Math 3201 *UNIT 5: Polynomial Functions* **NOTES**

Section 5.1 and 5.2:

Characteristics of Graphs and Equations of Polynomials Functions

What is a polynomial function?

***Polynomial Function*:**

- A function that contains only the operations of multiplication and addition with real-number coefficients, whole-number exponents, and two variables.

Polynomial functions are named according to their ***degree***.

Degree of a Function:

- Is equal to the ***highest*** exponent in the polynomial.

Examples of Polynomials:

Type of Polynomial	Standard Form	Specific Example	Degree	Graph
Constant	$f(x) = a$	$f(x) = 5x^0$	0	Horizontal line
Linear	$f(x) = ax + b$	$f(x) = 2x^1 + 1$	1	Line with slope a
Quadratic	$f(x) = ax^2 + bx + c$	$f(x) = 2x^2 - x + 1$	2	Parabola
Cubic	$f(x) = ax^3 + bx^2 + cx + d$	$f(x) = 2x^3 + 3x^2 - 2x + 1$	3	?? Will explore!

NOTES:

1. The terms in a polynomial function are normally written so that the powers are in ***descending*** order.
2. Vertical lines are not considered here because they are not functions since they do not satisfy the vertical line test. Vertical lines are not part of the polynomial family!

In previous grades, we have already explored Constant (Gr.9), Linear (Gr. 9 and L1), and Quadratic (L2) functions. We will review these functions and then explore cubic functions.

First however, we will need a few new terms:

Constant Term:

- The term in the polynomial that does not have a variable.

Leading Coefficient:

- The coefficient (the number in front) of the term with the greatest degree in a polynomial function in standard form. This is denoted as ***a***.
- For example, the leading coefficient of $f(x) = 2x^3 + 3x^2 - 4x + 1$ is 2.

Question 1:

For each polynomial, determine the degree, leading coefficient and constant term.

a) $f(x) = 3x^2 - 4x + 5$

Degree = _____ Leading coefficient = _____ Constant term = _____

b) $f(x) = -2x + 7$

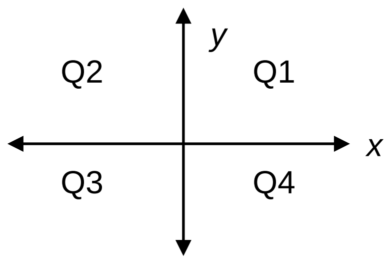
Degree = _____ Leading coefficient = _____ Constant term = ______

c) $f(x) = x^3 + 4x^2 - 6x - 9$

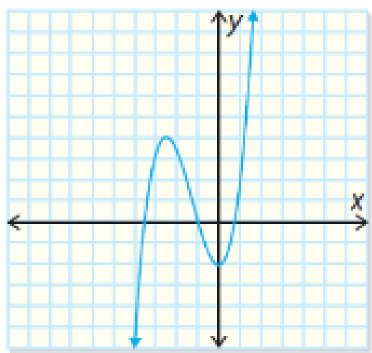
Degree = _____ Leading coefficient = _____ Constant term = ______

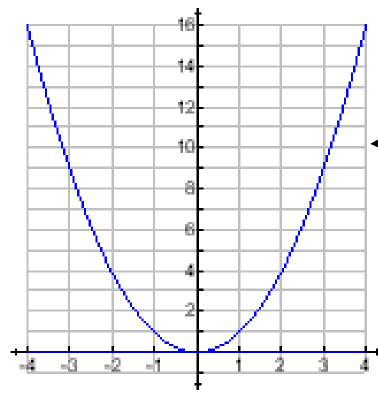
End Behaviour:

- The description of the shape of the graph, ***from left to right***, on the coordinate plane.
- The behaviour of the *y*-values as *x* becomes large in the positive or negative direction.
- Remember the coordinate plane is divided into four ***quadrants***.



