

Name: Answer Key
Teacher: ORAM

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YOU ARE TOLD BY THE SUPERVISOR TO BEGIN

Mathematics 3201

PRE-PUBLIC EXAMINATION

JUNE 9th, 2016

Value: 100 Marks

Duration: 3 Hours

General Instructions

This examination consists of two parts. Part I of the exam consists of selected response questions and Part II consists of constructed response questions.

Selected Response (50 marks)

Select the letter of the correct response from those provided. Place the CAPITAL LETTER of the correct response in the blank space provided on your Selected Response Answer Sheet.

Constructed Response (50 marks)

Answer ALL questions fully and concisely in the space provided. Show all work.

Student Checklist

The items below are your responsibility. Please ensure that they are completed.

- Write your name and teacher's name on the top of this page.
- Write your name, teacher's name, course name and number on the Selected Response Answer Sheet.
- Check the exam to ensure there are no missing pages. There are 26 pages in total.

ALL MATERIALS MUST BE PASSED IN WITH THIS EXAM. Use your time wisely. Good luck!

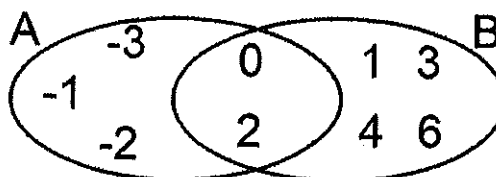
PART I

Total Value: 50 marks

Answer all items. Place the CAPITAL LETTER of the correct answer on the answer sheet provided.

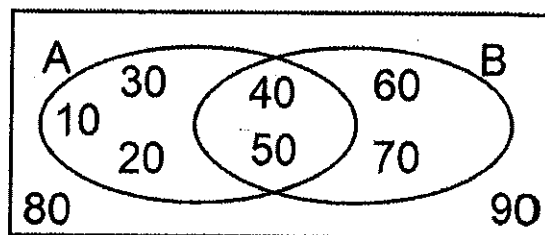
1. Identify the elements in the $A \cap B$, the intersection of A and B.

- (A) {1,3,4,6}
 (B) {0,2}
 (C) {-3,-2,-1,0,1,2,3,4,6}
 (D) {1,3,4,6}



2. Identify the elements in $(A \cup B)'$, the compliment of the union of A and B.

- (A) {10,20,30,40,50,60,70}
 (B) {40,50}
 (C) {10,20,30,60,70,80,90}
 (D) {80,90}



3. Which statement is true for sets A and B?

$$A = \{0,2,4,6,8,10\}$$

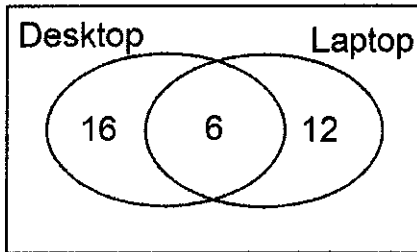
$$B = \{x|x = 3x, x \in \mathbb{N}\} \quad 3, 6, 9, 12$$

- (A) The intersection of A and B, $A \cap B = \{0,6\}$
 (B) A is a subset of B, $A \subset B$
 (C) The compliment of B, $B' = \{0,2,4,6,8,10\}$
 (D) The intersection of A and B, $A \cap B = \{6\}$

4. The table below shows the results of 34 people who own a computer. Which Venn Diagram represents the information in the table?

Computer	# of People
Desktop	22
Laptop	18

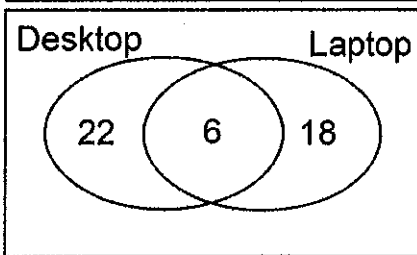
(A)



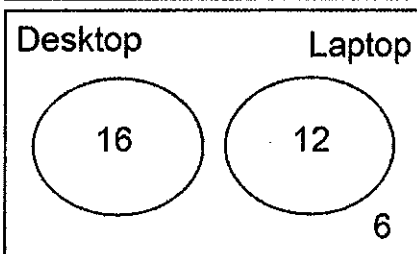
$$22 + 18 = 40$$

$$40 - 34 = 6$$

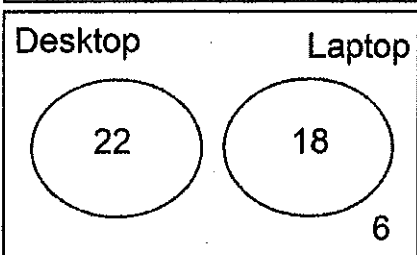
(B)



(C)



(D)



5. Which shows the total number of possible arrangements of 12 distinguishable objects taken 3 at a time?

- (A) ${}_{12}C_3$
- (B) ${}_3C_{12}$
- (C) ${}_{12}P_3$
- (D) ${}_3P_{12}$

no positions.

$${}_{12}C_3$$

6. Chris is selecting a new shirt. He can choose short sleeve or long sleeve. Each shirt comes in 3 colours and is either striped or plain. How many choices does he have?

Shirt
Long Sleeve
Short Sleeve

Colour
Red
Blue
Green

Pattern
Striped
Plain

- (A) 6
- (B) 7
- (C) 12
- (D) 24

$$2 \cdot 3 \cdot 2 = 12$$

7. A 3 digit password is created using the numbers 0 to 9. If repetition is permitted, how many passwords are possible if the last digit in the password must be 5?

- (A) 81
- (B) 90
- (C) 100
- (D) 500

$$\underline{10} \cdot \underline{10} \cdot \underline{5} = 500$$

8. A hockey team consists of 12 forwards and 8 defencemen. Determine the notation which indicates the number of ways in which a starting lineup of 3 forwards and 2 defencemen can be chosen.

- (A) ${}_{20}C_5$
- (B) ${}_{20}P_5$
- (C) ${}_{12}C_3 \times {}_8C_2$
- (D) ${}_{12}P_3 \times {}_8P_2$

positions

$${}_{12}P_3 \times {}_8P_2$$

9. A diner has a lunch special where customers have a choice of 5 soups, 4 sandwiches, and 3 desserts. How many meals are possible from this selection if you select an item from each category?

