UNIT 2: COUNTING METHODS

Section 2.1: Counting Principles

Example 1: (p. 68)

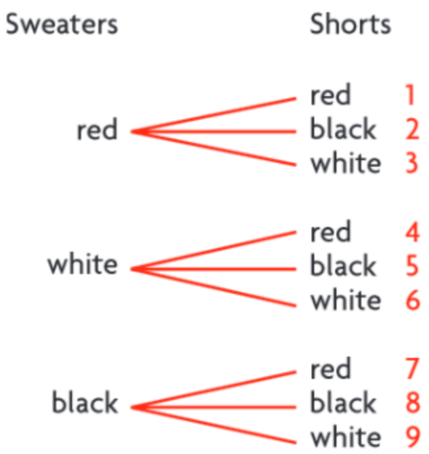
Hannah plays on her school soccer team. The soccer uniforms has:

- three different sweaters: red, white, and black and
- three different shorts: red, white, and black.

How many different variations of the soccer uniform can the coach choose from for each game?

In other words, what is the *sample space*, the different possible outcomes.

Strategy 1: Tree Diagram



NOTE: A tree diagram works but not an efficient method when working with a large sample space. →

Strategy 2: FUNDAMENTAL COUNTING PRINCIPLE (FCP)

If one task can be performed in *a* ways,

a second task can be performed in *b* ways,

and a third task can be performed in *c* ways,

then the number of ways to perform all the tasks together is: $\boxed{a \times b \times c}$

For the example above,

U = (# of sweaters) X (# of shorts)

= 3 × 3 = 9

If the coach plans on adding 2 different pairs of socks, black or white, how many variations of uniforms will there be?

U = 3 X 3 X 2 = 18

Example 2:

The school cafeteria advertises that it can serve up to 24 different meals consisting of one item from each of the three categories: ☆ ✓

Fruit:	Apples (A), Bananas(B) or Cantaloupe(C)
Sandwiches:	Roast Beef (R) or Turkey (T)
Beverages:	Lemonade (L), Milk (M), Orange Juice (O) or Pineapple Juice (P)

Is their advertising correct?

$\underset{\text{choices for fruit}}{\underline{3}} \times \underset{\text{choices for sandwich}}{\underline{2}} \times \underset{\text{choices for beverage}}{\underline{4}} = 24$

→

Distinguish between the words AND/OR

ways to choose a fruit and a sandwich and a beverage,

(MULTIPLY individual selections)

3 fruit choices x 2 sandwich choices x 4 beverage choices
= 24 possibilities

ways to choose a fruit or a sandwich or a beverage

(ADD individual selections)

3 fruit choices + 2 sandwich choices + 4 beverage choices
= 9 possibilities

Fundamental Counting Principle

→ ***Arrangements Without Restrictions***

Example 3:

A store manager has selected 4 possible applicants for two different positions at a department store. In how many ways can the manager fill the positions?

$\underline{4}$ × $\underline{3}$

\# of choices for position 1 and # of choices for position 2

\# of ways to fill the positions $\underline{12}$

Example 4:

How many ways can the letters in the word PENCIL be arranged?

Idea: We have 6 objects and 6 possible positions to occupy

$\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

$= 720$

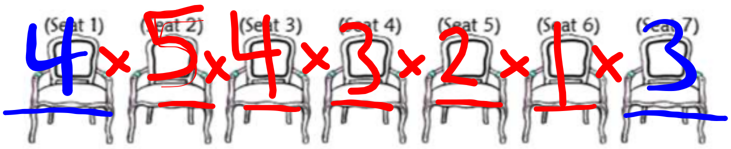
Fundamental Counting Principle

Arrangements With Restrictions

Example 5:

2 3

In how many ways can a teacher seat ~~4~~ boys and three girls in a row of 7 seats if a boy must be seated at each of the row?



$\underline{4} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{3} = \boxed{1440}$

Restriction: a boy must be in each end seat.

- Fill seats 1 and 7 first
- Then fill remaining seats

Example 6: (p. 69)

A luggage lock opens with the correct three-digit code. Each wheel rotates through the digits 0 to 9

10

a) How many different three-digit codes are possible (if repetition is allowed)?