This cubic function extends from Q3 to Q1



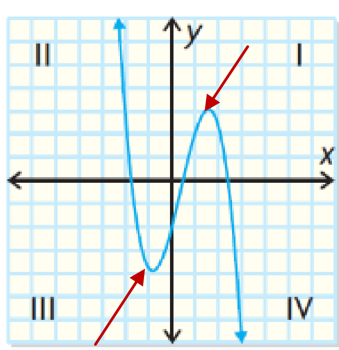


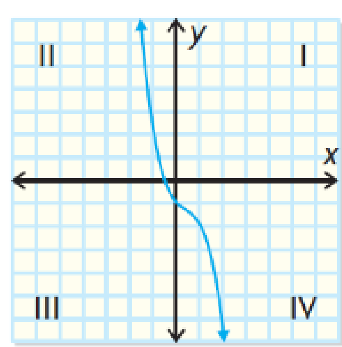
This quadratic function extends from Q2 to Q1

Turning Point:

- Any point where the graph of a function changes from increasing to decreasing or from decreasing to increasing.
- For example,

This curve has 2 turning points

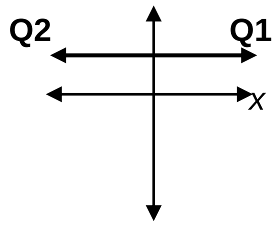
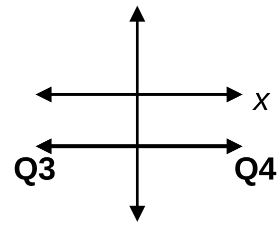




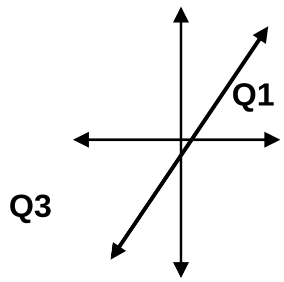
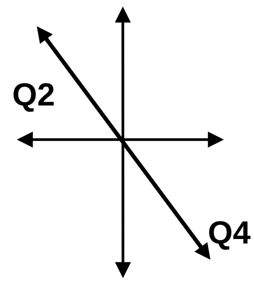
This curve does not have any turning points

So let's review!

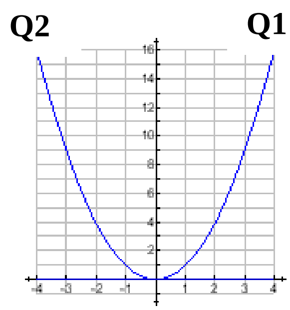
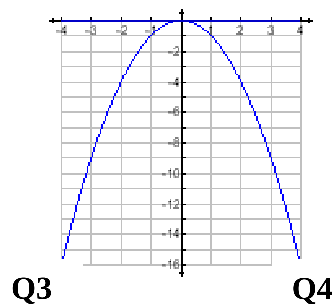
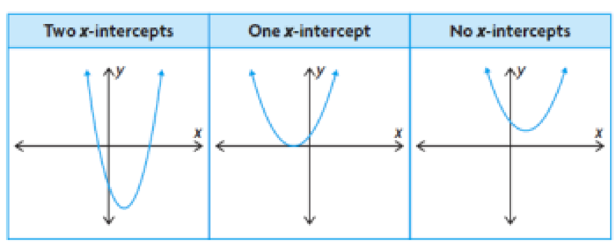
1. ***Constant Function,*** $f(x) = a$ ***(Degree = 0)***

Possible Sketches **Horizontal Lines**	If a is positive ($a > 0$)  	If a is negative ($a < 0$) 
End Behaviour	Line extends from Q2 to Q1	Line extends from Q3 to Q4
Number of *x*-intercepts	0 (exception: $y = 0$ in which case every point is on the x-axis)	
Number of *y*-intercepts	1 ($y = a$)	
Number of Turning Pts	0	
Domain	$\{x \mid x \in \Re\}$	
Range	$\{y \mid y = a, y \in \Re\}$	

2. *Linear Functions,* $f(x) = ax + b$ ***(Degree = 1)***

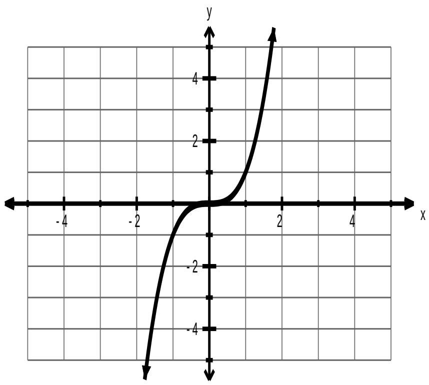
Possible Sketches **Oblique Lines**	If leading coefficient, *a*, is positive ($a > 0$)  	If leading coefficient, *a*, is negative ($a < 0$) 
End Behaviour	Line falls to the left and rises to the right Line extends from Q3 to Q1	Line falls to the right and rises to the left Line extends from Q2 to Q4
Number of *x*-intercepts	1	
Number of *y*-intercepts	1 ($y = b$)	
Number of Turning Pts	0	
Domain	$\{x \mid x \in \Re\}$	
Range	$\{y \mid y \in \Re\}$	

