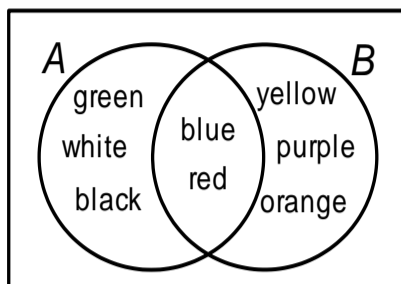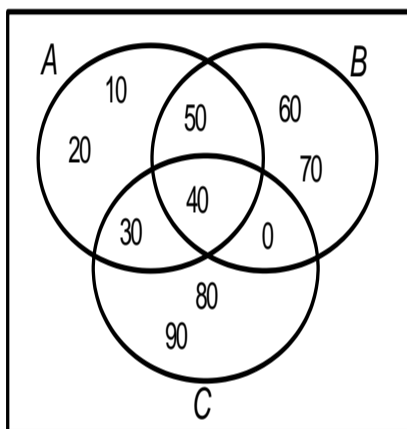Answer all items. Shade the letter of the correct answer on the computer scorable answer sheet.

1.   Given the Venn diagram below, what is the number of elements in both A and B, $n(A \cap B)$?

   *(A)   2
   (B)   3
   (C)   6
   (D)   8

2.   Given the Venn diagram below, which element(s) is (are) in sets A, B or C, $A \cup B \cup C$?

   (A)   {0, 30, 40, 50}
   (B)   {10, 20, 60, 70, 80, 90}
   (C)   {40}
   *(D)   {0, 10, 20, 30, 40, 50, 60, 70, 80, 90}

3.   There are 26 students in a classroom. 11 students have blonde hair (H), 16 have brown eyes (E), 6 do not have blonde hair or brown eyes, and 13 have blonde hair or brown eyes, but not both. How many of these students have both blonde hair and brown eyes, $H \cap E$?

   (A)   1
   *(B)   7
   (C)   13
   (D)   20

4. A is the set of positive even integers less than 12. B is the set of multiples of 3 between 4 and 20. Which element(s) is (are) not in the intersection of A and B, $(A \cap B)'$?

   (A)   {2, 4, 8, 9, 10, 15, 18}
   *(B)   {2, 4, 8, 9, 10, 12, 15, 18}
   (C)   {6}
   (D)   {6,12}

5. A student incorrectly wrote 4! = 12. To produce a correct solution for 4!, what operation should be applied to 12.

    (A)   add 2
    (B)   divide by 2
    *(C) multiply by 2
    (D)   subtract 2

6. Consider the word CAR. In how many different ways can the letters be arranged?

    (A)   1
    (B)   3
    (C)   4
    *(D) 6

7. A student must select a protective case for her new cell phone. She must choose a colour and a style for her case. Given the selections below, how many protective case choices does she have?

Case Colour	Case Style
Red	
Blue	hard
Green	
Black	
White	soft
Silver	

    (A)   8
    *(B) 12
    (C)   15
    (D)   30

8. Simplify: $\dfrac{(n-2)!}{n!}$

    *(A) $\dfrac{1}{n^2 - n}$

    (B) $\dfrac{1}{n^2 - 3n + 2}$

    (C) $n^2 - n$

    (D) $n^2 - 3n + 2$

9. In the grid below, a person must travel from *A* to *B* by only heading East (E) or South (S). One example of a route is shown representing three moves East followed by two moves South (EEESS). Under these rules, which represents the total number of possible routes that can be taken to get from *A* to *B*?
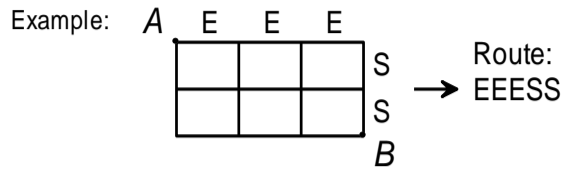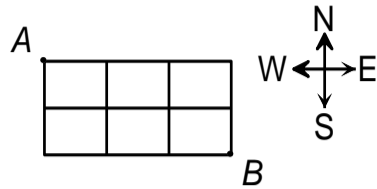
*(A) $\dfrac{5!}{3!2!}$

(B) $\dfrac{6!}{3!2!}$

(C) $5!$

(D) $6!$

10. There are 7 marbles in a bowl: 2 white, 3 green and 2 blue. If taken out one at a time, in how many different ways can all 7 marbles be taken out of the bowl?

