

Name: Solutions

Teacher: GRAM

**DO NOT OPEN THE EXAMINATION PAPER UNTIL
YOU ARE TOLD BY THE SUPERVISOR TO BEGIN**

Mathematics 3201

**PRE-PUBLIC EXAMINATION
JUNE 2014**

Value: 100 Marks

Duration: 3 Hours

General Instructions

This examination consists of two parts. Part I of the exam consists of selected response questions and Part II consists of constructed response questions.

Selected Response (50 marks)

Select the letter of the correct response from those provided. Place the **CAPITAL LETTER** of the correct response in the blank space provided on your Selected Response Answer Sheet.

Constructed Response (50 marks)

Answer ALL questions fully and concisely in the space provided. Show all work.

Student Checklist

The items below are your responsibility. Please ensure that they are completed.

- Write your name and teacher's name on the top of this page.
- Write your name, teacher's name, course name and number on the Selected Response Answer Sheet.
- Check the exam to ensure there are no missing pages. There are **19** pages in total.

ALL MATERIALS MUST BE PASSED IN WITH THIS EXAM. Use your time wisely.

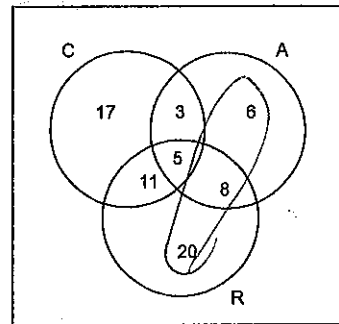
Good luck!

PART I
Total Value: 50 marks

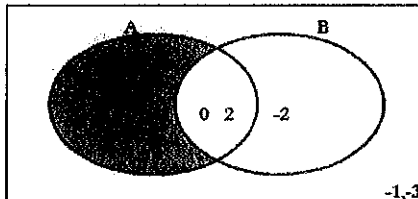
Answer all items. Place the CAPITAL LETTER of the correct answer on the answer sheet provided.

1. A summer camp offers canoeing (C), rock climbing (R), and archery (A). The following Venn diagram shows the types of activities the campers like. Determine $n((A \cup R) \setminus C)$.

- (A) 8
- (B) 13
- (C) 34
- (D) 53



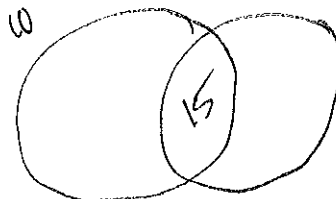
2. Which represents the shaded area in the Venn diagram below?



- (A) $A \setminus B$ (A minus B)
- (B) $A \cup B$ (A or B)
- (C) $B \setminus A$ (B minus A)
- (D) B' (not B)

3. There are 42 guests at a golf resort. Of these guests, 25 plan to go golfing (G) and 22 plan to play beach volleyball (V). There are 10 guests who do not plan to golf or play beach volleyball. How many guests plan to golf and play beach volleyball $n(G \cap V)$?

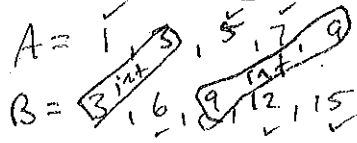
- (A) 15
- (B) 22
- (C) 37
- (D) 42



Handwritten calculations:
 $42 - 10 = 32$
 $25 + 22 = 47$
 $47 - 32 = 15$

4. A is the set of positive odd integers less than 11. B is the set of multiples of 3 between 2 and 17. How many elements are not in the intersection of A and B, $n((A \cap B)')$.

- (A) 1
 (B) 2
 (C) 6
 (D) 7



5. Which shows the total number of possible arrangements of 8 distinguishable objects taken 5 at a time?

- (A) 5C_8
 (B) 8C_5
 (C) 5P_8
 (D) 8P_5

perm $n = 8$
 $r = 5$

6. In how many ways can the letters in the word CART be arranged?

- (A) 1
 (B) 4
 (C) 12
 (D) 24

$4! = 24$

7. Which is a non-permissible value of n in the rational expression $\frac{(n+1)!}{(n+2)!}$?

- (A) -2
 (B) -1
 (C) 0
 (D) 1

$n+1 > 0 \quad n > -1 \checkmark$
 $n+2 > 0 \quad n > -2 \checkmark$ must work for both

8. Simplify: $\frac{(n-3)!}{(n-1)!}$

- (A) $\frac{1}{n^2-n}$
 (B) $\frac{1}{n^2-3n+2}$
 (C) n^2-n
 (D) n^2-3n+2

$$\frac{(n-3)!}{(n-1)(n-2)(n-3)!}$$

$$\frac{1}{n^2-2n-n+2}$$

$$\frac{1}{n^2-3n+2}$$

9. There are 8 marbles in a bag: 2 red, 3 white, and 3 blue. If taken out one at a time, in how many different ways can all 8 marbles be taken out of the bag?

