusive



b) Determine the probability that a randomly selected student skips breakfast but not lunch.

$$P(B \setminus L) = P(B) - P(B \cap L)$$
$$= 62 - 8$$
$$= 54\%$$

c) Determine the probability that a randomly selected student skips at least one of breakfast or lunch.

$$P(B \cup L) = P(B) + P(L) - P(B \cap L)$$
$$= 62 + 24 - 8$$
$$= 78\%$$

→

Example 6: (ex. 5, p. 174)

A car manufacturer keeps a database of all the cars that are available for sale at all the dealerships in Eastern Canada. For model A, the database reports that 43% have heated leather seats, 36% have a sunroof, and 49% have neither. Determine the probability of a model A car at a dealership having both leather seats and a sunroof.

$P(L \cup S) = 100 - 49$
$= 51\%$

$P(L \cup S) = P(L) + P(S) - P(L \cap S)$

$51 = 43 + 36 - P(L \cap S)$

$P(L \cap S) = 28$

Example 7:

The probability that Dana will make the hockey team is 2/3. The probability that she will make the swimming team is 3/4. If the probability of Dana making both teams is 1/2, determine the probability that she will make:

a) at least one of the teams.

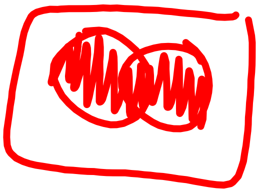
b) neither team.

a) $P(H \cup S) = P(H) + P(S) - P(H \cap S)$

$= \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$

$= \frac{8}{12} + \frac{9}{12} - \frac{6}{12}$

$\boxed{= \frac{11}{12}}$



b)

$1 - \frac{11}{12}$

$= \frac{1}{12}$

Practice Questions:

P. 176 - 180, # 3, 4, 5, 6, 7, 8, 12, 15

Section 3.5: Conditional Probability (complete after Section 3.6)

In the previous section we determined the probability of 2 independent events by multiplying their individual probabilities.

We will determine the probability of 2 dependent events in a similiar way.

Dependent events:

Events whose outcomes are affected by each other.

Ex: 2 cards drawn from a deck, without replacement.

Conditional probability: P(B|A)

The probability of an event, B, occurring, given that another event, A, has already occurred.

Dependent

NOTE: P(B|A) is NOT the same as B minus A, B\A.

$$\boxed{P(A \text{ and } B) = P(A) \times P(B|A)}$$

Using set notation, the formula is: $P(A \cap B) = P(A) \times P(B|A)$

Rearranging the formula for P(B|A) would give:

$$\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$$

→

Dependent

Example 1:

Cards are drawn from a standard deck of 52 cards (without replacement).

Calculate the probability of obtaining:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

a) a king, then another king

$$P(A) = \frac{1}{13} \text{ or } \frac{4}{52}$$

$$P(B|A) = \frac{3}{51}$$

$$= \frac{4}{52} \cdot \frac{3}{51}$$

$$= 0.0045$$

$$= 0.45\%$$

b) a club, then a heart

$$P(A) = \frac{1}{4} \text{ or } \frac{13}{52}$$

$$P(B) = \frac{13}{51}$$

$$P(A \cap B) = \frac{13}{52} \cdot \frac{13}{51}$$

$$= 0.0637$$

$$= 6.4\%$$

c) a black card, then a heart, then a diamond

$$\frac{26}{52} \cdot \frac{13}{51} \cdot \frac{13}{50} = \frac{4394}{132600} = 0.033$$

$$= 3.3\%$$

Example 2:

A computer manufacturer knows that, in a box of 100 chips, 3 will be defective. If Jocelyn draws 2 chips, at random, from a box of 100 chips, what is the probability that both of the chips will be defective?

$$\frac{3}{100} \cdot \frac{2}{99} = \frac{6}{990} = 0.006$$

$$= 0.6\%$$

Example 3:

A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

$$P(B \cap W) = P(B) \cdot P(W|B)$$

$$\frac{0.34}{0.47} = \frac{0.47 \cdot P(W|B)}{0.47}$$

$$P(W|B) = 0.72 \text{ or } 72\%$$

Example 4:

A hockey team has jerseys in three different colors. There are 4 green, 6 white and 5 orange jerseys in the hockey bag. Todd and Blake are given a jersey at random (without replacement). Students were asked to write an expression representing the probability that both jerseys are the same color. Which student correctly identified the probability and why?

✗ Tony	$\left(\frac{2}{4}\right)\left(\frac{2}{6}\right)\left(\frac{2}{5}\right)$
✗ Sam	$\left(\frac{2}{4}\right)+\left(\frac{2}{6}\right)+\left(\frac{2}{5}\right)$
Lesley	$\left(\frac{4}{15}\right)\left(\frac{3}{14}\right)+\left(\frac{6}{15}\right)\left(\frac{5}{14}\right)+\left(\frac{5}{15}\right)\left(\frac{4}{14}\right)$
✗ Dana	$\left(\frac{4}{15}\right)\left(\frac{4}{15}\right)+\left(\frac{6}{15}\right)\left(\frac{6}{15}\right)+\left(\frac{5}{15}\right)\left(\frac{5}{15}\right)$

→

Example 5: (ex. 3, p. 185)

According to a survey, 91% of Canadians own a cellphone. Of theses people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.