(A) 105
*(B) 210
(C) 420
(D) 5040

11. A soccer player has 17 attempts on net and 6 goals scored. What are the odds in favour of her scoring a goal on her next attempt?

*(A) $6:11$
(B) $6:17$
(C) $11:6$
(D) $17:6$

12. A committee of three people will be randomly chosen from a group of nine people; 5 females and 4 males. Which represents the probability of selecting a committee that has at least one male and at least one female member?

*(A) $\dfrac{\left(_4C_1 \times _5C_2\right)+\left(_4C_2 \times _5C_1\right)}{_9C_3}$

(B) $\dfrac{\left(_4C_0 \times _5C_3\right)+\left(_4C_3 \times _5C_0\right)}{_9C_3}$

(C) $\dfrac{\left(_9C_1 \times _5C_2\right)+\left(_9C_2 \times _5C_1\right)}{\left(_4C_3 \times _5C_3\right)}$

(D) $\dfrac{\left(_9C_1 \times _4C_2\right)+\left(_9C_2 \times _4C_1\right)}{\left(_4C_3 \times _5C_3\right)}$

13. *A* and *B* are mutually exclusive events. The probability that either *A* or *B* will occur, $P(A\chi B)$, is 56%. If the probability of *A* occurring, $P(A)$, is 17%, what is the probability of *B* not occurring, $P(B\text{N})$?

(A) 27%
(B) 39%
*(C) 61%
(D) 73%

14. You have a six-sided die with each side numbered one through six. You also have a coin with heads on one side and tails on the other. What is the probability of rolling a number greater than 4 with the die and tossing heads with the coin?

  (A) $\frac{1}{12}$

  *(B) $\frac{1}{6}$

  (C) $\frac{1}{4}$

  (D) $\frac{1}{3}$

15. A deck of 40 cards consists of 4 different coloured sets: red, blue, green and yellow. Each set is numbered from 0 to 9 as shown below. If two cards are randomly picked from the deck, what is the probability that the first card is blue or green and the second card is also blue or green?

Card Colour	Cards
red	0 1 2 3 4 5 6 7 8 9
blue	0 1 2 3 4 5 6 7 8 9
green	0 1 2 3 4 5 6 7 8 9
yellow	0 1 2 3 4 5 6 7 8 9