- (A) 280
 (B) 560
 (C) 6720
 (D) 40320

$$\frac{8!}{2!3!3!} = \frac{40320}{2 \cdot 6 \cdot 6} = \frac{40320}{72}$$

10. To get from his home to school, John must travel by only heading East (E) or South (S). One example of a route is given representing four moves East followed by three moves South (EEEESSS). Under these rules, which represents the total number of possible routes that can be taken to get from John's house to his school?

- (A) $\frac{7!}{4!3!}$
 (B) $\frac{12!}{4!3!}$
 (C) 7!
 (D) 12!

$$\frac{7!}{4!3!}$$

11. If the odds against the Toronto Maple Leafs winning their next game is 3:7. What is the probability that they will win their next game?

- ~~(A)~~ $\frac{3}{10}$
 (B) $\frac{3}{7}$
 (C) $\frac{7}{10}$
 (D) $\frac{7}{3}$

$$3 + 7 = 10$$

$$\frac{3}{10}$$

lose
win
fav = 7
unfav = 3

12. A committee of three people will be randomly chosen from a group of eight people; 5 females and 3 males. Which represents the probability of selecting a committee that has at least one male and at least one female member?

(A) $\frac{{}_3C_1 \times {}_5C_2 + {}_3C_2 \times {}_5C_1}{{}_8C_3}$

Total: $8C_3$

(B) $\frac{{}_3C_0 \times {}_5C_3 + {}_3C_3 \times {}_5C_0}{{}_8C_3}$

Case 1: 1M 2F
3C1 5C2

Case 2: 2F 1M
5C1 3C2

(C) $\frac{{}_8C_1 \times {}_5C_2 + {}_8C_2 \times {}_5C_1}{{}_3C_3 \times {}_5C_3}$

(D) $\frac{{}_8C_1 \times {}_3C_2 + {}_8C_2 \times {}_3C_1}{{}_3C_3 \times {}_5C_3}$

13. Shavana estimates that her probability of passing French is 0.7 and her probability of passing chemistry is 0.6. What is the probability that Shavana will fail French, but pass chemistry?

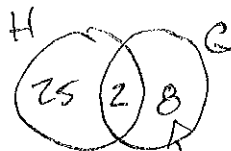
F. Pass 0.7 Fail 0.3
C Pass 0.6 Fail 0.4

- (A) 12%
(B) 18%
(C) 28%
(D) 42%

$0.3 \cdot 0.6 = 0.18$

14. There are 35 athletes in Charlie's Grade 12 class. Based on a survey he conducted, he knows that 27 of these students play hockey and 10 play golf. What is the probability that a student chosen at random will play golf only?

- (A) $\frac{2}{35}$
(B) $\frac{8}{35}$
(C) $\frac{2}{7}$
(D) $\frac{5}{7}$



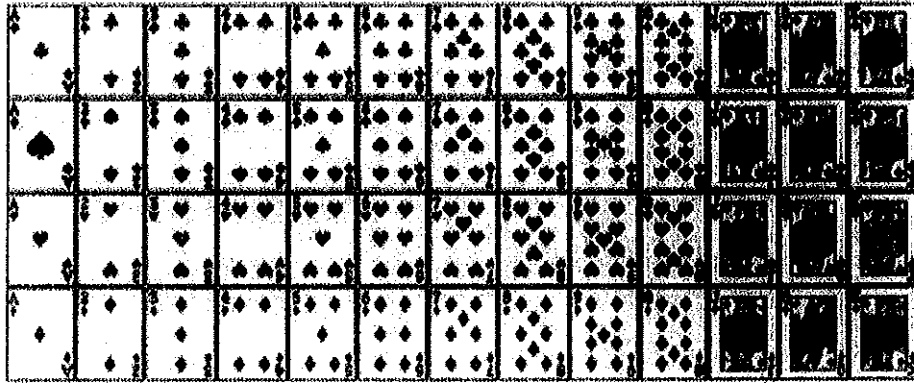
$27 + 10 = 37 - 35 = 2$

↑ only golf = few

35 = total

$\frac{8}{35}$

15. Students were asked to find the probability of choosing an ace from a standard deck of 52 cards and then choosing a king from the same deck of cards without replacement. Which student has the correct solution?