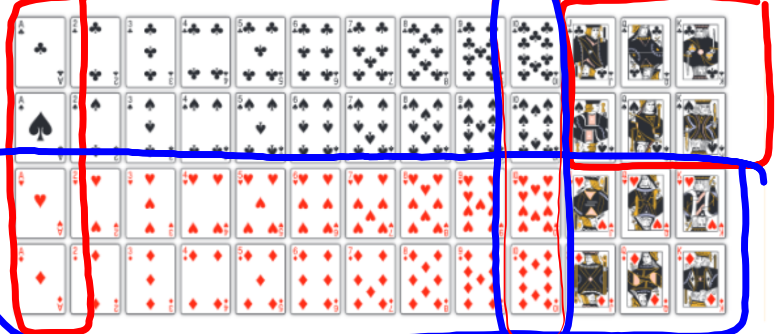
$\underline{10} \times \underline{10} \times \underline{10} = \boxed{1000}$

b) Suppose each digit can be used only once in a code. How many different codes are possible when repetition is NOT allowed?

$\underline{10} \times \underline{9} \times \underline{8} = \boxed{720}$

Example 7: FCP vs. Principle of Inclusion/Exclusion (ex.3, p. 70)

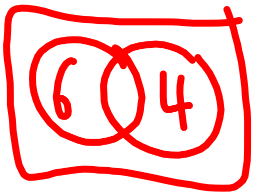
A standard deck of cards contains 52 cards as shown.



Count the number of possibilities of drawing a single card and getting:

a) either a black face card or an ace
A B

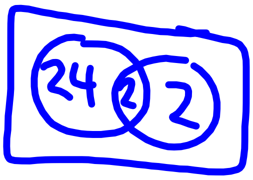
a)



$$n(A \cup B) = n(A) + n(B)$$
$$= 6 + 4 = \boxed{10}$$

b) either a red card or a 10
A B

b)



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 26 + 4 - 2$$
$$= \boxed{28}$$

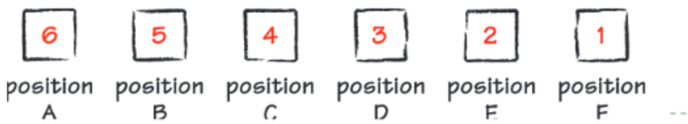
Practice Questions:
P. 73-75, #3,6,7,8,9ab,11ab,14,16ab

Section 2.2: Factorial Notation

Example 1:

There are 6 children in a group. How many different arrangements can be created as they form a line?

Idea: There are 6 different objects and 6 different positions to occupy.



Total number of permutations:

P = $6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 720$

Many examples involve arrangements where you multiply numbers decreasing by 1.

This is called FACTORIAL.

For example,

6 x 5 x 4 x 3 x 2 x 1, can be written as 6! and read as "6 factorial"

In general,

6(5)(4)(3)

$$\boxed{n! = n(n-1)(n-2)(n-3)........(2)(1) \quad \text{where} \quad n \in N}$$

$$\boxed{0! = 1}$$

Note the connection between the Fundamental Counting Principle and factorial notation n!

Example 2:

$x!$

In how many different ways can a set of 5 books be arranged on a shelf?

$5! = 5 \times 4 \times 3 \times 2 \times 1$
$= 120$

$\boxed{0! = 1}$

Complete the table. What pattern do you notice?

n	*n!*	*n*(*n* - 1)!
1	1	$1(0)! = 1(1) = 1$
2	2	$2(1)! = 2(1) = 2$
3	6	$3(2)! = 3(2) = 6$
4	24	$4(3)! = 4(6) = 24$
5	120	$5(4)! = 5(24) = 120$

$\boxed{n! = n(n-1)!}$

Example 3:

Evaluate the following:

NOTE: There is a factorial button on your calculator!

a) $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 5040$

b) $\frac{9!}{6!} = \frac{9 \times 8 \times 7 \times \cancel{6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{6 \times 5 \times 4 \times 3 \times 2 \times 1}}$
$= 504$

$= \frac{9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}}$

c) $\frac{12!}{9!3!}$

$= \frac{12 \times 11 \times 10 \times \cancel{9!}}{\cancel{9!}\ 3!}$

$= \frac{12 \times 11 \times 10}{6} = 220$

d) $\frac{100!}{97!}$

$= \frac{100 \times 99 \times 98 \times \cancel{97!}}{\cancel{97!}}$

$= 970200$

Example 4:

Identify and correct the error in the student's solution.