  (A) $\frac{1}{20}$

  (B) $\frac{19}{80}$

  *(C) $\frac{19}{78}$

  (D) $\frac{1}{4}$

16. What are the non-permissible values for the rational expression $\dfrac{3x}{5(4-x)(2x+1)}$?

  (A) $\left\{-4,\frac{1}{2}\right\}$

  (B) $\left\{-4,\frac{1}{2},5\right\}$

  *(C) $\left\{-\frac{1}{2},4\right\}$

  (D) $\left\{-\frac{1}{2},4,5\right\}$

17. What is the simplified form of $\dfrac{x^2}{x^2-5x}$, $x\neq 0,5$?

   (A) $-5x$

   (B) $-\dfrac{1}{5x}$

   *(C) $\dfrac{x}{x-5}$

   (D) $\dfrac{1}{1-5x}$

18. What expression is equivalent to $\dfrac{x+5}{x-4}$, $x\neq 4$?

   (A) $\dfrac{x^2+5x}{x^2-4x}$

   (B) $\dfrac{2x+10}{x-4}$

   (C) $\dfrac{3x+5}{3x-4}$

   *(D) $\dfrac{5x+25}{5x-20}$

19. Simplify: $\dfrac{12-4x}{2x^2-18}$

   (A) $\dfrac{-2}{x-3}$, $x\neq-3,3$

   *(B) $\dfrac{-2}{x+3}$, $x\neq-3,3$

   (C) $\dfrac{2}{x-3}$, $x\neq-3,3$

   (D) $\dfrac{2}{x+3}$, $x\neq-3,3$

20. Simplify: $\dfrac{6x}{9}\div\dfrac{4x^3}{3}$

   *(A) $\dfrac{1}{2x^2}$, $x\neq0$

   (B) $2x^2$, $x\neq0$

   (C) $\dfrac{9}{8x^4}$, $x\neq0$

   (D) $\dfrac{8x^4}{9}$, $x\neq0$
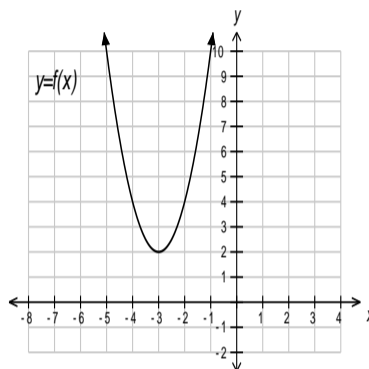
21. Simplify: $\dfrac{2x}{x+3} - \dfrac{5x}{2x+6}$

(A) $\dfrac{-3x}{-x-9}$, $x \neq -3$

(B) $\dfrac{-3x}{-x-3}$, $x \neq -3$

*(C) $\dfrac{-x}{2(x+3)}$, $x \neq -3$

(D) $\dfrac{x}{2(x+3)}$, $x \neq -3$

22. What is the range of the function $y = f(x)$ shown in the graph below?

(A) $\{y \mid y \leq -2, y \in R\}$

(B) $\{y \mid y \geq -2, y \in R\}$

(C) $\{y \mid y \leq 2, y \in R\}$

*(D) $\{y \mid y \geq 2, y \in R\}$

23. What is the y-intercept of the graph of the function $f(x) = 4x^3 + x^2 + 2x + 1$?
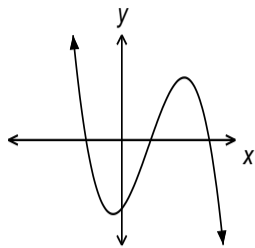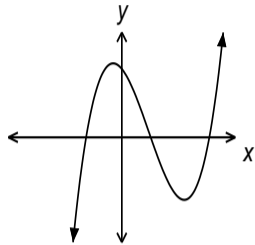
*(A) 1

(B) 2

(C) 3

(D) 4

24. Which graph best represents a function with the characteristics listed below?

- Three x-intercepts
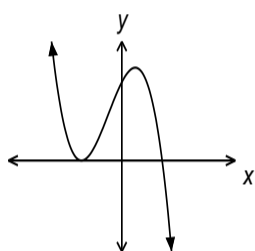- Extending from Quadrant II to Quadrant IV

*(A)

(B)

(C)

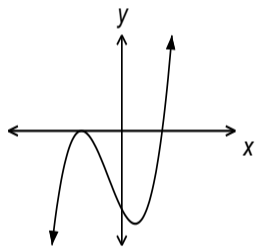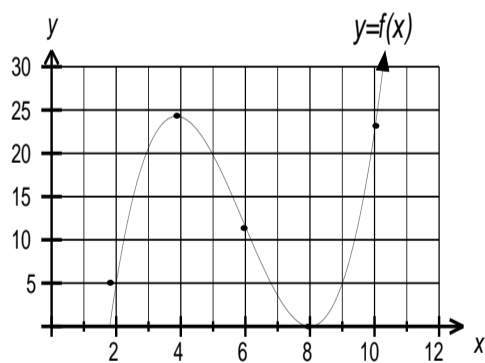

(D)

25. Given the table, the scatter plot and the curve of best fit of the polynomial $f(x)$, what is the value of $f(5)$?

x	y
2	5
4	24
6	12
8	0
10	23

(A)   2

(B)   9

(C)   18

*(D)   20

26. From which quadrants does the graph of $f(x) = x^3 + 3x^2 - 4$ extend?

    (A)   II to I
    (B)   II to IV
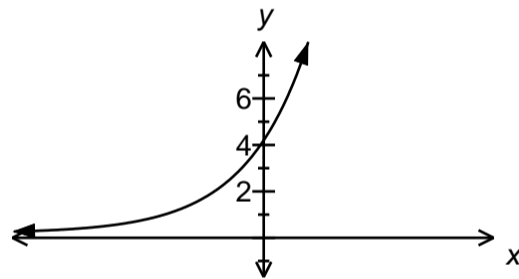   *(C)   III to I
    (D)   III to IV