Ace $\frac{4}{52} = \frac{1}{13}$
 King $\frac{4}{51}$

Student	Solution
Abby	$\left(\frac{1}{13}\right)\left(\frac{1}{13}\right)$
Devon	$\left(\frac{1}{13}\right) + \left(\frac{1}{13}\right)$
Robyn	$\left(\frac{1}{13}\right)\left(\frac{4}{51}\right)$ ✓
Ron	$\left(\frac{1}{13}\right) + \left(\frac{4}{51}\right)$

- (A) Abby
 (B) Devon
 (C) Robyn
 (D) Ron
16. What are the non-permissible value(s) of x in the rational expression $\frac{x+2}{3(x-2)(x-4)}$?
- (A) -2, 2, 4
 (B) 0
 (C) 2, 3, 4
 (D) 2, 4
- $x \neq 2 \quad x \neq 4$
17. What is the simplified form of $\frac{x^3}{x^3-6x^2}$, $x \neq 0, 6$?
- (A) $-6x^2$
 (B) $-\frac{1}{6x^2}$
 (C) $\frac{x}{x-6}$
 (D) $\frac{1}{x-6}$
- $\frac{x^3}{x^2(x-6)} = \frac{x}{x-6}$

18. What expression is equivalent to $\frac{x-1}{x+1}$, $x \neq 1$?

(A) $\frac{x^2-x}{x^2+x}$

(B) $\frac{2x-2}{x+1}$

(C) $\frac{2x-1}{2x+1}$

(D) $\frac{2x-2}{2x+2}$

mult/div by constant!

19. Simplify: $\frac{3x-9}{6x^2-54}$

(A) $\frac{-1}{2x-6}$, $x \neq -3, 3$

(B) $\frac{-1}{2x+6}$, $x \neq -3, 3$

(C) $\frac{1}{2x+6}$, $x \neq -3, 3$

(D) $\frac{1}{2x-6}$, $x \neq -3, 3$

$\frac{3(x-3)}{6(x^2-9)} = \frac{\cancel{3}(x-\cancel{3})}{2\cancel{6}(x-\cancel{3})(x+3)} = \frac{1}{2x+6}$

20. Simplify: $\frac{8}{3x} \div \frac{4x-4}{6x^2}$

(A) $\frac{16x-16}{9x^3}$, $x \neq 0, 1$

(B) $\frac{4x}{x-1}$, $x \neq 0, 1$

(C) $\frac{x-1}{4x}$, $x \neq 0, 1$

(D) $\frac{9x^3}{16x-16}$, $x \neq 0, 1$

$\frac{8\cancel{x}}{\cancel{3x}} \times \frac{\cancel{6x^2}}{\cancel{4x}(x-1)} = \frac{\cancel{4}x}{x-1}$

21. Simplify: $\frac{2x}{x+3} - \frac{5x}{2x+6}$

(A) $\frac{-3x}{-x-9}$, $x \neq -3$

(B) $\frac{-3x}{-x-3}$, $x \neq -3$

(C) $\frac{-x}{2(x+3)}$, $x \neq -3$

(D) $\frac{x}{2(x+3)}$, $x \neq -3$

22. How many turning points are on the graph of the function $f(x) = 2x^2 - x + 7$?

- (A) 0 (B) 1 (C) 2 (D) 3

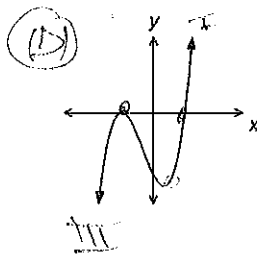
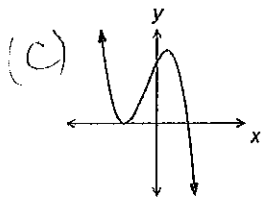
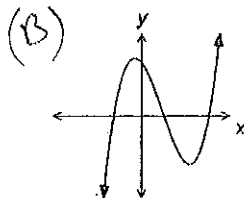
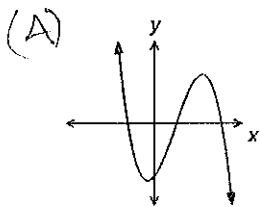
23. What is the y-intercept of the equation $f(x) = 2x^2 + 7x - 8$?

- (A) -8
- (B) 0
- (C) 7
- (D) 8

$$x = 0$$

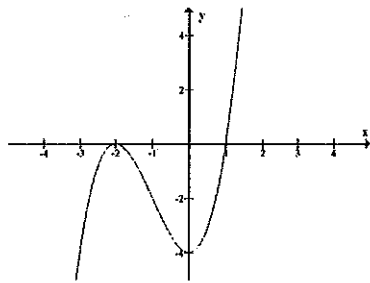
24. Which graph represents a function with the characteristics listed below?