$$\frac{640!}{638!4!}$$

$$\frac{640\times639\times638!}{638!4!}$$

$$\frac{640\times639}{4!}$$

$$160\ \frac{\cancel{640}\times639}{\cancel{4!}}$$

$$102\ 240$$

X

$$\frac{640\times639}{4\times3\times2\times1}$$

$$=\frac{640\times639}{24}$$

$$=17040$$

Example 5:

Simplify the following where $n \in N$:

a) $(n+3)(n+2)!$

$$(n+3)\cdot(n+2)(n+1)(n)\ldots 3\times2\times1$$

$$=(n+3)!$$

b) $\frac{3!(n+1)!}{2!(n-1)!}$

$$\frac{3\times\cancel{2\times1}(n+1)(n)\cancel{(n-1)\ldots3\times2}}{\cancel{2\times1}\ \cancel{(n-1)\ldots3\times2\times1}}$$

$$=3(n+1)(n)$$

$$=3(n^2+n)$$

$$=3n^2+3n$$

c) $\frac{(2n+1)!}{(2n-1)!}$

$$\frac{(2n+1)(2n)\cancel{(2n-1)!}}{\cancel{(2n-1)!}}$$

$$=(2n+1)(2n)$$

$$=4n^2+2n$$

d) $\frac{(n-5)!}{(n-3)!}$

$$=\frac{\cancel{(n-5)(n-6)\ldots2\cdot1}}{(n-3)(n-4)\cancel{(n-5)\ldots2\cdot1}}$$

$$=\frac{1}{(n-3)(n-4)}$$

$$=n^2-4n-3n+12$$

$$=n^2-7n+12$$

Example 6:

Solve the following where $n \in N$:

a) $\frac{(n+2)!}{(n+1)!}=10$

$$\frac{(n+2)\cancel{(n+1)(n)\ldots\times2\times1}}{\cancel{(n+1)(n)\ldots\times2\times1}}=10$$

$$n+2=10$$

$$\boxed{n=8}$$

b) $\frac{n!}{(n-2)!}=90$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}=90$$

$$n(n-1)=90$$

$$n^2-n=90$$

$$n^2-n-90=0$$

$$(n-10)(n+9)=0$$

$$\boxed{n=10} \qquad n=-9$$

~~Practice Questions:~~

~~p. 81-83, #5acef,3bc,4,6abef,11bcd,7,8,12,13,14~~

Section 2.3: Permutations When All Objects Are Distinguishable

Permutation

↳ ***arranging all or part of a set of distinguishable objects where ORDER is IMPORTANT***

What are the possible permutations of the letters A, B and C ?

ABC, ACB, BAC, BCA, CAB, CBA

Example 1:

How many ways are there to arrange 3 people of a group of 5 in a line?

FCP: $\underline{5} \times \underline{4} \times \underline{3} = 60$

This expression can be written as: $\frac{5!}{2!} = \frac{5!}{(5-3)!} = \boxed{{}_5P_3}$

Total

Formula to determine the number of permutations of n different elements taken r at a time:

\# we use

${}_nP_r$

$${}_nP_r = \frac{n!}{(n-r)!}$$

- arranging a subset of items
- only some of the items are used in the arrangement

OR ${}_nP_n$ if all objects are used!

$${}_5P_5 = \boxed{\frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1}} = 5!$$

Example 2:

If there are 7 members on the student council, how many ways can the council select 3 students to be the president, vice-president and the treasurer?

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7\times6\times5\times\cancel{4!}}{\cancel{4!}}$$

$$= 7\times6\times5$$

$$= 210$$

$$\frac{5040}{24} = 210 \text{ or}$$

Example 3:

In how many ways can 6 people be arranged in a line for a photograph?

$$\boxed{{}_nP_r = \frac{n!}{(n-r)!}} \qquad {}_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} \longleftarrow \text{\#ways to count an empty set}$$

$$0! = 1$$

$${}_6P_6 = 6! = 720$$

Example 4:

26

10 A code consists of three letters chosen from A to Z and three digits chosen from 0 to 9, with no repetitions of letters or numbers. Determine the total number of possible codes.

$$\underline{26}\times\underline{25}\times\underline{24}\times\underline{10}\times\underline{9}\times\underline{8}$$

$$= 11\,232\,000$$

$${}_{26}P_3 \cdot {}_{10}P_3 = 15\,600 \cdot 720$$

$$= 11\,232\,000$$

Permutations

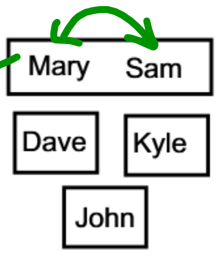
With Constraints

- two or more objects must be placed together
- two or more objects cannot be placed together
- certain objects must be placed in certain positions

Example 5:

a) How many ways can a group of 5 people be arranged in a line if two of them are good friends, Mary and Sam, and ~~want~~ need to sit together.