27. Which function passes through the point $(1, -7)$?

    *(A) $f(x) = -x^3 - 3x^2 + x - 4$

    (B) $f(x) = -x^3 - 2x^2 + x - 7$

    (C) $f(x) = x^3 + 2x^2 - 4$

    (D) $f(x) = x^3 + 3x^2 - 7$

28. Which is a decreasing exponential function?

    (A) $f(x) = \frac{1}{3}\left(\frac{5}{2}\right)^x$

    (B) $f(x) = 0.5(1.5)^x$

    (C) $f(x) = \frac{3}{2}(1)^x$

    *(D) $f(x) = 2\left(\frac{3}{4}\right)^x$

29. Which exponential function best represents the graph shown?

    (A) $f(x) = \left(\frac{1}{4}\right)^x$

    (B) $f(x) = (4)^x$

    (C) $f(x) = 4\left(\frac{1}{4}\right)^x$

    *(D) $f(x) = 4(4)^x$

30. The population of a strain of bacteria growing in a Petri dish is modeled by the function $P(t) = 3000(2)^{\frac{t}{4}}$ where $P(t)$ represents the number of bacteria and t represents the time in hours after the initial count. How much time will it take for the number of bacteria to reach 12 000?

    (A)   4 h
    *(B)   8 h
    (C)   16 h
    (D)   32 h

31. Solve for x: $2^{3x+1} = 4^{2x-1}$

    (A)   −3
    (B)   −2
    (C)   2
    *(D)   3

32. Which is true of the table given below?

x (years)	0	3	6	9	12
y (amount)	10	20	40	80	160

	Initial Amount	Amount Growth
*(A)	10	doubles every three years
(B)	10	triples every two years
(C)	20	doubles every three years
(D)	20	triples every two years

33. The function that models the decay of carbon-14 is $A(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$, where $A(t)$ is the number of grams of carbon-14 present at time t, in years. Which statement is true?

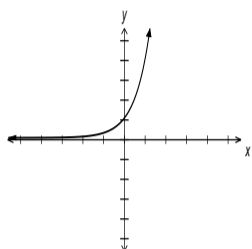   (A)   The amount of carbon-14 doubles every 5730 years.
    (B)   There are 50 g of carbon-14 present initially.
    (C)   14 g will be present after 50 years.
    *(D)   50 g of carbon-14 will be present after 5730 years.

34. Which graph best represents $y = 2\ln x$?
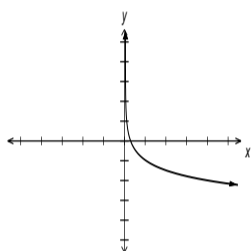
(A)
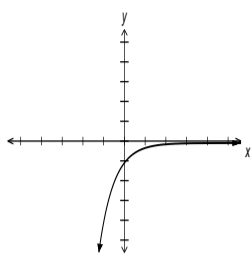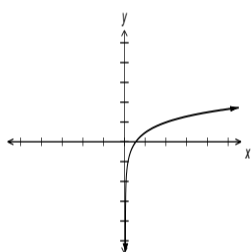

(B)

(C)

*(D)

35. What is $\log_3 32 - 2\log_3 4$ written as a single logarithm?

*(A)　　$\log_3 2$

(B)　　$\log_3 4$

(C)　　$\log_3 16$

(D)　　$\log_3 24$

36. Evaluate:　　$\log_3\left(\frac{1}{243}\right)$

(A)　　−81

*(B)　　−5

(C)　　5

(D)　　81

37. What is the logarithmic form of $C = 5^d$?

*(A)　　$d = \log_5 C$

(B)　　$d = \log_c 5$

(C)　　$C = \log_5 d$

(D)　　$C = \log_d 5$

38. Solve for x: $4^{x+1} = 7$

(A) $\frac{\log 4}{\log 7} - 1$

*(B) $\frac{\log 7}{\log 4} - 1$

(C) $\frac{\log 4 - 1}{\log 7}$

(D) $\frac{\log 7 - 1}{\log 4}$

39. The equation $A(t) = A_\circ\left(\frac{1}{2}\right)^{\frac{t}{3}}$ represents a radioactive sample after t years. How much time will it take for 15% of the sample to remain?