- (A) ${}_5P_1 \times {}_4P_1 \times {}_3P_1$
- (B) ${}_5C_1 \times {}_4C_1 \times {}_3C_1$
- (C) ${}_{12}P_3$
- (D) ${}_{12}C_3$

Comb.

10. Solve for n:

$$\frac{n!}{(n-2)!} = 12, \quad \text{where } n \in I$$

- (A) -4
- (B) -3
- (C) 3
- (D) 4

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 12$$

$$n^2 - n - 12 = 0 \quad (n-4)(n+3) \quad \begin{matrix} n=4 \\ n \neq -3 \end{matrix}$$

11. Identify the numeric expression which indicates the probability of getting heads if a fair coin is tossed 3 times.

- (A) $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$
- (B) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
- (C) $2 + 2 + 2$
- (D) $2 \times 2 \times 2$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

12. The probability of getting a snow day tomorrow is 40%. What are the odds in favor of a snow day tomorrow?

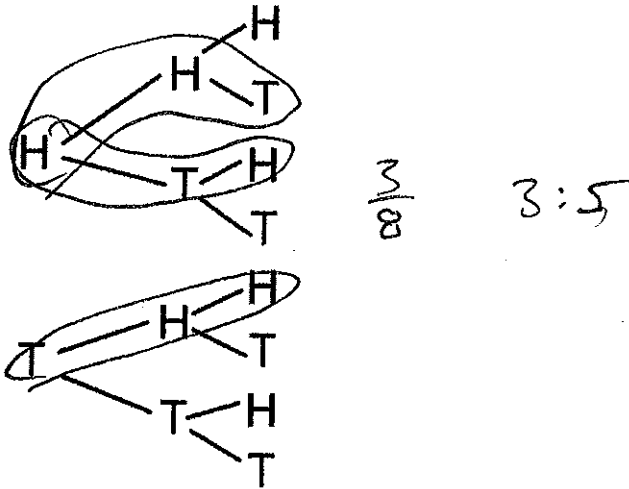
- (A) 2:3
- (B) 2:5
- (C) 3:2
- (D) 5:2

$$40\% : 60\%$$

$$2 : 3$$

13. A fair coin is tossed 3 times. Given the tree diagram below what are the odds in favor of flipping at least 2 heads?

- (A) 1:4
- (B) 1:3
- (C) 3:8
- (D) 3:5



14. If you have 5 nickels, 3 dimes, and 2 quarters, what is the probability of selecting (without replacement) 2 coins that have a combined value of 50 cents?

- (A) $\frac{1}{168}$
- (B) $\frac{1}{50}$
- (C) $\frac{1}{45}$
- (D) $\frac{1}{25}$

$$\frac{2}{10} \cdot \frac{1}{9} = \frac{2}{90} = \frac{1}{45}$$

15. If $n(U) = 20$ and the percent of elements in set A is 50%, the percent of elements in B is 60%, and the number of elements in the complement of the union of A and B, $n(A \cup B)' = 2$, what is $n(A \cap B)$, the number of elements in the intersection of A and B?

- (A) 0
- (B) 2
- (C) 4
- (D) 8

18 in $A \cup B$

10 in A

12 in B

$$10 + 12 = 22$$

$$22 - 18 = 4 = n(A \cap B)$$

16. Which rational expression is equivalent to

$$\frac{3 \cdot 2}{x+2}, x \neq -2$$

- (A) $\frac{3x}{x^2+2}$
(B) $\frac{3x}{3x^2+2x}$
(C) $\frac{6}{2x+2}$
(D) $\frac{6}{2x+4}$

17. What is the lowest common denominator of

$$\frac{x}{x-2} + \frac{3}{2x-3}$$

- (A) $-2(2x)$
(B) $(x-2)(2x-3)$
(C) $(3x-5)$
(D) $(2x^2+6)$

18. Simplify:

$$\frac{2x-8}{3x^2-48} = \frac{2(x-4)}{3(x-4)(x+4)} = \frac{2}{3(x+4)}$$

- (A) $\frac{2}{3(x+4)}$
(B) $\frac{2}{3(x-4)}$
(C) $\frac{1}{3x-6}$
(D) $\frac{2(x-4)}{3x^2-48}$

19. Simplify:

$$\frac{2x+1}{x-2} \div \frac{(2x+1)(x-2)}{4}$$

- (A) $\frac{4}{(x-2)^2}$
- (B) $\frac{(2x+1)^2}{4}$
- (C) $\frac{4}{2x-4}$
- (D) $\frac{4x+2}{4}$

$$\frac{\cancel{2x+1}}{x-2} \times \frac{4}{\cancel{(2x+1)}(x-2)}$$
$$\frac{4}{(x-2)^2}$$

20. Simplify:

$$3 \cdot \frac{3}{2x} + \frac{8 \cdot 2}{3x} \quad \text{LCD } 6x$$

- (A) $\frac{2}{x}$
- (B) $\frac{25}{6}$
- (C) $\frac{25}{6x}$
- (D) $\frac{11}{5x}$

$$\frac{9 + 16}{6x} = \frac{25}{6x}$$

21. Solve:

$$\frac{2}{3x+5} = \frac{5}{4x+2}$$

- (A) -3
- (B) $-\frac{12}{7}$
- (C) $\frac{12}{7}$
- (D) 3

$$2(4x+2) = 5(3x+5)$$

$$8x+4 = 15x+25$$

$$\frac{-21}{7} = 7x$$

$$-3 = x$$

22. Determine the y-intercept and end behavior of

$$f(x) = -3x^3 + 4x^2 + 6x - 3$$

(-3) y, int

	y-intercept	end behaviour
(A)	-3	QII - QIV
(B)	-3	QIII - QI
(C)	3	QII - QIV
(D)	3	QIII - QI

-neg cubic

23. What is the maximum number of turning points that could be on the graph of a function of the form

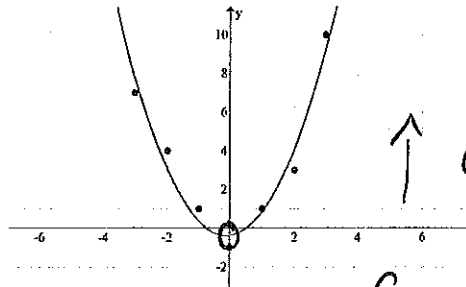
$$f(x) = ax^3 + bx^2 + cx + d$$

- (A) 0
- (B) 1
- (C) 2
- (D) 3

cubic 2 max.