3. ***Quadratic Functions,*** $f(x) = ax^2 + bx + c$ ***(Degree = 2)***

Possible Sketches **Parabolas**	If leading coefficient, *a,* is positive ($a > 0$) Parabola opens up and has a minimum 	If leading coefficient, *a,* is negative ($a < 0$) Parabola opens down and has a maximum 
End Behaviour	Parabola rises to the left and rises to the right Extends from Q2 to Q1	Parabola falls to the left and falls to the right Extends from Q3 to Q4
	NOTE: The graphs have the SAME behavior to the left and right.	
Range (depends on vertex and opening)	$\{y \mid y \geq \min, y \in \Re\}$	$\{y \mid y \leq \max, y \in \Re\}$
Number of *x*-intercepts	 2, 1 or 0	
Number of *y*-ints	1 ($y = c$)	
Number of Turning Points	1	
Domain	$\{x \mid x \in \Re\}$	

Let's now explore Cubic Functions of the form $y = ax^3 + bx^2 + cx + d$!

1. $y = x^3$



Leading Coefficient: ________

Constant Term: ___________

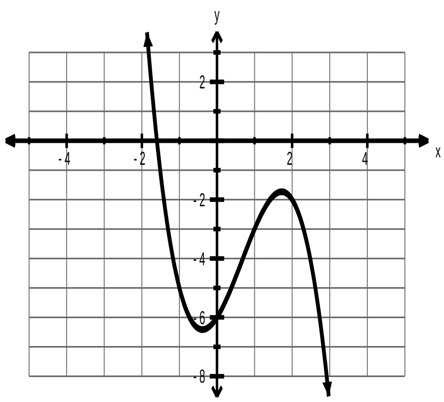
End Behaviour: ___________

# of Turning Points: ________

# of x-intercepts: __________

y-intercept: _______________

2. $y = -x^3 + 2x^2 + 2x - 6$



Leading Coefficient: ________

Constant Term: ___________

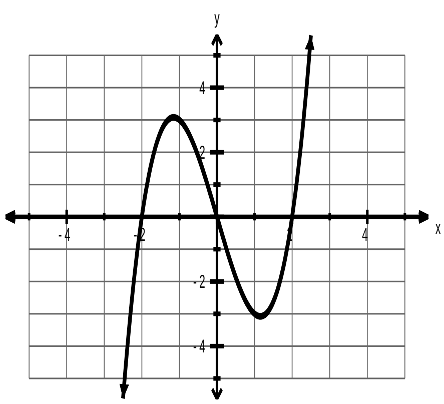
End Behaviour: ___________

# of Turning Points: ________

# of x-intercepts: __________

y-intercept: _______________

3. $y = x^3 - 4x$



Leading Coefficient: ________

Constant Term: ___________

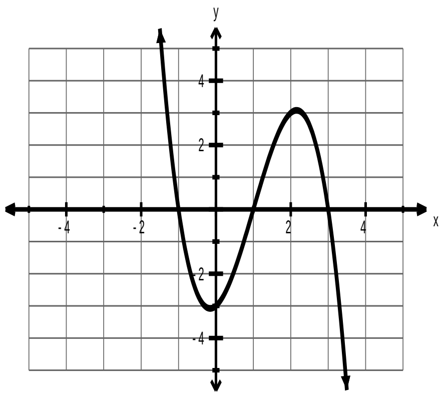
End Behaviour: ___________

# of Turning Points: ________

# of x-intercepts: __________

y-intercept: _______________

4. $y = -x^3 + 3x^2 + x - 3$



Leading Coefficient: ________

Constant Term: ___________

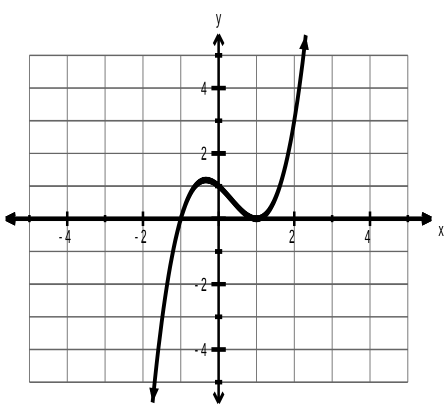
End Behaviour: ___________

\# of Turning Points: ________