~~$_5P_5 = 5!$~~



$_2P_2$

★ when certain items are to be kept together, treat the joined item as if they were only one object.

$$_4P_4 = \frac{4!}{0!} = 4! = 24$$

$$_2P_2 = 2! = 2$$

$$_4P_4 \cdot {}_2P_2 = 24 \cdot 2 = \boxed{48}$$

b) How many ways can a group of 5 people be arranged in a line if Mary and Sam should not sit together.

Complement

total # of arrangements with no restrictions — arrangements with Mary and Sam together

$$_5P_5 - 48$$

$$= 5! - 48$$

$$= 120 - 48 = \boxed{72}$$

Example 6:

At a used car lot, seven different car models are to be parked close to the street for easy viewing.

a) The three red cars must be parked so that there is a red car at each end and the third red car is exactly in the middle. How many ways can the seven cars be parked?

$\underline{3} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{2} \times \underline{1} \times \underline{1}$

$= 144$

$_3P_3 \cdot {}_4P_4$

$= 3! \cdot 4! = 6 \cdot 24 = \boxed{144}$

b) The three red cars must be parked side by side. How many ways can the seven cars be parked?

$_5P_5 \cdot {}_3P_3$

$= 5! \cdot 3!$

$= 120 \cdot 6 = \boxed{720}$

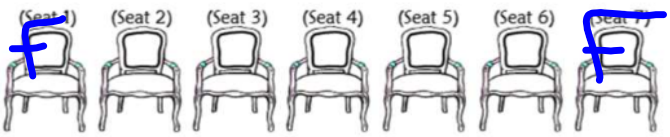
Arrangements Involving Cases

Some problems have more than one case. Calculate the number of arrangements for each case and then **add** up the values for all cases to obtain the total.

Key Words: At Least, At Most , Either

Example 7:

Determine the number of arrangements of 4 girls and 3 boys in a row of seven seats if the ends of the rows must be either both female or both male.



Girls

4 girls, 3 boys

Boys

both

Case for Females | Case for Males

$_4P_2 \cdot {_5P_5}$

$= \frac{4!}{(4-2)!} \cdot 5!$

$= \frac{4!}{2!} \cdot 5!$

$= 12 \cdot 120$

$= 1440$

$_3P_2 \cdot {_5P_5}$

$= \frac{3!}{(3-2)!} \cdot 120$

$= \frac{3!}{1!} \cdot 120$

$= 6 \times 120$

$= \boxed{720}$

Example 8: (Ex. 3, p. 88)

Tania needs to create a password for a social networking website she registered with. The password can use any digits from 0 to 9 and/or any letters of the alphabet. The password is ***case sensitive***, so she can use both lower and upper-case letters. A password must be at least 5 characters to a maximum of 7 characters, and each character can be used only once in the password. How many different passwords are possible?

10

52

# of choices: $10 + 2(26)$

$= 10 + 52$

$= 62$

Case 1 (5)

$_{62}P_5 = \frac{62!}{(62-5)!}$

$= \frac{62!}{57!} = 62 \cdot 61 \cdot 60 \cdot 59 \cdot 58 \cdot 57$

Case 2 (6)

$_{62}P_6$

$= \frac{62!}{(62-6)!}$

$= \frac{62!}{56!}$

Case 3 (7)

$_{62}P_7$

$= \frac{62!}{55!}$

Total: $\frac{62!}{57!} \cdot \frac{62!}{56!} \cdot \frac{62!}{55!} = 2\,523\,690\,780\,000$

Example 9:

A social insurance number (SIN) in Canada consists of a nine-digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position in the number, how many SINs can be created if each digit can be repeated? How does this compare with the number of SINs that can be created if no repetition is allowed?