24. Given the table of values and graph, which statement would be TRUE about the a and c values in the quadratic regression?

x	y
-3	7
-2	4
-1	1
0	-1
1	1
2	3
3	10

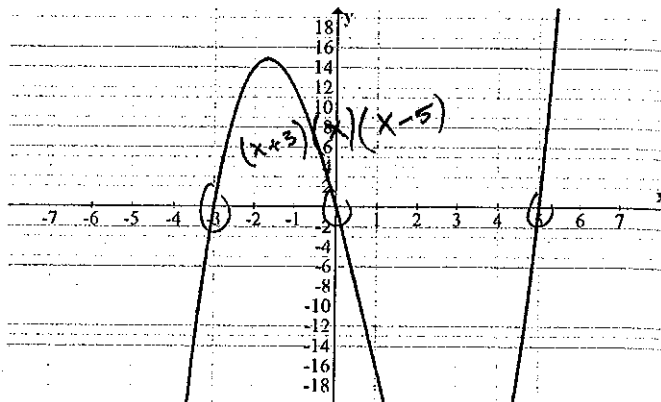


*↑ a positive (>0)
c (negative y, int) (<0)*

- (A) $a > 0, c > 0$
- (B) $a < 0, c < 0$
- (C) $a < 0, c > 0$
- (D) $a > 0, c < 0$

25. Which polynomial function is represented by the graph?

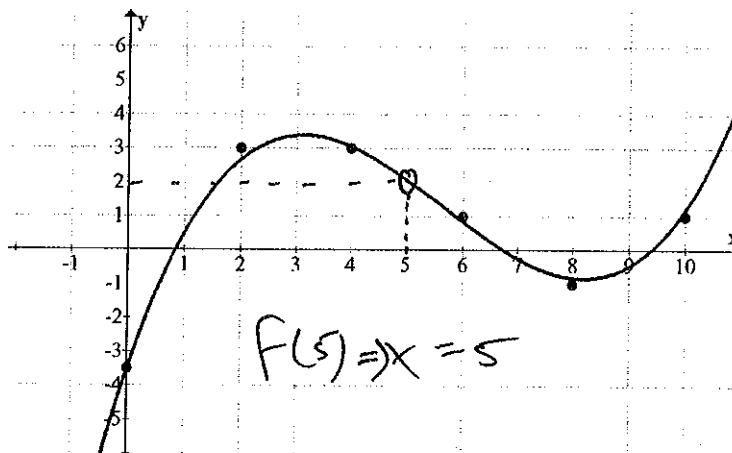
- (A) $x^3 - 15x$
 - (B) $x^3 + 15x$
 - (C) $x^3 - 2x^2 - 15x$
 - (D) $x^3 + 2x^2 - 15x$
- $\times (x^2 + 2x - 15)$
 $(x+3)(x-5)$



* 26. Given the scatter plot and equation of the cubic regression function what is $f(5)$?

- (A) 1.5
- (B) 2
- (C) 11.5
- (D) 12

$$f(x) = 0.0655x^3 - 1.112x^2 + 5.04x - 3.5$$



omit
 * 27. What is the vertex and y-intercept of $f(x) = 3(x+2)^2 - 5$?

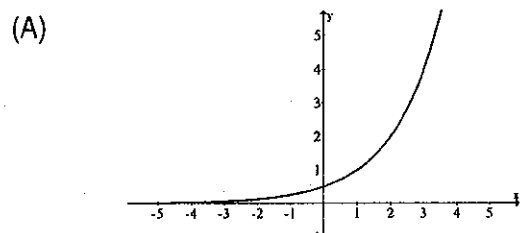
	Vertex	y-intercept
(A)	$(-2, 5)$	-17
(B)	$(2, 5)$	-17
(C)	$(-2, 5)$	-11
(D)	$(2, 5)$	-11

y int $\rightarrow x=0 \rightarrow$
 vert. $(-2, -5)$

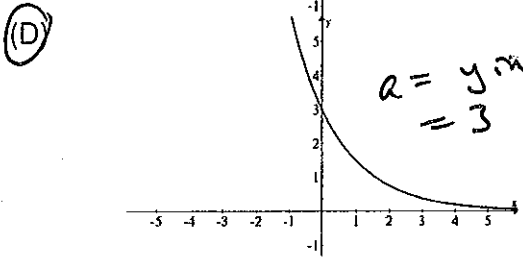
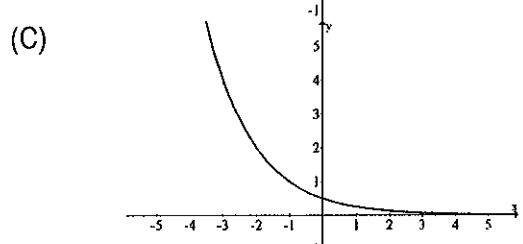
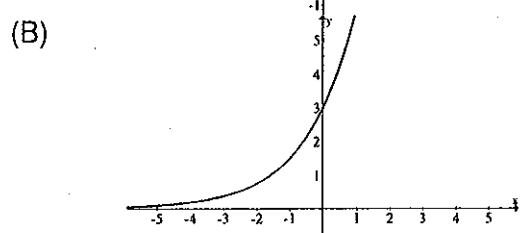
28. Given the equation $P(t) = 5(3)^{\frac{t}{4}}$, where $P(t)$ represents the population of bacteria over time, what was the initial population and growth rate of the bacteria?

	Initial Amount	Growth Rate
(A)	3	4
(B)	3	5
<input checked="" type="radio"/> (C)	5	3
(D)	5	4

29. Which graph matches the equation $f(x) = 3\left(\frac{1}{2}\right)^x$?



decreasing.