\# of x-intercepts: __________

y-intercept: ________________

5. $y = x^3 - x^2 - x + 1$



Leading Coefficient: ________

Constant Term: ___________

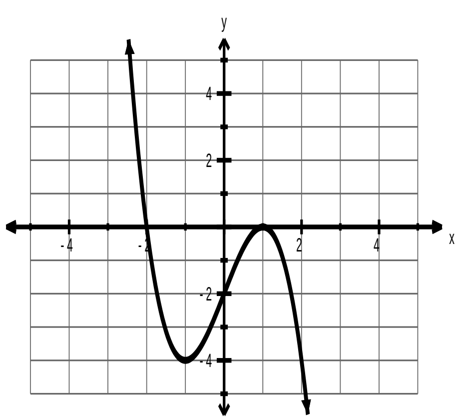
End Behaviour: ___________

\# of Turning Points: ________

\# of x-intercepts: __________

y-intercept: ________________

6. $y = -x^3 + 3x - 2$



Leading Coefficient: ________

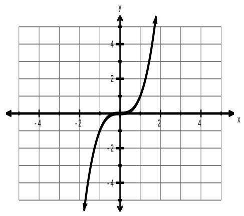
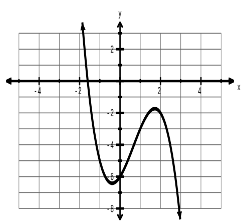
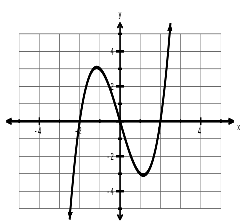
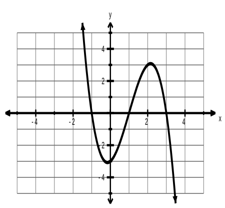
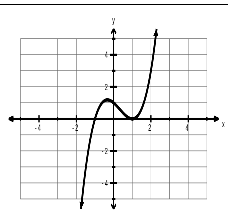
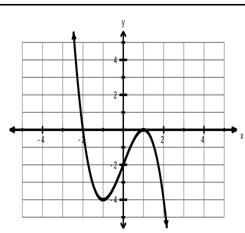
Constant Term: ___________

End Behaviour: ___________

\# of Turning Points: ________

\# of x-intercepts: __________

y-intercept: ________________

Function	**Sketch**	**Leading Coeff.(*a*)**	**Cons. Term (d)**	**End Behav.**	**Number Turn. Pts**	**# of x-ints.**	**y-int.**
a) $y = x^3$							
b) $y = -x^3 + 2x^2 + 2x - 6$							
c) $y = x^3 - 4x$							
d) $y = -x^3 + 3x^2 + x - 3$							
e) $y = x^3 - x^2 - x + 1$							
f) $y = -x^3 + 3x - 2$							

Investigation Questions:

1. How are the sign of the leading coefficient and the end behavior related?

2. How are the degree of the function and the number of *x*-intercepts related?

3. How is the *y*-intercept related to the equation?

4. How many turning points can a cubic function have?

5. In general, how are the number of turning points and the degree related?

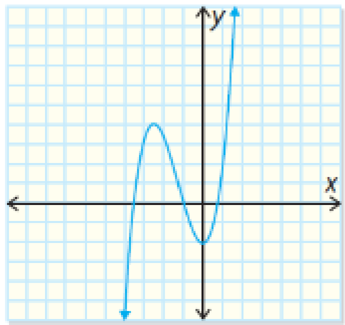
6. What is the domain and range for cubic functions?

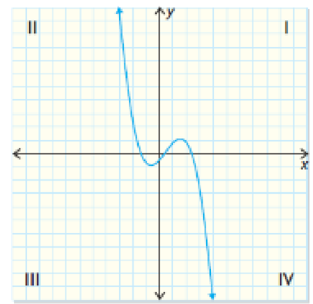
7. Explain why quadratic functions have maximum or minimum values, but cubic polynomial functions have only turning points?