(A) 0.7 years
(B) 0.9 years
*(C) 8.2 years
(D) 10.0 years

40. What is the midline equation for the graph shown below?
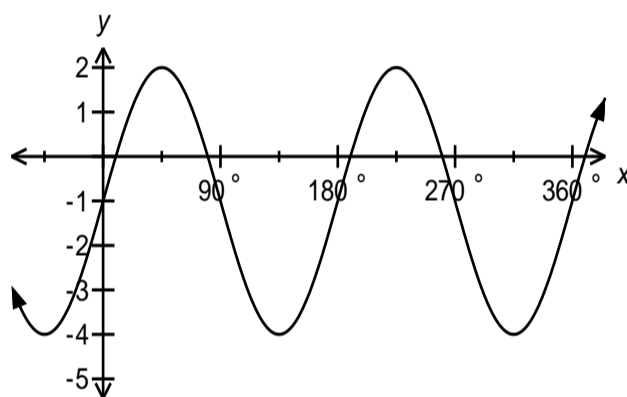
(A) $y = -4$
*(B) $y = -1$
(C) $y = 0$
(D) $y = 2$

41. What are the amplitude and maximum value for the function
$f(x) = 2\sin 3(x + 60°) + 1$?

	Amplitude	Maximum Value
*(A)	2	3
(B)	2	4
(C)	3	3
(D)	3	4

42. The graph of which function has a period of 180°?

(A) $y = 3\cos\frac{1}{2}x - 1$

(B) $y = 3\cos(x - 180°) - 1$

(C) $y = 4\cos(x + 180°) + 1$

*(D) $y = 4\cos 2x + 1$

43. What is $\frac{4\pi}{9}$ radians in degrees?

    (A)   45E

    *(B)   80E

    (C)   160E

    (D)   405E

44. What is the domain of the function $y = 4\cos x + 2$?

    (A) $\{x \mid -2 \le x \le 6, x \in R\}$

    *(B) $\{x \in R\}$

    (C) $\{y \mid -2 \le y \le 6, y \in R\}$

    (D) $\{y \in R\}$

45. The graph of the function $y = 4\cos 3x$ has its amplitude doubled and its period halved. Which represents the new function?

    (A) $y = 2\cos\frac{3}{2}x$

    (B) $y = 2\cos 6x$

    (C) $y = 8\cos\frac{3}{2}x$

    *(D) $y = 8\cos 6x$

46. The interest rate on the loan shown in the chart below is 5% compounded monthly. How much of the second payment is the interest toward the loan?

Payment Period (month)	Payment ($)	Principal Paid ($)	Balance ($)
0			15,000
1	450	387.50	14,612.50
2	450	389.11	14,223.39
3	450	390.74	13,832.65

    (A) $59.26

    *(B) $60.89

    (C) $62.50

    (D) $182.65

47. 312 bi-weekly payments are required to pay off a loan. How many years does this represent?

    (A)   6 years

    *(B)   12 years

    (C)   13 years

    (D)   26 years

48. $A = 2000(1.08)^4$ represents a bank loan that is compounded annually. What is the interest rate?

    (A)   2%
    (B)   4%
    (C)   6%
   *(D) 8%

49. Which represents the lowest interest that would be paid?

	Interest rate	Compounded
(A)	12%	daily
*(B)	12%	monthly
(C)	19%	daily
(D)	19%	monthly

50. A student repaid a total of $2880.27 for a loan including the principal and interest. If the interest rate was 9% compounded monthly for 4 years, what was the principal amount of the loan, to the nearest dollar?
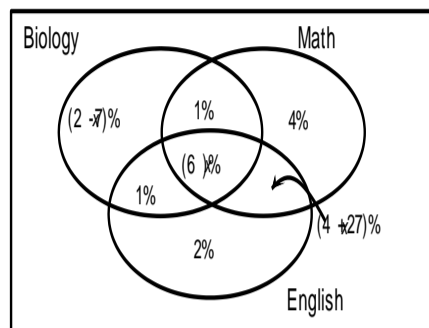
   *(A) $2012
    (B) $2040
    (C) $2633
    (D) $2795

Answer **ALL** items in the space provided. Show **ALL** workings.