30. Solve for x:

$$5^{x-3} = \frac{1}{25}$$

- (A) -6
- (B) 0
- (C) 1
- (D) 6

$$5^{x-3} = 5^{-2}$$

$$x-3 = -2$$

$$x = 1$$

31. Which of the following is true of the two equations?

$$\text{Function \#1: } f(x) = \left(\frac{1}{2}\right)^x$$

$$\text{Function \#2: } g(x) = \left(\frac{5}{4}\right)^x$$

- (A) Both are increasing
- (B) Both are decreasing
- (C) Function 1 is increasing and Function 2 is decreasing
- (D) Function 1 is decreasing and Function 2 is increasing

32. Which is true of the table given below?

Years (x)	-2	0	2	4	6
Amount (y)	4	12	36	108	324

2 years
Initial triple

- | | Initial Amount | Growth rate |
|--------------------------------------|----------------|---------------------------|
| (A) | 4 | Triples every two years |
| (B) | 4 | Doubles every three years |
| <input checked="" type="radio"/> (C) | 12 | Triples every two years |
| (D) | 12 | Doubles every three years |

33. An investment can be modelled by the following growth function, where x represents the time in months.

$$y = 2000(1.012)^x$$

What is the value of the investment after two years?

- (A) 370.78
- (B) 2048.29
- (C) 2662.95
- (D) 48 576.00

$$7.12 = 24$$

$$2000(1.012)^{24}$$

34. Write the following equation in logarithmic form.

$$10^4 = x$$

- (A) $x = \log 4$
- (B) $x = \log 10$
- (C) $4 = \log x$
- (D) $10 = \log_4 x$

$$\log_{10}(x) = 4$$

35. Given the formula $\beta = 10(\log I + 10)$, where β is the sound level in decibels and I is the sound's intensity in watts per square metre. If the intensity is 0.5 W/m^2 find the sound level to the nearest whole number.

- (A) 9
- (B) 11
- (C) 97
- (D) 120

$$10(\log(0.5) + 10)$$

36. Evaluate:

- (A) 2
- (B) 3
- (C) 10
- (D) 12

$$\log_4 2 + 5 \log_4 2$$

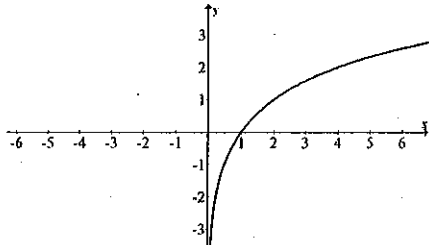
$$= \log_4 2 + \log_4 32$$

$$= \log_4 64$$

$$= \frac{\log 64}{\log 4} = 3$$

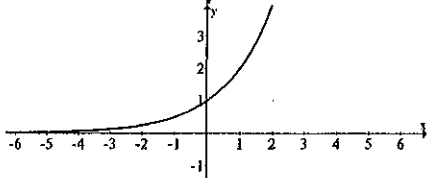
37. Which graph represents $y = \log_2 x$?

(A)

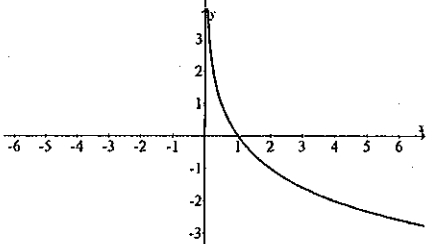


inc log.

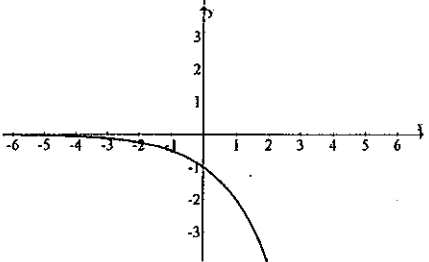
(B)



(C)



(D)



38. Solve for x:

$$2^{x-1} = 6$$

(A) $\log_2 6 - 1$

(B) $\log_2 6 + 1$

(C) $\log_6 2 - 1$

(D) $\log_6 2 + 1$

$$(x-1)(\log 2) = \frac{\log 6}{\log 2} + 1$$

$$\log_2 6 + 1$$

39. Identify the step in which the error occurs in the simplification of the following expression.

$$\frac{1}{2} \log_3 81 - \left(2 \log_3 3 + \log_3 \frac{1}{3} \right)$$

Step 1 $\log_3 81^{\frac{1}{2}} - \left(\log_3 3^2 + \log_3 \frac{1}{3} \right)$

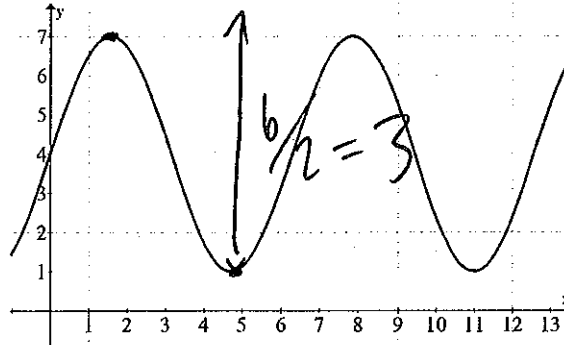
Step 2 $\log_3 9 - \left(\log_3 9 + \log_3 \frac{1}{3} \right)$

Step 3 $\log_3 9 - (\log_3 3)$

Step 4 $\log_3 6$ $\div \text{to } \frac{9}{3} = 3$

- (A) Step 1
- (B) Step 2
- (C) Step 3
- (D) Step 4

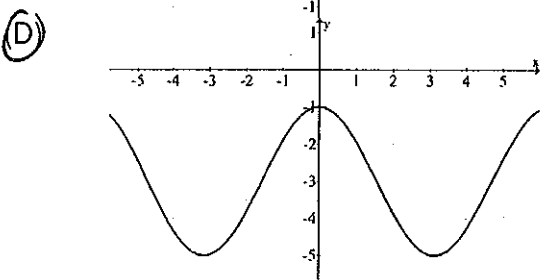
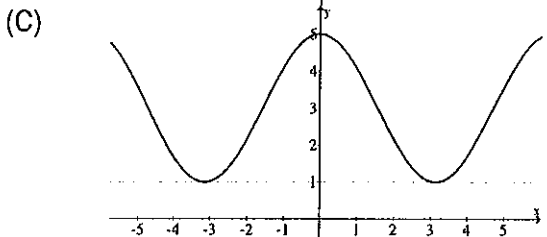
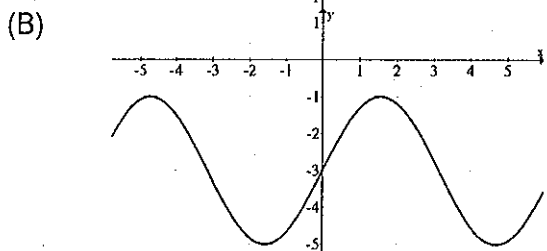
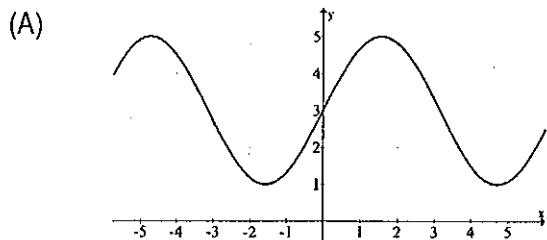
40. What is the amplitude of the sinusoidal function depicted below?