- Two x-intercepts
- Extending from Quadrant III to Quadrant I

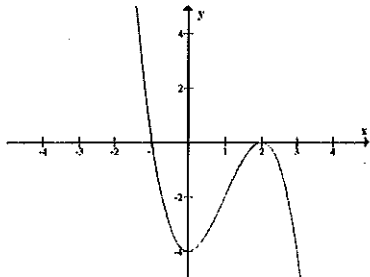


25. Which graph match the equation $f(x) = -3x^2 - 4$ *quad*
open & y int @ -4
down

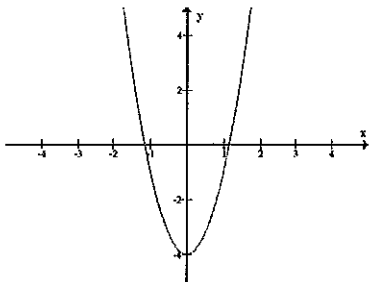
(A)



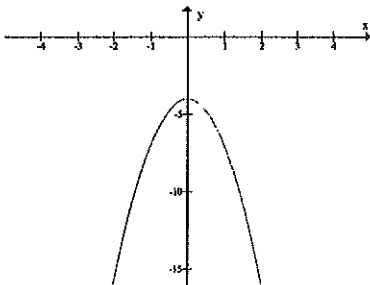
(B)



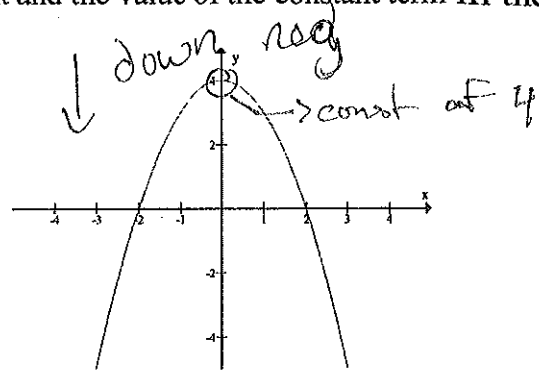
(C)



(D)



26. Determine the sign of the leading coefficient and the value of the constant term in the graph below.



- (A) negative and 0
 (B) negative and 4
 (C) positive and 0
 (D) positive and 4
27. From which quadrants does the graph of $f(x) = -x^3 + 3x^2 - 4$ extend?

II to IV

- (A) II to I
 (B) II to IV
 (C) III to I
 (D) III to IV
28. Which is an increasing exponential function?

- (A) $f(x) = \frac{1}{3} \left(\frac{5}{2}\right)^x$ $b > 1$
 (B) $f(x) = 0.5(0.5)^x$
 (C) $f(x) = \frac{3}{2}(1)^x$
 (D) $f(x) = 2\left(\frac{3}{4}\right)^x$

29. Which equation describes a graph with a y-intercept of 4 that doubles in amount every 5 years?

- (A) $y = 4 \left(\frac{1}{2}\right)^{\frac{t}{5}}$
 (B) $y = 5 \left(\frac{1}{2}\right)^{\frac{t}{4}}$
 (C) $y = 4(2)^{\frac{t}{5}}$
 (D) $y = 5(2)^{\frac{t}{4}}$

30. The population of a strain of bacteria growing in a Petri dish is modeled by the function $P(t) = 6000(2)^{\frac{t}{7}}$, where $P(t)$ represents the number of bacteria and t represents the time in hours after the initial count. How much time will it take for the number of bacteria to reach 24 000?

(A) 7 h
 (B) 14 h
 (C) 28 h
 (D) 56 h

$24000 = \frac{6000}{6000} (2)^{\frac{t}{7}}$
 $4 = (2)^{\frac{t}{7}}$
 $2^2 = 2^{\frac{t}{7}}$
 $2 = \frac{t}{7} \quad t = 14$

31. Solve for x: $3^{7x+1} = 27^{x-1}$
- (A) -1
 (B) $-\frac{1}{2}$
 (C) $\frac{1}{2}$
 (D) 1
- $3^{3(x-1)}$
 $7x+1 \quad 3x-3$
 $4x = -4$
 $x = -1$

32. What is the equation that represents a \$5000 loan with an annual interest rate of 4% that is compounded quarterly?

(A) $A = 0.01(5000)^n$
 (B) $A = 0.04(5000)^n$
 (C) $A = 5000(1.01)^n$
 (D) $A = 5000(1.04)^n$