Value

3   51.   200 students wrote exams in Math, Biology and English. The Venn Diagram below represents the percentage of those who wrote the exams. Algebraically determine the percentage of students who wrote all three exams, and determine the number of students that this represents.

$$(2x-7)+1+4+6x+1+(4x+27)+2=100$$

$$12x+28=100$$

$$12x+28-28=100-28$$

$$12x=72$$

$$\frac{12x}{12}=\frac{72}{12}$$

$$x=6$$

The percentage of students who wrote all three exams is $6\times 6 = 36\%$.

The number of students who wrote all three exams is $0.36\times 200 = \boxed{72}$

2   52.(a) In how many ways can a teacher seat four girls and two boys in a row of six seats if the two boys must be seated next to each other?

$$2!\times 5!=240$$

3   52.(b) Algebraically solve for *n*: $_nP_2 = 12$

$$\frac{n!}{(n-2)!} = 12$$

$$\frac{n(n-1)\,(n-2)(n-3)\times\cdots\times 2\times 1}{(n-2)(n-3)\times\cdots\times 2\times 1} = 12$$

$$n(n-1) = 12$$

$$n^2 - n = 12$$

$$n^2 - n - 12 = 0$$

$$\text{sum} = -1; \quad \text{product} = -12$$

#'s are $-4 \,\&\, 3$

$$n^2 - 4n + 3n - 12 = 0$$

$$(n^2 - 4n)(+3n - 12) = 0$$

$$n(n-4) + 3(n-4) = 0$$

$$(n-4)(n+3) = 0$$

ZPP

$$n - 4 = 0 \ \text{ OR } \ n + 3 = 0$$

$$n = 4 \ \text{ OR } \ \cancel{n = -3}$$

$$n = 4$$

2   52.(c) Four students are to be chosen from a group of 12 to fill the positions of president, vice-president, treasurer and secretary. In how many ways can this be accomplished?

Order matters so this question involves permutations.

$$_{12}P_4 = 11880$$

3   53.(a) If a 5-digit number is generated at random from the digits 2, 3, 4, 5 and 8 (with no repetition), what is the probability that it will be an odd number?

odd numbers $= 4 \times 3 \times 2 \times 1 \times 2 = 48$;   # 5-digit numbers $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

$$P(\text{odd number}) = \frac{48}{120} = 0.40$$

3   53.(b) A person will be randomly selected from a group to draw a marble from a bag. The odds of selecting a female from the group is 7:8 and the odds of drawing a red marble from the bag are 1:3. What is the probability of a non-red marble being drawn from the bag by a male from the group?

$$\text{odds in favour of (female)} = \frac{7}{8}; \quad P(\text{female}) = \frac{7}{15}; \quad P(\text{male}) = \frac{8}{15}$$

$$\text{odds in favour of (red)} = \frac{1}{3}; \quad P(\text{red}) = \frac{1}{4}; \quad P(\text{not red}) = \frac{3}{4}$$

$$P(\text{not red \& male}) = P(\text{not red}) \times P(\text{male}) = \frac{3}{4} \times \frac{8}{15} = \frac{24}{60} = \frac{2}{5} = 0.40$$

4   54.(a) Simplify and state restrictions: $\dfrac{1-x^2}{8-8x} \div \dfrac{3x+3}{2(3x-1)}$

$$\frac{1-x^2}{8-8x} \div \frac{3x+3}{2(3x-1)} = \frac{(1+x)(1-x)}{8(1-x)} \div \frac{3(x+1)}{2(3x-1)} = \frac{(1+x)(1-x)}{8(1-x)} \times \frac{2(3x-1)}{3(x+1)}$$

$$= \frac{\cancel{(1+x)}\,\cancel{(1-x)}}{\underset{4}{\cancel{8}}\,\cancel{(1-x)}} \times \frac{\overset{1}{\cancel{2}}(3x-1)}{3\cancel{(x+1)}} = \frac{3x-1}{12}; \quad x \neq \pm 1, \frac{1}{3}$$

2   54.(b) Pat and Chris can paint the house in 5 hours if they work together. Pat is a professional painter and can paint twice as fast as Chris. How long would it take Pat to paint the house by himself?