Repetition

$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$

$= 10^9$

$= 1\,000\,000\,000$

No Repetition

$_{10}P_9 = \frac{10!}{(10-9)!} = \frac{10!}{1!} = 10! = 3\,628\,800$

$1\,000\,000\,000 - 3\,628\,800 = 996\,371\,200$

Example 10:

Solve equations using $_nP_r$ where $n > 0,\ n > r$

a) Solve: $_nP_2 = 30$

$\frac{n!}{(n-2)!} = 30$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 30$

$n(n-1) = 30$

$n^2 - n = 30$

$n^2 - n - 30 = 0$

$(n-6)(n+5) = 0$

$\boxed{n=6}$ $n=-5$

b) Solve: $_{n-1}P_2 = 12$

$\frac{(n-1)!}{(n-1-2)!} = 12$

$\frac{(n-1)!}{(n-3)!} = 12$

$\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 12$

$(n-1)(n-2) = 12$

$n^2 - 2n - n + 2 = 12$

$n^2 - 3n + 2 = 12$

$n^2 - 3n - 10 = 0$

$(n-5)(n+2) = 0$

$\boxed{n=5}$ $n=-2$

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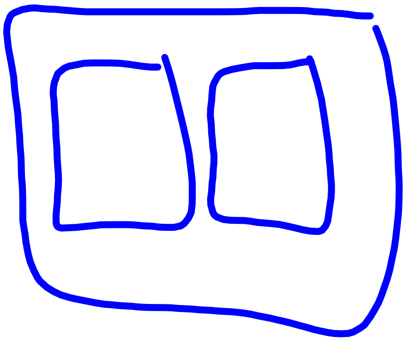
Example 11:

Mary has a set of posters to arrange on her bedroom wall. she can only fit 2 posters side by side. If there are 72 ways to choose and arrange 2 posters, how many posters does she have in total?

$${}_nP_2 = 72$$



$$\frac{n!}{(n-2)!} = 72$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 72$$

$$n(n-1) = 72$$

$$n^2 - n = 72$$

$$n^2 - n - 72 = 0$$

$$\rightarrow (n-9)(n+8) = 0$$

$$n = 9 \qquad n = -8$$

Practice Questions:

p.93-94, #1acde,3,5,8,9,10abc,13ab,14ab,15ab

Section 2.4: Permutations When Objects are Identical

A permutation of 'n' elements taken 'n' at a time (${}_nP_n$ or $n!$) is affected if one or more elements in the set are IDENTICAL.

For example,

If a set of 3 marbles consists of 2 identical green marbles and 1 blue marble, the set {G1, G2, B} is identical to {G2, G1, B}. This configuration is counted as two different arrangements instead of one.

Therefore, it must be removed from the total count by dividing out repetitions ($\frac{3!}{2!}$)

> In general,
>
> The number of permutations of 'n' objects containing 'a' identical objects of one kind and 'b' identical objects of another kind and so on is: $\frac{n!}{a!b!...}$

Dividing n! by a! and b! eliminates arrangements that are the same and that would otherwise be counted multiple times.

Example 1:

If there are 9 different cookies (4 chocolate chip, 3 oatmeal and 2 raisin), in how many different orders can you eat all of them if you eat one at a time?

$$\frac{9!}{4!\,3!\,2!} = \frac{9\times8\times7\times\cancel{6}\times5\times\cancel{4!}}{\cancel{4!}\,\cancel{3!}\,2!} = \frac{9\times8\times7\times5}{2}$$

$$= 1260$$

Example 2:

How many different ways can you arrange the letters in the word MATHEMATICS?

$$\frac{11!}{2!\,2!\,2!} = \frac{39\,916\,800}{8} = \boxed{4\,989\,600}$$

Example 3: (ex. 2, p. 101)

How many ways can the letters in CANADA be arranged, if the first letter must be N and the last letter must be C?

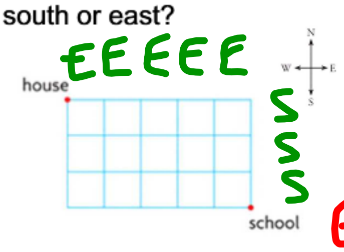
N _ _ _ _ C

$$\frac{4!}{3!} = \frac{4\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}} = \boxed{4}$$

Example 4: (ex. 3, p. 102)

Julie's home is three blocks north and five blocks west of her school. How many routes can Julie take from home to school if she always travels either south or east?