$P(C) = 0.91$

$P(S|C) = 0.42$

$P(C \cap S) = 0.91 \cdot 0.42$

$= 0.38$

Example 6: (ex. 4, p. 186)

Hillary is the coach of a junior ultimate team. Based on the team's record, it has a 60% chance of winning on clam days and a 70% chance of winning on windy days. Tomorrow, there is a 40% chance of high winds. There are no ties in ultimate. What is the probability that Hillary's team will win tomorrow?

$P(\text{windy}) = 0.4$

$P(\text{win}) = 0.7$ $\quad P(\text{windy} \cap \text{win}) = 0.4(0.7) = \boxed{0.28}$

$P(\text{Lose}) = 0.3$ $\quad P(\text{windy} \cap \text{Lose}) = 0.4(0.3) = 0.12$

$P(\text{calm}) = 0.6$

$P(\text{win}) = 0.6$ $\quad P(\text{calm} \cap \text{win}) = 0.6(0.6) = \boxed{0.36}$

$P(\text{Lose}) = 0.4$ $\quad P(\text{calm} \cap \text{Lose}) = 0.6(0.4) = 0.24$

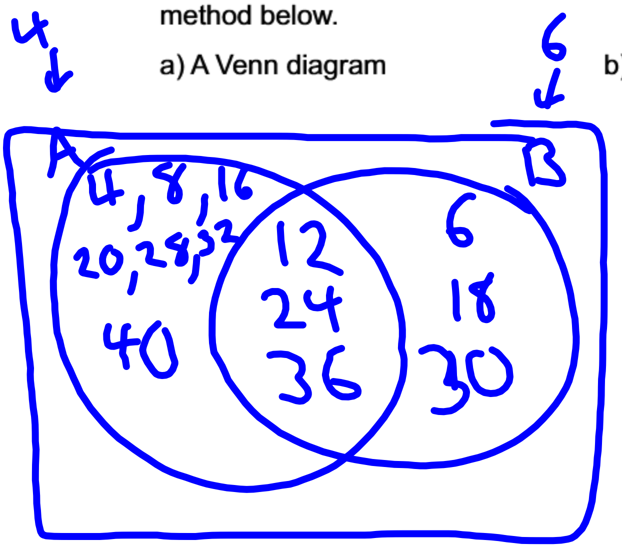
$P(\text{win}) = 0.28 + 0.36$

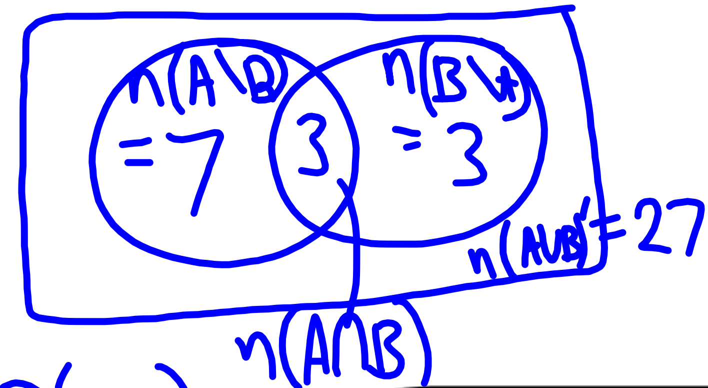
$= 0.64$ or 64%

Example 7: (ex. 2, p. 184)

Nathan asks Riel to choose a number between 1 and 40 and then say one fact about the number. Riel says that the number he chose is a multiple of 4. Determine the probability that the number is also a multiple of 6, using each method below.

a) A Venn diagram





$P(B|A) = \frac{3}{10}$

b) A formula

$P(A\cap B) = P(A)\cdot P(B|A)$

$\frac{P(A\cap B)}{P(A)} = P(B|A)$

$P(A) = \frac{10}{40}$ $P(A\cap B) = \frac{3}{40}$

$P(B|A) = \frac{\frac{3}{40}}{\frac{10}{40}}$

$= \frac{3}{40} \div \frac{10}{40}$

$= \frac{3}{\cancel{40}} \times \frac{\cancel{40}}{10}$

$P(B|A) = \frac{3}{10}$

Same!! :)

Practice Questions:

P. 188 - 191, # 1, 4, 7, 9, 10, 16, 18, 19

Section 3.6: Independent Events (Complete before Section 3.5)

In previous grades, we calculated the probability of 2 independent events.

Recall that if one event does not affect the probable outcome of the other event, then the events are independent.

If one event does affect the other, then the events are dependent, and we will use conditional probability in Section 3.5 to calculate the probability of both events occurring.

In a situation with replacement, independent events are created.

In a situation without replacement, dependent events are created.