	Time taken to paint the house (hrs).	Fraction of the house painted in 1hr.
Pat	t	$\dfrac{1}{t}$
Chris	$2t$	$\dfrac{1}{2t}$
Both	5	$\dfrac{1}{5}$

$$\frac{1}{t}+\frac{1}{2t}=\frac{1}{5}$$
$$\text{LCD}=10t$$
$$10t\left(\frac{1}{t}\right)+10t\left(\frac{1}{2t}\right)=10t\left(\frac{1}{5}\right)$$
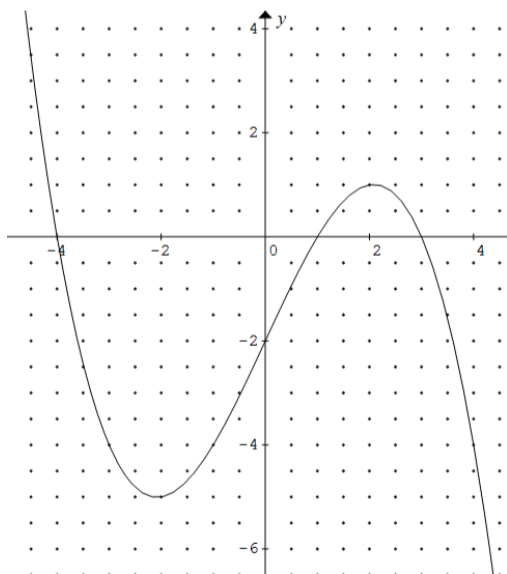$$10+5=2t$$
$$15=2t$$
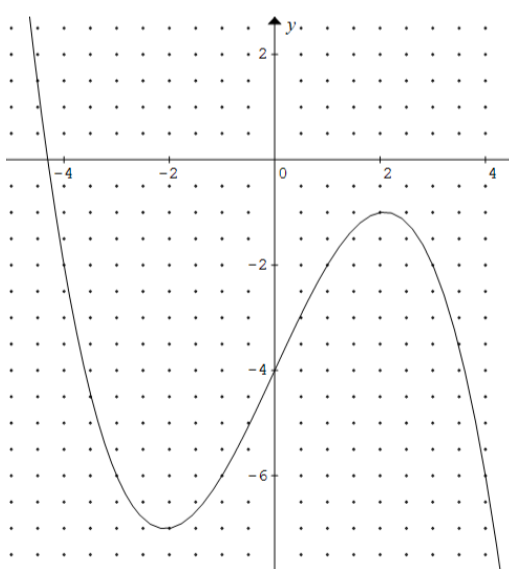$$\frac{15}{2}=\frac{2t}{2}$$
$$t=7.5\text{hrs}$$

It would take Pat 7.5 hrs to paint the house by himself.

2   55.(a) Sketch two possible graphs that are different, yet are both cubic functions with negative leading coefficients and negative y-intercepts. Explain why the graphs you have sketched are different.

Graph 1:   Graph 2:

The graphs have different numbers of x-intercepts.

4   55.(b) Given the function $f(x) = 2x^3 + 5x^2 - 3x - 4$, complete the table to describe its characteristics.

(3)   (i)

y-intercept	-4 or (0,-4)
end behaviour (left and right)	Q3 to Q1 Rises to right. Falls to left.
Max # of possible x-intercepts	3

(1)   (ii)   Explain why the graph of this function is not a parabola.

The degree is 3 and not 2. The end behavior is not correct for a parabola. The maximum possible number of *x*-intercepts for a parabola is 2.

3   56.(a) Algebraically solve for *x*: $\sqrt{3} = 27^{4x+1}$

$$3^{\frac{1}{2}} = \left(3^3\right)^{4x+1}$$

$$3^{\frac{1}{2}} = 3^{3(4x+1)}$$

$$3^{\frac{1}{2}} = 3^{12x+3}$$

$$\Rightarrow \frac{1}{2} = 12x+3$$

$$2\left(\frac{1}{2} = 12x+3\right)$$

$$1 = 24x+6$$

$$1-6 = 24x+6-6$$

$$-5 = 24x$$

$$x = \frac{-5}{24}$$

4   56.(b) Nora is about to invest $5000 in an account that pays 6% interest a year compounded monthly for the next 3 years. A different financial institution offers 6.5% interest a year compounded semi-annually for the next 3 years. Write a function that models the growth of Nora's investment for each situation. Should Nora invest her money in this financial institution instead? Explain why or why not.

$$i = \frac{0.06}{12} = 0.005; \quad n = 3 \times 12 = 36 \qquad\qquad i = \frac{0.065}{2} = 0.0325; \quad n = 3 \times 2 = 6$$

$$A_1 = 5000(1.005)^{36} = 5983.40 \qquad\qquad A_2 = 5000(1.0325)^6 = 6057.74$$

Nora should invest with institution 2 as she will have more money after 3 years if she invests with this institution.