EEEEE



S S S

$$\frac{8!}{5!\,3!}$$

East → 5! 3! ← South

$$= \frac{8\cdot7\cdot\cancel{6}\cdot\cancel{5!}}{\cancel{5!}\cdot\cancel{3!}}$$

$$= 8\cdot7$$

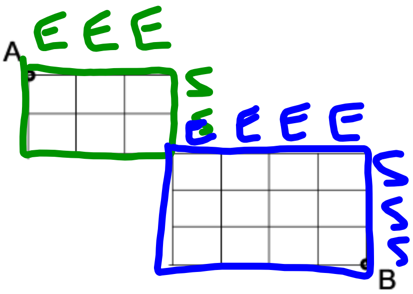
$$= \boxed{56}$$

→

Example 5:

Determine the number of routes there are to get from point A to point B, if you travel only south or east?

a)



$$\frac{5!}{3!\,2!} \cdot \frac{7!}{4!\,3!}$$

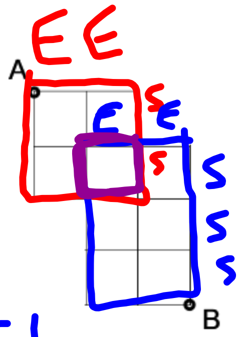
$$= \frac{5\cdot 4\cdot \cancel{3!}}{\cancel{3!}\,2!} \cdot \frac{7\cdot 6\cdot 5\cdot \cancel{4!}}{\cancel{4!}\,\cancel{3!}}$$

$$= \frac{20}{2} \cdot 35$$

$$= 10 \cdot 35$$

$$= \boxed{350}$$

b)



$$= \frac{4!}{2!\,2!} \cdot \frac{5!}{2!\,3!}$$

$$= \frac{4\cdot 3\cdot \cancel{2!}}{2!\,\cancel{2!}} \cdot \frac{5\cdot 4\cdot \cancel{3!}}{2!\,\cancel{3!}}$$

$$= \frac{12}{2} \cdot \frac{20}{2}$$

$$= 6 \cdot 10$$

$$= \boxed{60}$$

$$2! = \boxed{2}$$

$$\frac{60}{2} = \boxed{30}$$

Practice Questions:

p.104-107, #4,5,6bd,7ab,9a,10,(11ab),12,15ab,16,17ab

Section 2.5/2.6: Combinations

Combination

→ ***arranging all or part of a set of distinguishable objects where ORDER does NOT matter***

Think about: Students choose two letters from the list A, B, C

\# of arrangments: AB BA AC CA BC CB

Verify using permutations: $_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3! = \boxed{6}$

order in which the letters are chosen is not important:

AB is same as BA
AC is same as CA
BC is same as CB

each group of 2 permutations is just 1 combination

\# of combinations: 3 combinations

In general, given a set of 'n' objects, taken 'r' at a time, the number of possible combinations is:

$$\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}} \quad \text{or} \quad \boxed{{}_nC_r = \frac{{}_nP_r}{r!}}$$

This may be denoted as $\binom{n}{r}$ read as "n choose r"

Example 1:

Identify each of the following as a permutation or a combination.

a) A fruit salad consisting of apples, grapes and strawberries.

combination

b) The combination to a safe is 4-7-2.

permutation

Example 2:

In a lottery, 6 numbers from 1 to 49 are selected.

Compare the number of possibilities whether order matters (permutation) or whether order does not matter (combination).

$$_{49}P_6 = \frac{49!}{(49-6)!} = \frac{49!}{43!} = 10\,068\,347\,520$$

$$_{49}C_6 = \frac{49!}{6!(49-6)!} = \frac{49!}{6!\,43!} = 13\,983\,816$$

NOTE: The # of combinations is LESS than the # of permutations.

Example 3:

There are 10 members of student council. How many ways can 4 of the members be chosen to serve on the dance committee?

$$_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4!\,6!} = \frac{10\cdot9\cdot8\cdot7\cdot\cancel{6!}}{4!\,\cancel{6!}}$$

$$= \frac{5040}{24} = \boxed{210}$$

Example 4:

An ice cream parlour serves 10 flavours of ice cream. A large sundae has 3 scoops of ice cream. How many different ice-cream combinations are there if each scoop in the sundae is a different flavour?

$$_{10}C_3 = \frac{10!}{3!\,7!} = \frac{10\cdot9\cdot8\cdot\cancel{7!}}{3!\,\cancel{7!}} = \frac{720}{6} = \boxed{120}$$

$$\frac{5!}{4!(5-4)!}$$

Example 5:

A team consists of 9 players: 5 male and 4 female.

a) How many different 4-person teams does the coach have to choose from for an all-male competition?

$$_5C_4 = \frac{5!}{4!\,1!} = \boxed{5}$$

b) How many different 4-person teams does the coach have to choose from, with 2 males and 2 females, for a mixed competition?