- (A) 3
- (B) 4
- (C) 6
- (D) 7

41. What is the graph of the function?

$$y = 2 \cos x - 3$$



start at max
mid -3
amp 2

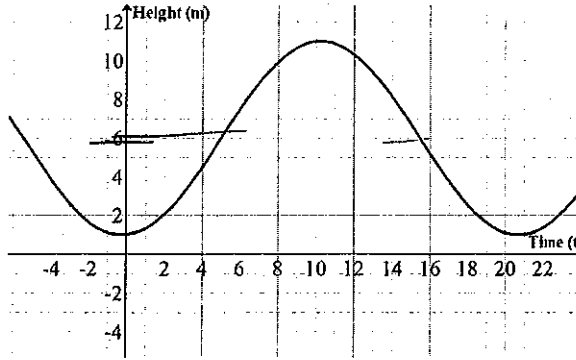
42. What is $\frac{3\pi}{8}$ radians written using degree measure?

- (A) 33.8°
- (B) 67.5°
- (C) 212°
- (D) 665.5°

$$\frac{3\pi}{8} \times \frac{180}{\pi} = 67.5$$

43. Given the graph of a Ferris wheel, what is the height of the centre of the Ferris wheel?

- (A) 5 m
- (B) 6 m
- (C) 10 m
- (D) 11 m



44. The function $h(t) = 6 \sin[10(t - 90)] + 8$ represents the height of a person, in metres, on a Ferris wheel over time. What is the height of a person, in metres, after 38 seconds?

- (A) 5.9
- (B) 14.0
- (C) 46.2
- (D) 177.7

$$\begin{aligned} \text{min } 8 - 6 &= 2 \\ \text{max} &= 8 + 6 = 14 \end{aligned}$$

45. Determine the range of

$$y = 3 \sin(x - 2) + 5$$

- (A) $\{y | y \geq 0, y \in \mathbb{R}\}$
- (B) $\{y | y \geq 5, y \in \mathbb{R}\}$
- (C) $\{y | -2 \leq y \leq 8, y \in \mathbb{R}\}$
- (D) $\{y | 2 \leq y \leq 8, y \in \mathbb{R}\}$

$$\begin{aligned} 5 - 3 \\ 5 + 3 \\ 2 - 8 \end{aligned}$$

46. Given the following amortization table determine the amount owing after 5 years.

Amortization Schedule				
0				\$200,000.00
1	\$1,297.20	\$1,125.00	\$172.20	\$199,827.80
2	\$1,297.20	\$1,124.03	\$173.16	\$199,654.64
3	\$1,297.20	\$1,123.06	\$174.14	\$199,480.50
4	\$1,297.20	\$1,122.08	\$175.12	\$199,305.38
5	\$1,297.20	\$1,121.09	\$176.10	\$199,129.28
6	\$1,297.20	\$1,120.10	\$177.09	\$198,952.18

- (A) \$176.10
- (B) \$1 121.09
- (C) \$1 297.20
- (D) \$199 129.28

47. Bill is buying a new home that costs \$220 000. Using the screen provided, what will be the number of years to repay the loan?

- (A) 3
- (B) 12
- (C) 30
- (D) 36

$N=360$ payments
 $I\% = 4.75$
 $PV = 220000$
 $PMT = -1147.62414$
 $FV = 0$
 $P/Y = 12$ comp.
 $C/Y = 12$
 $PMT: [] [] [] []$ BEGIN

$$\frac{360}{12} = 30 \text{ y.}$$

48. Which of the following will depreciate over time?

- (A) New home
- (B) New car
- (C) Rare coin collection
- (D) Rare stamp collection

49. How much would Kim have to invest at 1.2% compounded annually for 15 years to have a future value of \$59 796.77?

- (A) \$10 924.65
- (B) \$50 000
- (C) \$53 389.97
- (D) \$71 513.07

$$n = 15 \quad i = 1.2$$

$$\frac{A}{(1+i)^n} = \frac{59796.77}{(1.012)^{15}}$$

50. If $A = 10\,000(1.013)^{12}$, what is the interest rate as a percentage (%)?

- (A) 0.13
- (B) 1.013
- (C) 1.3
- (D) 13

$$1.013 - 1 = 0.013 \times 100 = 1.3\%$$

PART II

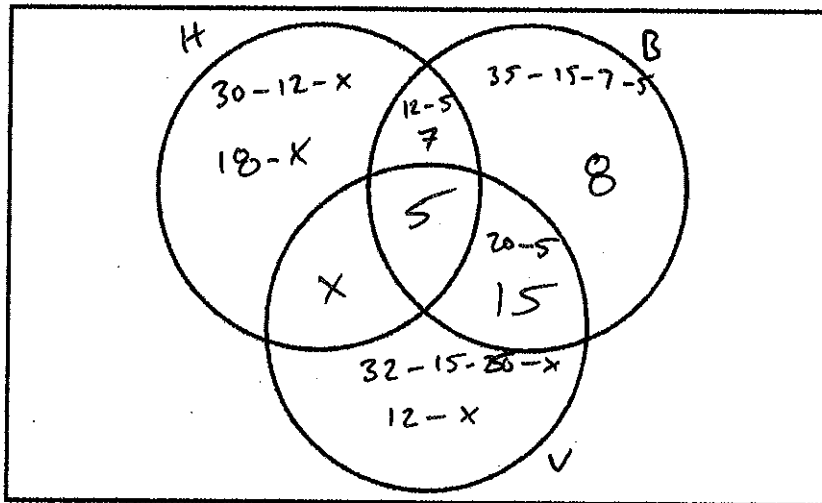
Total Value: 50%

Value

2 51. 60 student participate in school sports.