4. ***Cubic Functions,*** $f(x) = ax^3 + bx^2 + cx + d$ ***(Degree = 3)***

<table>
<tr><td>Possible Sketches

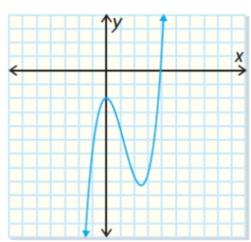
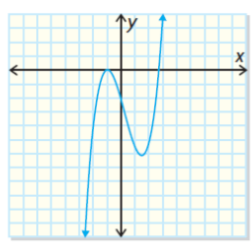
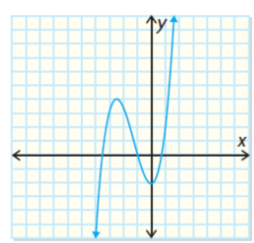
Sideways “S”</td><td colspan="2">If leading coefficient, a, is positive (a > 0)

Q1

Q3</td><td colspan="2">If leading coefficient, a, is negative (a < 0)

Q2

Q4</td></tr>
<tr><td rowspan="2">End Behaviour</td><td colspan="2">Curve falls to the left and rises to the right

Extends from Q3 to Q1</td><td colspan="2">Curve rises to the left and falls to the right

Extends from Q2 to Q4</td></tr>
<tr><td colspan="4">NOTE:The graphs have OPPOSITE behaviors to the left and right.

They have similar behavior to the behavior of linear functions.</td></tr>
<tr><td>Number of x-intercepts</td><td colspan="4">One x-intercept:

Two x-intercepts:

Three x-intercepts:
</td></tr>
<tr><td>Number of y-intercepts</td><td>1 ($y = d$)</td><td colspan="2">Domain</td><td>$\{x \mid x \in \Re\}$</td></tr>
<tr><td>Number of Turning Points</td><td>2 or 0</td><td colspan="2">Range</td><td>$\{y \mid y \in \Re\}$</td></tr>
</table>

***** Refer to page 276 and page 286 for a summary.*****

Summary Points for Polynomials of Degree 3 or less:

- The graph of a polynomial function is continuous
- Degree determines the shape of graph
- Degree = max # of x-intercepts
- There is only one y-intercept for every polynomial and it is equal to the constant term
- The maximum number of turning points is one less than the degree. That is, a polynomial of degree n, will have a maximum of $n - 1$ turning points.
- The end behavior of a line or curve is the behavior of the y-values as x becomes large in the positive or negative direction. For linear and cubic functions the end behavior is opposite to the left and right, while it is the same for quadratic functions.

ASSIGN:

p. 277, #1 – 4

(You can refer to the graphs at the end of the text to answer question 3 if you do not have graphing technology)

EXAMPLES:

1. Determine the following characteristics of each function:

   - number of possible *x*-intercepts
   - *y*-intercept
   - domain and range
   - number of possible turning point
   - end behavior

   a. $f(x) = -3x + 2$

   b. $f(x) = 4x^2 - 6x + 3$

   c. $f(x) = -2x^3 - 3x^2 + 2x - 1$

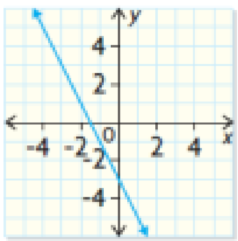
   d. $f(x) = 4x^3 + 2x$

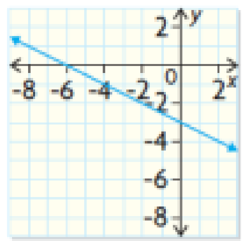
2. Match each graph with the correct polynomial function.