4   57.(a) Algebraically solve for *x*: $5^{x-1} - 8^{x+1} = 0$

$$5^{x-1} = 8^{x+1}$$

$$\log 5^{x-1} = \log 8^{x+1}$$

$$(x-1)\log 5 = (x+1)\log 8$$

$$x\log 5 - \log 5 = x\log 8 + \log 8$$

$$x\log 5 - x\log 8 - \log 5 + \log 5 = x\log 8 - x\log 8 + \log 8 + \log 5$$

$$x\log 5 - x\log 8 = \log 8 + \log 5$$

$$x(\log 5 - \log 8) = \log 8 + \log 5$$

$$\frac{x(\log 5 - \log 8)}{\log 5 - \log 8} = \frac{\log 8 + \log 5}{\log 5 - \log 8}$$

$$x \approx -7.85$$

3   57.(b) The pH scale is used to measure the acidity of a solution. The pH, $p(x)$, is defined by the equation $p(x) = -\log x$, where the concentration of hydrogen ions, x, in a solution is measured in moles per litre (mol/L).

(1)   (i)   Black coffee has a pH of 5. What is its hydrogen ion concentration?

$$5 = -\log x$$
$$-1\big(5 = -\log_{10} x\big)$$
$$-5 = \log_{10} x$$
$$\Rightarrow 10^{-5} = x$$
$$x = 0.00001 \text{mol/L}$$

(2)   (ii)   Baking soda has a pH of 9. In terms of concentration, how much more acidic is black coffee than baking soda?

$$9 = -\log x$$
$$-1\big(9 = -\log_{10} x\big)$$
$$-9 = \log_{10} x$$
$$\Rightarrow 10^{-9} = x$$

The coffee is $\dfrac{10^{-5}}{10^{-9}} = 10^{-5-(-9)} = 10^4 = 10000$ times more acidic than the baking soda.

6   58.(a) Use the sinusoidal function shown below to answer the questions that follow.
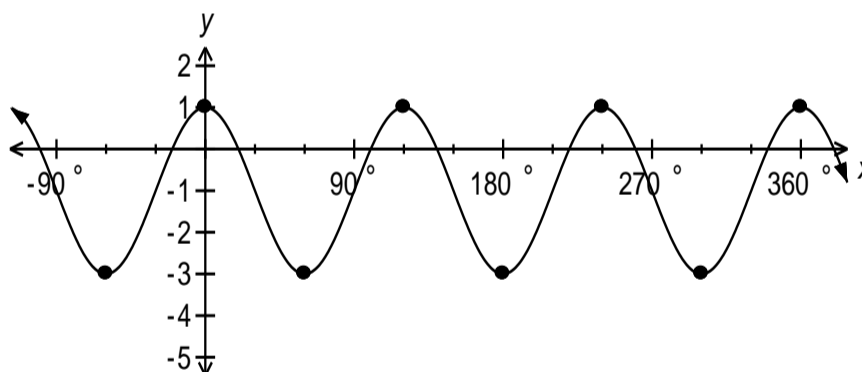
(4)   (i)   Determine the amplitude, period, equation of midline and the range.

$$A = 2; \quad p = 120°; \quad y = -1; \quad \{y \mid -3 \le y \le 1\} \text{ or } [-3,1]$$

(2)   (ii)   Use the information from part (i) to determine a function that represents the graph in the form $y = a\cos b(x) + d$.

$$y = 2\cos 3x - 1$$

3   59.   Pat borrowed $2500 at a rate of 8% compounded monthly for 3 years. How much interest will Pat be charged for borrowing the money?

$$i = \frac{0.08}{12} = 0.00\overline{6}; \quad n = 3 \times 12 = 36$$

$$A = 2500\left(1.00\overline{6}\right)^{36} = 3175.59$$

$$\text{Interest} = 3175.59 - 2500 = \boxed{\$675.59}$$