- 30 play hockey
- 32 play volleyball
- 35 play basketball
- 12 play hockey and basketball
- 20 play volleyball and basketball
- 5 play all three sports

How many students play only volleyball?



$$60 = (18 - x) + (x) + (12 - x) + 7 + 5 + 15 + 8$$

$$60 = 65 - x$$

$$-5 = -x$$

$$5 = x$$

$$\text{just Volley} = 12 - x = 7.$$

- 3 52. (a) Algebraically solve for n:

$${}_{n-2}C_2 = 15$$

$$\frac{n-2!}{(n-4)! 2!} = 15$$

$$\frac{(n-2)(n-3)(\cancel{n-4})! \cdot \cancel{2!}}{(\cancel{n-4})! 2!} = 15 \cdot 2!$$

$$n^2 - 3n - 2n + 6 = 30$$

$$n = 8 \checkmark$$

$$n^2 - 5n - 24 = 0$$

$$n = 8 \text{ extraneous}$$

$$(n-8)(n+3)$$

52. (b) In Newfoundland and Labrador a license plate number consists of a three letters and three digits (ABC 123).

1

- (i) How many license plates are possible if repetition is allowed?

$$\underline{26} \quad \underline{26} \quad \underline{26} \quad \underline{10} \quad \underline{10} \quad \underline{10} = 17576000$$

1

- (ii) Determine how the result would change in 52.(b)(i), if repetition is NOT allowed and zero can not be the 1st digit.

$$\underline{26} \quad \underline{25} \quad \underline{24} \quad \underline{9} \quad \underline{9} \quad \underline{8} = 10108800$$

53. (a) A Math 3201 class consists of 6 boys and 8 girls. The teacher wants a group of 5 to go to a Math Competition.

- 1 (i) What is the probability of the group consisting of 5 girls?

$$\text{Total } {}^{14}C_5 = 2002$$

$$\text{Fav } {}^8C_5 = 56$$

$$P = \frac{\text{fav}}{\text{tot}} = \frac{56}{2002} = \frac{4}{143} = 0.03$$

- 1 (ii) What is the probability that the team will have 3 boys?

$$\text{Tot } {}^{14}C_5 = 2002$$

$$\text{fav } {}^8C_2 {}^6C_3 = 28 \cdot 20 = 560$$

$$P = \frac{\text{fav}}{\text{tot}} = \frac{560}{2002} = 0.28$$

- 2 (iii) What is the probability that the team will have at most 3 boys?

$$3B2G \quad 2B3G \quad 1B4G \quad 0B5G$$

$$\text{Tot} = {}^{14}C_5 = 2002$$

$$\begin{aligned} \text{fav} &: {}^6C_3 {}^8C_2 + {}^6C_2 {}^8C_3 + {}^6C_1 {}^8C_4 + {}^8C_5 \\ &= 28 \cdot 20 + 15 \cdot 56 + 6 \cdot 70 + 56 \\ &= 560 + 840 + 420 + 56 \\ &= 1876 \end{aligned}$$

$$P = \frac{\text{fav}}{\text{tot}} = \frac{1876}{2002} = \frac{134}{143} = 0.94$$

53. (b) A 5 digit password is created by choosing from the digits 0 to 9 and the 26 letters of the alphabet.

2 (i) If letters are not case sensitive and repetition is not allowed, what is the probability that the password starts and ends with a vowel?

26 letters
10 digits
5 vowels

Total: $\underline{36} \quad \underline{35} \quad \underline{34} \quad \underline{33} \quad \underline{32} = \frac{45239040}{60466176}$

Fav: $\underline{5} \quad \underline{34} \quad \underline{33} \quad \underline{32} \quad \underline{4} = 718080$

$$P = \frac{\text{fav}}{\text{tot}} = \frac{718080}{\frac{45239040}{60466176}} = 0.0188$$

2 (ii) does the answer in 53.(b)(i) change if the password is case sensitive? If so, what is the new probability of the password starting and ending with a vowel?

Yes, double amount of letters. caps & lower case

Tot: $\underline{62} \quad \underline{61} \quad \underline{60} \quad \underline{59} \quad \underline{58} = \frac{916132832}{776520240}$

Fav: $\underline{5} \quad \underline{60} \quad \underline{59} \quad \underline{58} \quad \underline{4} = 4106400$

$$P = \frac{\text{fav}}{\text{tot}} = \frac{4106400}{\frac{916132832}{776520240}} = 0.00545$$

54. (a) Ken and Joe are mowing a lawn. When Joe mows the lawn by himself he is 0.5 hours faster than when Ken mows the lawn by himself.

3 (i) If it takes them 3 hours to mow the lawn together, how long does it take Joe by himself?

$x = \text{Joe}$
 $x + 0.5 = \text{Ken}$
 $3 = \text{total}$

$$\frac{1}{x} + \frac{1}{x+0.5} = \frac{1}{3}$$

$$3x + 1.5 + 3x = 3x^2 + 0.5x$$

$$6x + 1.5 = 3x^2 + 0.5x$$

$$3x + 1.5 + 3x$$

$$= x^2 + 0.5x$$

$$6x$$

$$x^2 - 5.5x + 1.5$$

$$x \approx 5.76$$

$$0 = \text{~~3x^2 - 4.5x - 1.5~~}$$

$$= \text{~~3x^2 - 4.5x - 1.5~~}$$

$$= \text{~~3(x^2 - 1.5x - 0.5)~~}$$

$$= 3x^2 - 4.5x - 1.5$$

$$= 3(x^2 - 1.5x - 0.5)$$

$$x = \frac{1.5 \pm \sqrt{(1.5)^2 - 4(1)(-0.5)}}{2(1)}$$

$$= \frac{1.5 \pm 2.06}{2}$$

$$x = -\frac{1}{4} \quad x = 1.75 \checkmark$$

$$x = \frac{3.56}{2} \quad x = \frac{1.56}{2}$$

1 (ii) How would the equation change if Joe can mow the lawn twice as fast as Ken?