M F

$$_5C_2 \cdot {}_4C_2$$

$$= \frac{5!}{2!\,3!} \cdot \frac{4!}{2!\,2!}$$

$$= \frac{5\cdot4}{2} \cdot \frac{4\cdot3}{2}$$

$$= 10 \cdot 6$$

$$= \boxed{60}$$

Example 6:

The student council decides to form a sub-committee of 5 members to plan their Christmas Concert. There are a total of 11 student members: 5 males and 6 females.

a) Determine how many different ways the sub-committee can consist of exactly three females.

2 M 3 F

$$_5C_2 \cdot {_6C_3} = \frac{5!}{2!3!} \cdot \frac{6!}{3!3!} = \frac{5 \cdot 4}{2} \cdot \frac{6 \cdot 5 \cdot 4}{6} = 10 \cdot 20 = \boxed{200}$$

b) Determine how many different ways the sub-committee can consist of at least three females.

Case 1 2 M, 3 F

$= \boxed{200}$

Case 2 1 M, 4 F

$$_5C_1 \cdot {_6C_4} = \frac{5!}{1!4!} \cdot \frac{6!}{4!2!} = 5 \cdot 15 = \boxed{75}$$

Case 3 0 M, 5 F

$$_5C_0 \cdot {_6C_5} = 1 \cdot \frac{6!}{5!1!} = 1 \cdot 6 = \boxed{6}$$

$$200 + 75 + 6 = \boxed{281}$$

c) Determine how many different ways the sub-committee can consist of at least one female.

Only case not considered 5 M, 0 F

$$_{11}C_5 - {_5C_5} = \frac{11!}{5!6!} - \frac{5!}{5!0!} = 462 - 1 = \boxed{461}$$

Example 7:

Solve:

a) $_{n-2}C_2 = 36$

$$\frac{(n-2)!}{2!(n-2-2)!} = 36$$

$$\frac{(n-2)!}{2!(n-4)!} = 36$$

$$\frac{(n-2)(n-3)\cancel{(n-4)\cdots 3\cdot 2\cdot 1}}{2!\cancel{(n-4)(n-5)\cdots 3\cdot 2\cdot 1}} = 36$$

$$= \frac{(n-2)(n-3)}{2}$$

$$= \frac{n^2-3n-2n+6}{2} = 36$$

$$= \frac{n^2-5n+6}{2} = 36$$

$$= n^2-5n+6 = 72$$

$$= n^2-5n-66=0$$

$$= (n-11)(n+6)=0$$

$$= \boxed{n=11} \quad n-6$$

b) $_{n+1}C_1 = 20$

$$\frac{(n+1)!}{1!(n+1-1)!} = 20$$

$$\frac{(n+1)!}{1(n)!} = 20$$

$$\frac{(n+1)\cancel{(n)\cdots 3\cdot 2\cdot 1}}{\cancel{n\cdots 3\cdot 2\cdot 1}} = 20$$

$$n+1 = 20$$

$$\boxed{n=19}$$

Practice Questions: p.110, #1abcd

p.118-120, #4acdf, 5, 10, 11abcde, 12, 15a

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Section 2.7: Solving Counting Problems

Remember:

Permutation

→ order matters (selection of objects)

Examples:
- password or code
- selecting a group to be president, vice-president, treasurer
- awarding medals to 1st place, 2nd place, 3rd place