Example 1:

Determine if events A and B are independent or dependent.

a) Event A: drawing a queen from a standard deck of cards

Event B: drawing a king from the remaining cards in the same deck.

Dependent

b) Event A: rolling a 5 on a die

Event B: rolling a 3 on the same die

Independent

Your Turn:

Classify the following events as either independent or dependent.

a) The experiment is rolling a die and flipping a coin. The first event is rolling a six and the second event is obtaining tails.

Independent

b) The experiment is rolling a pair of dice. The first event is rolling an odd number on one die and the second event is rolling an even number on the other dice.

Independent

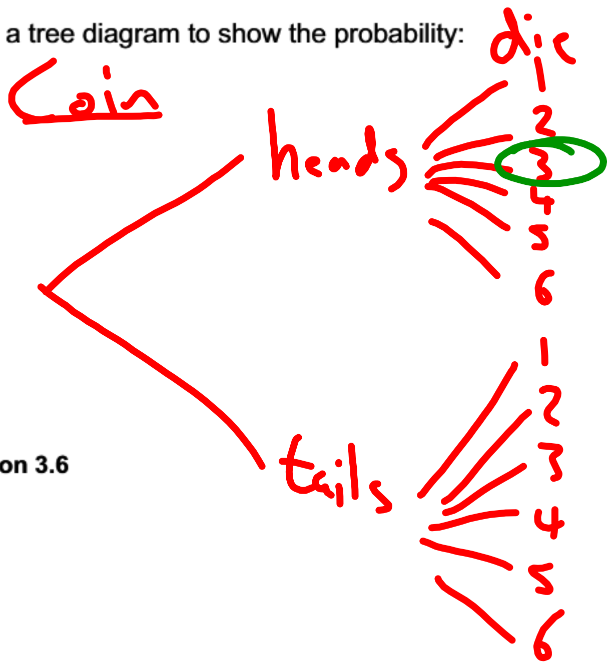
c) The experiment is dealing 5 cards from a standard deck. The first event is that the first card dealt is a spade, the second event is that the second card is a spade, and the third event is that the third card is a spade and so on.

Dependent

Example 2:

Determine the probability of rolling a 3 on a die and tossing heads on a coin. (independent events)

Draw a tree diagram to show the probability:



$\boxed{\frac{1}{12}}$

From the tree diagram, we can see the probability of rolling a 3 and tossing heads P(3 and H) is : $\frac{1}{12}$

• What is the probability of rolling a 3 on a die, (P(3))?

$1/6$

• What is the probability of tossing heads on a coin, (P(H))?

$1/2$

• What is the value of P(3) × P(H)?

$1/6 \cdot 1/2 = 1/12$

• What do you notice about the value P(3) × P(H) and the value from the tree diagram (P(3 and H))?

They are the same!

Summary: When events are independent of each other, the probability of event B does not depend on the probability of event A occurring.

In such cases, $\boxed{P(A \text{ and } B) = P(A) \times P(B)}$

Example 3:

If you have a pair of dice, what is the probability of rolling an odd number on one die and rolling an even number of the other die?

$P(\text{Odd}) = \frac{3}{6} \text{ or } \frac{1}{2}$

$P(\text{even}) = 3/6 \text{ or } 1/2$

$P(\text{odd} \cap \text{even})$

$= P(\text{odd}) \cdot P(\text{even})$

$= 1/2 \cdot 1/2 = \boxed{1/4}$

Example 4:

Jane encounters 2 traffic lights on her way to school. There is a 55% chance that she will encounter a red light at the first light, and a 40% chance that she will encounter a red light on the second light. If the traffic lights operate on separate timers, determine the probability that both lights will be red on her way to school.

$P(A \cap B) = P(A) \cdot P(B)$

$= 0.55 \cdot 0.4$

$= 0.22$ or 22%

Example 5:

Tanya estimates that her probability of passing French is 0.7 and her probability of passing Chemistry is 0.6. Determine the probability that Tanya will:

a) Pass both French and Chemistry:

$0.7 \cdot 0.6 = 0.42$ or $\boxed{42\%}$

b) Pass French but fail Chemistry: 0.4

$0.7 \cdot 0.4 = 0.28$ or $\boxed{28\%}$

c) Fail both French and Chemistry: 0.3

$0.3 \cdot 0.4 = 0.12$ or $\boxed{12\%}$

→

Example 6: (ex. 2, p. 194)

All 1000 tickets for a charity raffle have been sold and placed in a drum. there will be two draws. the first draw will be for the grand prize, and the second draw will be for the consolation prize. After each draw, the winning tickets will be returned to the drum so that it might be drawn again. Max has bought five tickets. Determine the probability, to a tenth of a percent, that he will win at least one prize.

<u>Case 1</u> Doesn't Win either (2) $\rightarrow \frac{995}{1000}$

$= 0.995$

$0.995 \cdot 0.995$

≈ 0.99

<u>Case 2</u>: Wins at least 1

$1 - 0.99 = 0.01$ or $\boxed{1\%}$

Practice Questions:

P. 198 - 200, #1, 2, 5, 6, 8, 12, 13