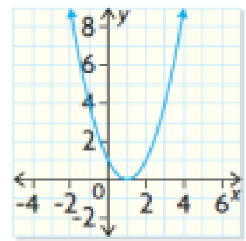
$a(x) = -x^3 + 4x^2 - 2x - 2$ $\qquad$ $b(x) = x^2 - 2x - 2$

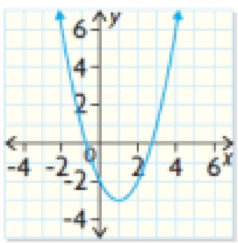
$c(x) = x^3 - 2x^2 - x - 2$ $\qquad$ $d(x) = -\frac{1}{2}x^3 - 3$

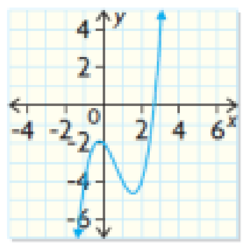
$e(x) = x^2 - 2x + 1$ $\qquad$ $f(x) = -2x - 3$

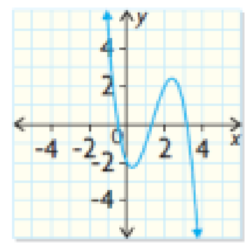
i)


iii)


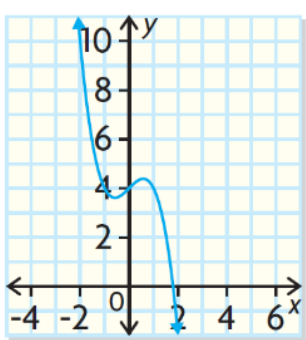
v)


ii)


iv)


vi)


3. Determine the degree, the sign of the leading coefficient, and the constant term for the polynomial function represented by each graph.

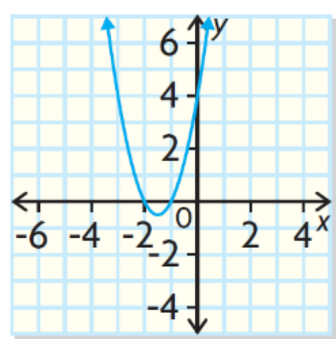


a.

Degree:

Sign:

Constant Term:



b.

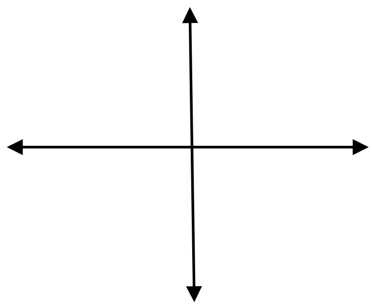
Degree:

Sign:

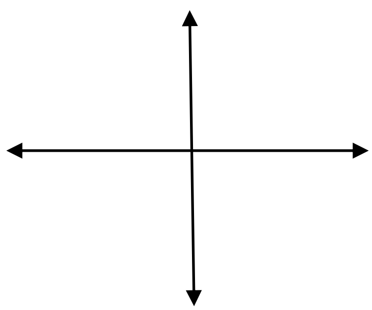
Constant Term:

4. Sketch a possible graph of polynomial functions that satisfy each set of characteristics.

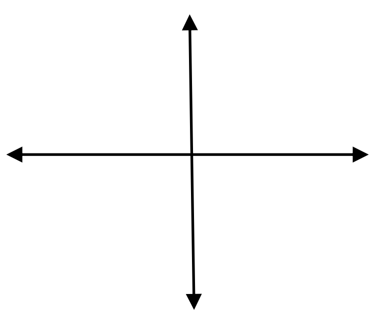
   a. Degree 2, one turning point which is a minimum, constant term of – 4



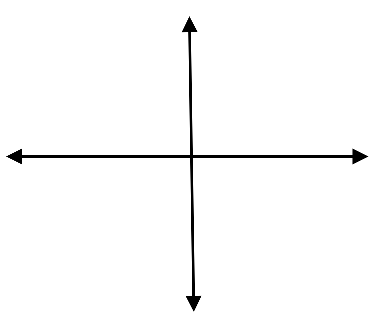
   b. Two turning points (one in Q3 and one in Q1), negative leading coefficient, constant term of 3



   c. Degree 1, positive leading coefficient, constant term of – 2



   d. Cubic, three *x*-intercepts, positive leading coefficient



5. Write a polynomial function that satisfies each set of characteristics.
    a. Extending from QIII to QIV, one turning point, y-intercept of 5
    b. Extending from QIII to QI, y-intercept of -4
    c. Degree 1, increasing, y-intercept of -3
    d. Two turning points, y-intercept of 7
    e. Range of $y \geq 2$ and y-intercept of 2

ASSIGN:

p. 287, #1 – 4, 6 – 13