$2x = \text{Ken}$
 $x = \text{Joe}$

$$\frac{1}{2x} + \frac{1}{x} = \frac{1}{3}$$

54. (b) Jimmy incorrectly simplified the following rational expression:

$$\begin{aligned} & \frac{x}{3x+6} - \frac{x+2}{2} \\ &= \frac{x}{3(x+2)} - \frac{x+2}{x} \\ &= \frac{x}{3(x+2)} - \frac{x+2}{2} \\ &= \frac{x}{3} - \frac{1}{2} \\ &= \frac{2x-3}{6} \end{aligned}$$

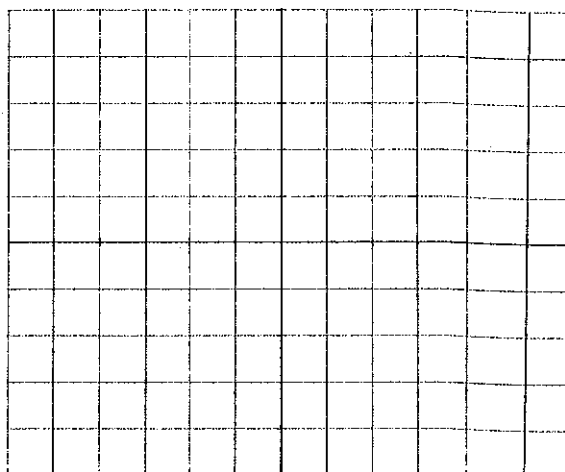
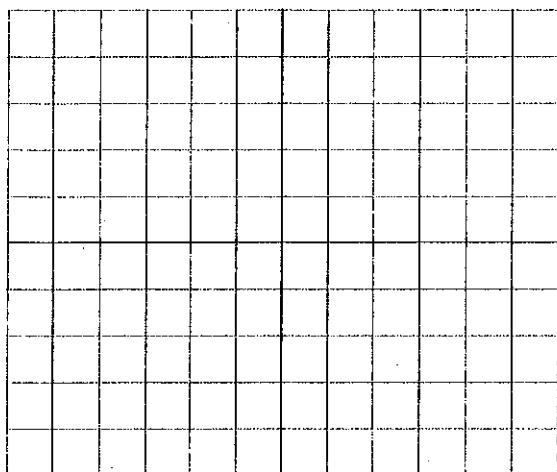
- 1 (i) Identify and explain any error he had in his simplification.

cannot cancel through diagonal for a subtraction.

- 1 (ii) Provide a correct solution to Jimmy's problem.

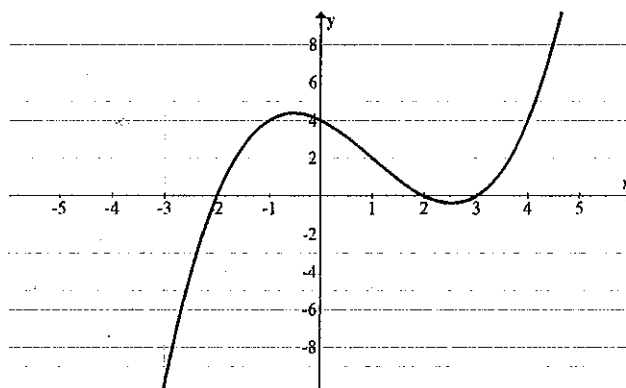
$$\begin{aligned} & \frac{x-2}{3(x+2)-2} - \frac{x+2}{2} \quad \text{LCD} = 6(x+2) \\ &= \frac{2x - \cancel{3(x+2)}}{6(x+2)} = \frac{-3x^2 + 10x + 12}{6(x+2)} \\ &= \frac{2x - (3x^2 + 12x + 12)}{6(x+2)} \end{aligned}$$

- 2 55. (a) Sketch two different polynomial graphs which have the same end behavior, number of turning points, and y-intercept



varryin answers.

55. (b) Given the graph of the polynomial function below

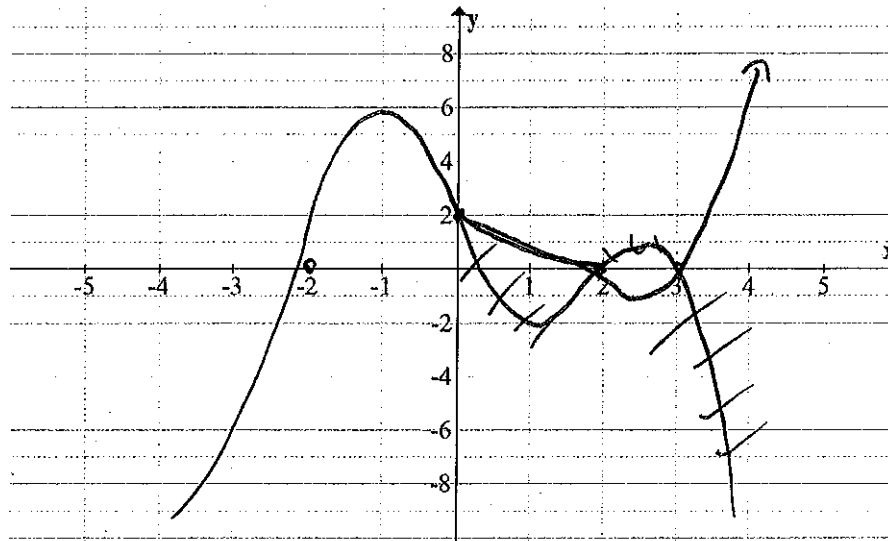


- 2 (i) Determine the end behavior, the number of turning points, y-intercept, and degree.

End behaviour	<i>III to I</i>
# of turning points	<i>2</i>
Y-intercept	<i>(0, 4)</i>
Degree	<i>3 (cubic)</i>

2

- (ii) If the y-intercept of the graph given was changed to 2 while keeping the x-intercepts the same, sketch the resulting graph and describe how the change in the y-intercept affects the appearance of the graph.



3 56. (a) Solve:

$$\sqrt{\frac{1}{3}} = 9^{5x+1}$$

$$\frac{1}{3} = 9^{10x+2}$$

$$3^{-1} = 3^{2(10x+2)}$$

$$-1 = 20x + 4$$

$$-5 = 20x$$

$$\frac{-5}{20} = x$$

$$-\frac{1}{4} = x$$

- 2 56. (b) Given the following formulas representing the temperature in two objects

$$\text{Object 1: } A(t) = 90 \left(\frac{1}{2}\right)^{\frac{t}{5}}$$

$$\text{Object 2: } A(t) = 90 \left(\frac{1}{3}\right)^{\frac{t}{5}}$$

Determine which object will cool more quickly and explain why.

object 2 lose $\frac{2}{3}$ of it's heat in 5 hours whereas object 1 loses $\frac{1}{2}$ its heat in the same time.

object 2 cools faster.