$\frac{4\%}{4} = 0.01$

33. Which is true of the table given below?
- initial* $\frac{160}{320} = \frac{1}{2}$

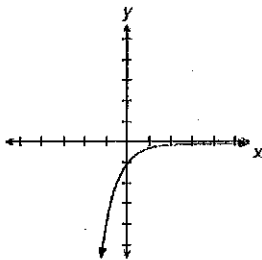
x (years)	0	3	6	9	12
y (amount)	320	160	80	40	20

- Initial Amount** **Amount Growth**
- (A) 160 reduces to half every three years
 (B) 160 reduces to one third every two years
 (C) 320 reduces to half every three years
 (D) 320 reduces to one third every two years

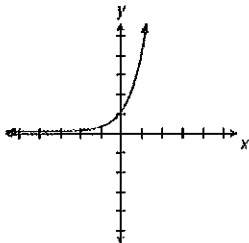
34. Which graph represents $y = -2 \ln x$?

No y int.

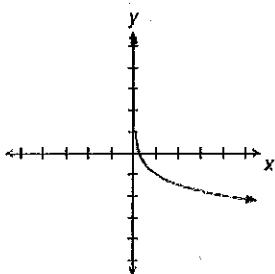
(A)



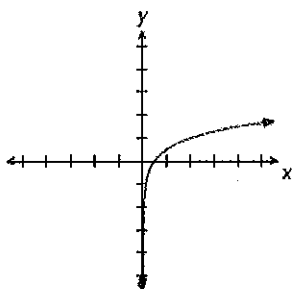
(B)



(C)



(D)



35. What is $3 \log_5 3 - \log_5 9$ written as a single logarithm?

- (A) $\log_5 1$
- (B) $\log_5 3$
- (C) $\log_5 81$
- (D) $\log_5 243$

$$\log_5 3^3 - \log_5 9 = \log_5 \left(\frac{27}{9}\right) = \log_5 (3)$$

36. Evaluate: $\log_{\frac{1}{3}} 81$

- (A) -27
- (B) -4
- (C) 4
- (D) 27

$$\frac{\log(81)}{\log(\frac{1}{3})} = -4$$

37. What is the exponential form of $d = \log_c 5$?

- (A) $c = 5^d$
- (B) $5 = c^d$
- (C) $d = 5^c$
- (D) $5 = d^c$

$$c^d = 5$$

38. Solve for x: $3^{x-1} = 8$

- (A) $\frac{\log 3}{\log 8} + 1$
- (B) $\frac{\log 3+1}{\log 8}$
- (C) $\frac{\log 8}{\log 3} + 1$
- (D) $\frac{\log 8+1}{\log 3}$

$$(x-1) \log 3 = \frac{\log 8}{\log 3} + 1$$

39. The equation $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{5}}$ represents a radioactive sample after t years. How much time will it take for 70% of the sample to decay?

- (A) 2.6 years
- (B) 3.0 years
- (C) 7.0 years
- (D) 8.7 years

$$0.7 = 1 \left(\frac{1}{2}\right)^{\frac{t}{5}}$$

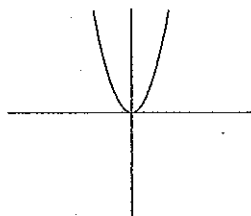
$$\log(0.7) = \frac{t}{5} \log\left(\frac{1}{2}\right)$$

$$\frac{\log(0.7)}{\log(\frac{1}{2})} = \frac{t}{5}$$

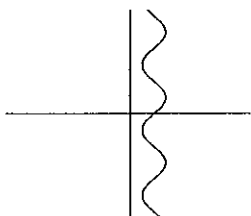
$$t = 8.7$$

40. Which graph represents a function that is periodic and sinusoidal?

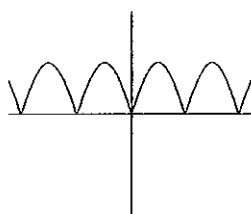
(A)



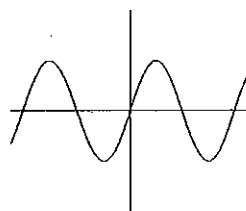
(B)



(C)



(D)



41. What is the amplitude and maximum value for the function

$$f(x) = 3 \sin 2(x - 90^\circ) + 1$$

max = 1 + 3 = 4

(A)	2	3
(B)	2	4
(C)	3	3
(D)	3	4

42. The graph of which function has a period of 90° ?

- (A) $y = 3 \cos \frac{1}{4}x - 1$
 (B) $y = 3 \cos(x - 90^\circ) - 1$
 (C) $y = 4 \cos(x + 90^\circ) + 1$
 (D) $y = 4 \cos 4x + 1$