Combination

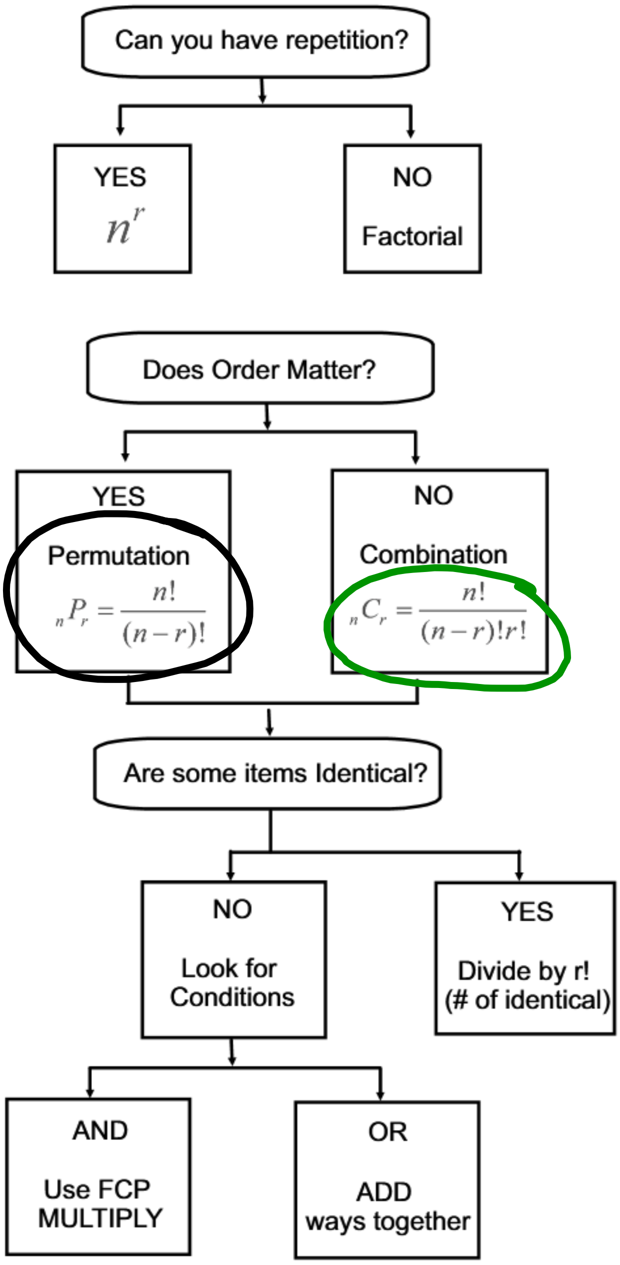
→ order does not matter (selection of objects)

Examples:
- lottery
- pick a team of 5 people from a group of 10
- taking three dogs for a walk
- choose 3 desserts from a menu

The first question you should ask yourself when solving counting problems is:

DOES ORDER MATTER??

Flow Chart to help answer questions!



Example 1: (ex. 1, p. 122)

A piano teacher and her students are having a group photograph taken. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of nine chairs for this pose?

teacher	boys	girls
1	3	5

BTG

$$n = (1! \cdot 3! \cdot 5!) \cdot 3!$$
$$= 1 \cdot 6 \cdot 120 \cdot 6$$
$$= \boxed{4320}$$

Example 2: (ex. 2, p. 123)

Combination problems are common in computer science. Suppose there is a set of 10 different data items represented by {a, b, c, d, e, f, g, h, i, j} to be placed into four different memory cells in a computer. Only 3 data items are to be placed in the first cell, 4 data items in the second cell, 2 data items in the third cell, and 1 data item in the last cell. How many ways can the 10 data items be placed in the four memory cells?

Cell 1 Cell 2 Cell 3 Cell 4

$${}_{10}C_3 \cdot {}_7C_4 \cdot {}_3C_2 \cdot {}_1C_1$$
$$= \binom{10}{3} \cdot \binom{7}{4} \cdot \binom{3}{2} \cdot \binom{1}{1}$$
$$= 120 \cdot 35 \cdot 3 \cdot 1$$
$$= \boxed{12600}$$

Example 3: (ex. 3, p. 124)

How many different five-card hands that contain at most one black card can be dealt to one person from a standard deck of playing cards?

Case 1: 1 Black, 4 Red

$_{26}C_1 \cdot {}_{26}C_4$

$= 26 \cdot 14950$

$= 388\,700$

Case 2: All Red

$_{26}C_5$

$= 65\,780$

Total

$$\begin{array}{r} 388\,700 \\ +\ \ 65\,780 \\ \hline \boxed{454\,480} \end{array}$$

Practice Questions:

p. 126-127, #1abc,3ab,4,5ab,6,10,11ab,13,14