57. (a) The formula for pH is $P(x) = -\log x$, where x is the concentration of the Hydrogen Ion and $P(x)$ is the pH of the solution.

Common Substances	pH
Hydrochloric Acid (1 mol/L)	0.0
Gastric Juices	1.0
Lemon Juice	2.2
Vinegar	2.4
Tomato Juice	4.1
Urine	6.0
Pure Water	7.0
Blood	7.4
Baking Soda Solution	8.4
Toothpaste	9.9
Milk of Magnesia	10.5
Household Ammonia	11.9
Sodium Hydroxide (1 mol/L)	14.0

↑ ACID
 NEUTRAL
 ↓ ALKALINE

- 2 (i) Determine the pH and identity of an unknown substance, if the hydrogen ion concentration is 0.00398.

$$p(x) = -\log(0.00398) = 2.4$$

the sub. is vinegar.

- 2 (ii) Using the pH from the table, what is the hydrogen ion concentration of toothpaste?

$$\text{tooth paste pH} = 9.9$$

$$x = 10^{-\text{pH}} = 10^{-9.9}$$

$$x = 1.26 \times 10^{-10} \text{ mol/L}$$

57. (b) The equation $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{4}}$ represents mass, in grams (g), of a radioactive sample where the half-life is 4 years.

- 1 (i) If the initial mass is 60g, what is the amount remaining after 12 years?

$$\begin{aligned} A(12) &= 60 \left(\frac{1}{2}\right)^{\left(\frac{12}{4}\right)} \\ &= 7.5 \text{ g.} \end{aligned}$$

- 3 (ii) If the initial mass of the substance was 240g how long will it take to reach 60 grams?

$$\frac{60}{240} = \frac{240}{240} \left(\frac{1}{2}\right)^{\frac{x}{4}}$$

$$\frac{\log\left(\frac{1}{4}\right)}{\log\left(\frac{1}{2}\right)} = \frac{x}{4} \log\left(\frac{1}{2}\right)$$

$$\log\left(\frac{1}{2}\right)$$

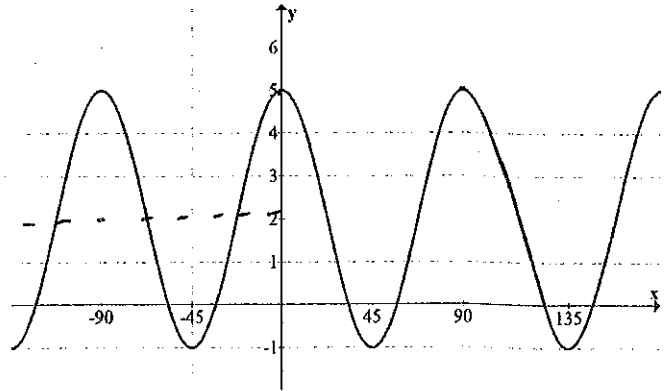
$$2 = \frac{x}{4}$$

$$x = 8$$

8 years

4 57. (a) Use the sinusoidal function shown to complete the following table.

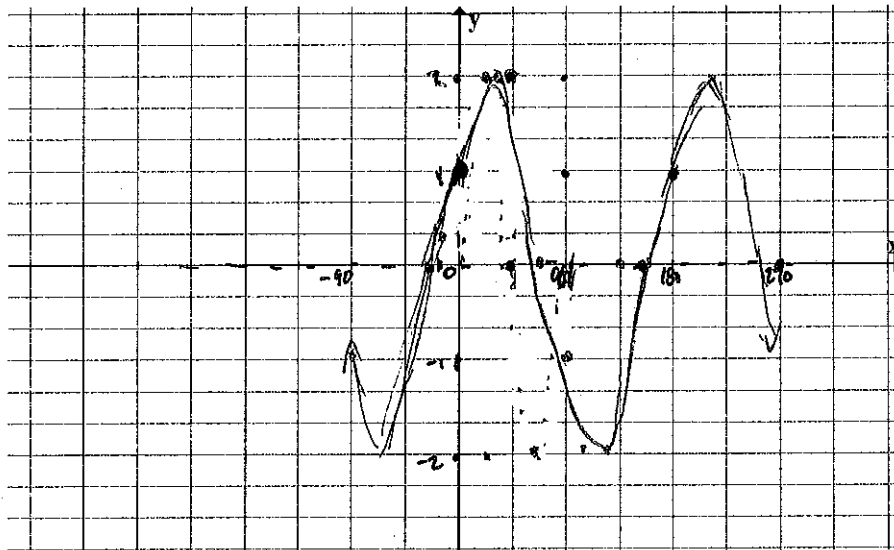
Amplitude	3
Period	90°
Equation of Midline	$y = 2$
Maximum Value	5
Minimum Value	-1



2 58. (b) Sketch a sinusoidal function with the following characteristics:

- Domain: $\{x \mid -90^\circ \leq x \leq 270^\circ\}$
- Period: 180°
- Range: $\{y \mid -2 \leq y \leq 2, y \in \mathbb{R}\}$
- Y-intercept: $(0, 1)$

mid is \emptyset



- 4 59. Mary borrows \$1200 at 8% interest compounded annually. Doug borrows \$1000 at 10% interest compounded annually. Write an exponential equation for each option and determine which is the best, if borrowed over 7 years.

$$\text{Mary: } A = 1200(1.08)^7$$

$$A = 2056.59$$
$$- 1200$$

$$\text{int} = 856.59$$

$$\text{Doug: } A = 1000(1.10)^7$$

$$= 1948.72$$

$$\text{int} = 948.72$$

Mary had better deal
more up front less total
interest.

Math 3201 Information Sheet

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$${}_n C_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$${}_n P_r = \frac{n!}{(n-r)!}$$

$$\frac{n!}{a!b!c!\dots}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + rt) \qquad A = P(1 + i)^n$$