$\frac{90}{360} = \frac{1}{4} \rightarrow \text{recip}$ 4 in eq

43. What is the value of $\frac{5\pi}{6}$ in degrees?

- (A) 150°
 (B) 170°
 (C) 216°
 (D) 300°

$\frac{5\pi}{6} \cdot \frac{180}{\pi} = \frac{900}{6} = 150^\circ$

44. What is the range of the function $y = 8 \cos x - 4$?

- (A) $\{x \mid -12 \leq x \leq 4, x \in R\}$
 (B) $\{x \in R\}$
 (C) $\{y \mid -12 \leq y \leq 4, y \in R\}$
 (D) $\{y \in R\}$

Min $-4 - 8 = -12$
 Max $-4 + 8 = 4$

45. The graph of the function $y = 4 \cos 3x$ has its amplitude halved and its period doubled. Which represents the new function?

- (A) $y = 2 \cos \frac{3}{2}x$
 (B) $y = 2 \cos 6x$
 (C) $y = 8 \cos \frac{3}{2}x$
 (D) $y = 8 \cos 6x$

$\frac{4}{2} = 2$
 $3 \cdot \frac{1}{2} = \frac{3}{2}$

$= 2 \Rightarrow \frac{1}{2} \text{ recip}$

46. The interest rate on the loan shown in the chart below is 5% compounded monthly. How much of the third payment is interest?

Payment Period (month)	Payment (\$)	Principal Paid (\$)	Balance (\$)
0			15,000
1	450	387.50	14,612.50
2	450	389.11	14,223.39
3	450	390.74	13,832.65

- (A) \$59.26
 (B) \$60.89
 (C) \$62.50
 (D) \$182.65

$$450 - 390.74 = 59.26$$

47. 24 semi-annual payments are required to pay off a loan. How many years does this represent?

- (A) 1
 (B) 2
 (C) 12
 (D) 24

$$24 / 2 = 12$$

48. The equation $A = 1000(1.005)^{36}$ represents a bank loan that is compounded monthly. How long has interest accumulated on the loan?

- (A) 1.5 years
 (B) 3 years
 (C) 5 years
 (D) 36 years

$$n = 36$$

$$\frac{36}{12} = 3$$

12

49. Which represents the highest interest that would be paid?

(A)	12%	daily
(B)	12%	monthly
(C)	19%	daily
(D)	19%	monthly

high %
w/ most payments

50. A student repaid a total of \$2536.32 for a loan including the principal and interest. The interest rate was 8% compounded quarterly for 2 years. What was the principal amount of the loan to the nearest cent?

- (A) \$1864.27
- (B) \$2164.72
- (C) \$2174.48
- (D) \$2437.83

$$2536.32 = A_0 (1.02)^8$$

$$n = 2 \times 4$$

$$i = \frac{8\%}{4} = 1.02$$

PART II
Total Value: 50 marks

Answer **ALL** items in the space provided. Show **ALL** workings.

Value

3

51. 40 members in a sports club were surveyed:

23 play ball hockey

24 play tennis

18 play golf

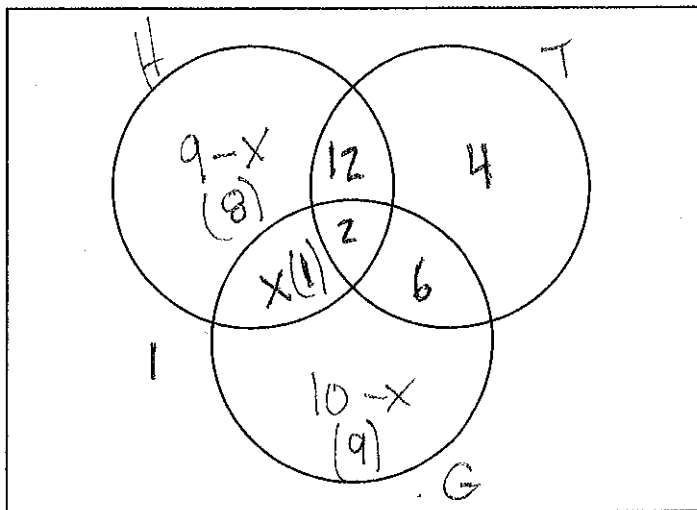
14 play tennis and ball hockey

8 play tennis and golf

2 play all three sports

1 member makes the refreshments and does not play any sport

Determine the number of people who play ball hockey only.



$$39 = (9-x) + 12 + 2 + 6 + 4 + x + 10-x$$

$$39 = 31 - x$$

$$-2 = -x$$

$$x = 2$$

8 people play only Hockey

- 2 52.(a) In Newfoundland and Labrador a license plate consists of a letter-letter-letter-digit-digit arrangement such as CXT 132. How many license plate arrangements are possible if no letter can be repeated but numbers can? (There are 26 letters in the alphabet)

$$\underline{26} \quad \underline{25} \quad \underline{24} \quad \underline{10} \quad \underline{10} \quad \underline{10} = 15600000$$

- 2 52.(b) How many different arrangements of the letters in the word MISSISSIPPI are possible?

$$\frac{11!}{4!4!2!} = \frac{39916800}{1152} = 34650$$

11 total
4 i's
4 s's
2 p's

- 3 52.(c) A committee of six (6) members must be chosen from a group of four (4) males and seven (7) females. In how many ways can the committee be chosen if there must be three (3) males and three (3) females on it?

$$\begin{aligned} 1 \text{ case} &= 4C_3 \times 7C_3 \\ &= 4 \times 35 \\ &= 140 \end{aligned}$$

- 2 53.(a) When considering the total arrangements of the letters in the word MOUTH, what is the probability of the vowels (O, U) being together?

O U together 2 ways O U, U O

4 options M, T, H, O U

$$\text{total} = 4! \cdot 2 = 24 \cdot 2 = 48.$$

- 3 53.(b) John is sometimes late for school in the mornings. Based on his track record, there is an 85% chance of being on time when he remembers to set his alarm clock and a 40% chance when he doesn't. Tomorrow there is a 60% chance that John will set his alarm clock. What is the probability that he will be on time for school tomorrow?

Clock $\begin{cases} 85\% \text{ on time} \\ 15\% \text{ late} \end{cases}$ no clock $\begin{cases} 40\% \text{ on time} \\ 60\% \text{ late} \end{cases}$

Tomo $\begin{cases} 60\% \text{ clock} \\ 40\% \text{ no clock} \end{cases}$

$$60\% \cdot 85\% + 40\% \cdot 40\%$$

$$0.6 \times 0.85 + 0.4 \times 0.4$$

$$0.51 + 0.16 = 0.66\%$$

- 4 54.(a) Simplify and state any restrictions: $\frac{9-x^2}{x^2} \div \frac{x+3}{3x}$

$$= \frac{(3-x)(3+x)}{x^2} \cdot \frac{3x}{x+3}$$

$$= \frac{3-x}{x}, \quad x > 0, -3$$

$$x^2 \rightarrow x \geq 0$$

$$3x \rightarrow x > 0$$

$$x+3 \rightarrow x > -3$$

$$x \geq 0, -3$$

- 2 54.(b) Tyrone can mow a lawn in 3 hours. When Tyrone and Devon mow the same lawn together they can finish it in $\frac{12}{7}$ hours. How long would it take Devon to mow this lawn on his own?

$$\frac{1}{T_y} + \frac{1}{D} = \frac{1}{\text{total}}$$

$$\frac{1}{3} + \frac{1}{t} = \frac{1}{12/7}$$

$$\frac{1 \cdot t}{3t} + \frac{1 \cdot 3}{t} = \frac{12 \cdot 3}{7}$$

$$t + 21 = 36$$

$$21 = 36 - t$$

$$\frac{21}{35} = t$$

$$\frac{3}{7} = t$$

- 3 55.(a) A spherical balloon is being inflated. The surface area (SA), in square meters, is related to the time in minutes as show in the table below.

Time (s)	0	1	2	3	4
SA (m^2)	13	28	50	79	113

Using a graphing calculator, the quadratic regression function give as follows:

QuadReg:

$$y = ax^2 + bx + c$$

$$a = 3.214$$

$$b = 12.243$$

$$c = 12.829$$

Write the quadratic function that best models this data and then use the function to find the time at which the balloon has a surface area of $40 m^2$. Be sure to include what each variable represents.

~~responsible~~

~~oops...~~

$$y = 3.214x^2 + 12.243x + 12.829$$

$$40 = 3.214x^2 + 12.243x + 12.829$$

solve w/ quad formula

$$a = 3.214 \quad b = 12.243 \quad c = -27.171$$

$$x = \frac{-12.243 \pm \sqrt{(12.243)^2 - 4(3.214)(-27.171)}}{2(3.214)}$$

$$x = \frac{-12.243 \pm \sqrt{499.2}}{6.248}$$

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$$x = 16s \quad x = -5.5s \text{ (extraneous)}$$

y = surface area
x = time

55.(b) Given the function $f(x) = -4x^3 + 2x^2 - 3x + 56$, complete the table to describe its characteristics.

3

i)

y-intercept	($x=0$) 56
end behaviour (left and right)	(neg cubic) quad II to quad IV
Max # of possible x-intercepts	(cubic) 3

1

ii) Explain why the graph of this function has the range $\{y \in R\}$.

a cubic covers all possible values of y . It's two ends extend towards positive ∞ and negative ∞ .

4

56.(a) Algebraically solve for x :

$$\sqrt{\frac{1}{2}} = 4^{5x+1}$$

$$\left(\frac{1}{2}\right)^{\frac{1}{2}} = 4^{5x+1}$$

$$(2)^{-\frac{1}{2}} = 4^{5x+1}$$

$$2^{-\frac{1}{2}} = 2^{2(5x+1)}$$

$$-\frac{1}{2} = 10x + 2$$

$$-\frac{5}{2} = \frac{10x}{10}$$

$$-\frac{5}{20} = x$$

$$-\frac{1}{4} = x$$

56.(b) The amount of a substance present after x hours is given in the table below.

x	Time (hours)	0	1	2	3	4
y	Amount (g)	2000	1000	500	250	125

- 2 (i) Use this information to create an equation of the form $y = ab^x$ that models the data.

$$a = \text{initial } (x=0) \\ = 2000$$

$$y = 2000 \left(\frac{1}{2}\right)^x$$

or

$$y = 2000 (0.5)^x$$

$$b = \text{rate} = \frac{y_2}{y_1} = \frac{1000}{2000} = \frac{1}{2} \text{ or } 0.5$$

- 2 (ii) Use this equation to find the time at which the amount present is 31.25 g.

$$\frac{31.25}{2000} = \frac{2000}{2000} (0.5)^x$$

$$0.015625 = (0.5)^x$$

$$\frac{\log(0.015625)}{\log 0.5} = x \frac{\log(0.5)}{\log(0.5)}$$

- 4 57.(a) Algebraically solve for x : $2^{x-1} - 3^{x+2} = 0$

$$(x-1) \log(2) - (x+2) \log(3) = 0$$

$$x \log(2) - \log(2) - x \log(3) - 2 \log(3) = 0 \quad + \log(2) + 2 \log(3)$$

$$x \log(2) - x \log(3) = \log(2) + 2 \log(3)$$

$$\frac{(-0.1761)x}{-0.1761} = \frac{1.2553}{-0.1761}$$

$$x = -7.1285$$

- 3 57.(b) Andy deposits \$1000 into a savings account that pays 3% per year, compounded monthly. Determine how long it will take the investment to be worth \$5000.

$$A = A_0 (1+i)^n \quad i = 0.03 \div 12 = 0.0025 \quad n = 12t$$

$$A = 1000(1.0025)^{12t}$$

$$\frac{5000}{1000} = \frac{1000}{1000} (1.0025)^{12t}$$

$$\frac{\log(5)}{\log(1.0025)} = 12t \frac{\log(1.0025)}{\log(1.0025)}$$

$$\frac{644.58}{12} = \frac{12t}{12}$$

$$53.7 = t$$

about 54 years.

- 2 58.(a) (i) Create a sinusoidal function of the form $y = a \sin b(x-c) + d$ with an amplitude of 3, a period of 180° , a phase shift of 90° to the left, and a midline at $y = 6$.

$$a = 3$$

$$c = 90 \text{ left } \xrightarrow{\text{opp}} +90^\circ$$

$$b = 2 \left(\frac{180}{360} = \frac{1}{2} \cdot \text{recip.} = 2 \right)$$

$$d = 6$$

$$y = 3 \sin 2(x+90^\circ) + 6$$

- 2 (ii) Use this function to determine the range of the graph produced by the function.

$$\text{Mid} = 6 \quad \text{max} = 6 + 3 = 9$$

$$\text{Min} = 6 - 3 = 3$$

$$\text{Range: } R = \{y \mid 3 \leq y \leq 9, y \in \mathbb{R}\}$$

or $[3, 9]$ or use words to describe.

- 3 59. Sara borrowed \$5000 at a rate of 6% compounded monthly for 4 years. How much interest will Sara be charged for borrowing the money?

$$A = A_0 (1+i)^n \quad i = 0.06 \div 12 = 0.005 \quad n = 4 \times 12 = 48$$

$$A = 5000(1.005)^{48}$$

$$A = 6352.45$$

$$\text{Interest: } A - A_0 = 6352.45 - 5000$$