Mathematics 3201 Unit 7: Logarithmic Functions Name:_____

Selected Response: Write the UPPERCASE letter of the correct answer in the correct blank on page 5. (18 marks)

1. What is the end behavior of $y = \log x$?

   (A) I to II
   (B) I to IV
   (C) II to I
   (D) IV to I

2. What is the end behavior of $y = \ln x$?

   (A) I to II
   (B) I to IV
   (C) II to I
   (D) IV to I

3. What is the end behavior of $y = -6 \ln x$?

   (A) I to II
   (B) I to IV
   (C) II to I
   (D) IV to I

4. What is true of the graph of $y = -\ln x$?

   (A) x-intercept is $(-1,0)$; no y-intercept
   (B) x-intercept is $(1,0)$; no y-intercept
   (C) no x-intercept is; y-intercept$(0,-1)$
   (D) no x-intercept is; y-intercept $(0,1)$

5. What is the domain of $y = 7 \ln x$?

   (A) $\{x|x > 0; x \epsilon R\}$
   (B) $\{x|x > 7; x \epsilon R\}$
   (C) $\{x|x < 0; x \epsilon R\}$
   (D) $\{x|x < 7; x \epsilon R\}$

6. What is the range of 4 log x?

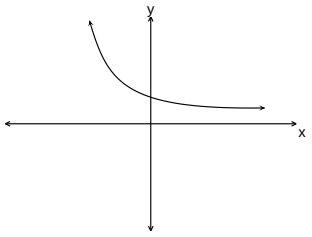
(A) $\{y|y > 0; y \epsilon R\}$
(B) $\{y|y > 4; y \epsilon R\}$
(C) $\{y|y \epsilon R\}$
(D) $\{y|y < 4; y \epsilon R\}$

7. Which function will have the fastest decrease in x-values

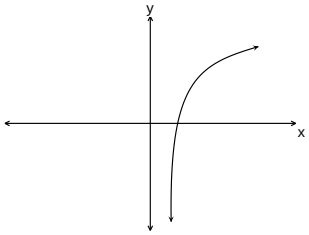
(A) y = -3 log x
(B) y = 2 log x
(C) y = $\frac{-1}{3}$ log x
(D) y = $\frac{1}{2}$ log x

8. Which graph best represents the function $y = \ln x$?

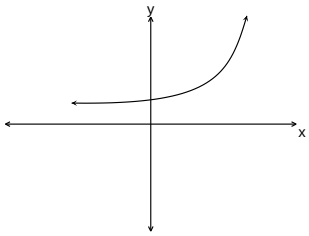
(A)



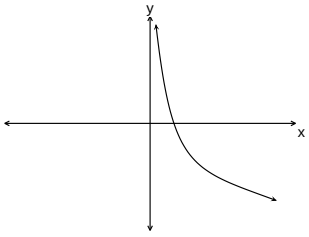
(B)



(C)

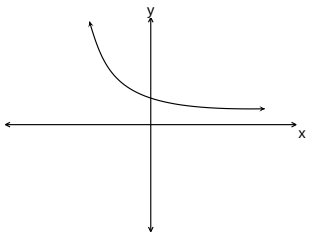


(D)

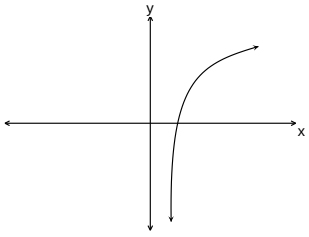


9. Which graph best represents the function $y = 7\log x$?

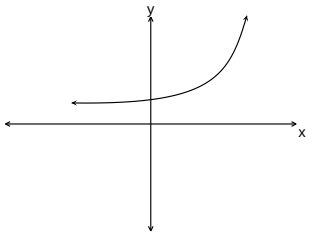
(A)



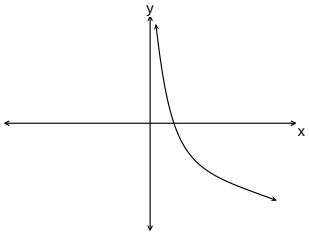
(B)



(C)



(D)



10. What is the logarithmic form of $\left(\sqrt{2}\right)^6 = 8$?

(A) $\log_{\sqrt{2}} 6 = 8$

(B) $\log_8 6 = \sqrt{2}$

(C) $\log_{\sqrt{2}} 8 = 6$

(D) $\log_6 8 = \sqrt{2}$

11. Given $7^x = 14$, which best approximates *x*?

(A) 1.15

(B) 1.36

(C) 2

(D) 7

12. Given $7^x + 5 = 25$, what is the approximate value of x?

(A) 0.83
(B) 1.30
(C) 1.54
(D) 1.75

13. Which logarithmic equation correctly represents the exponential equation $10^7 = x$?

(A) $x = \log 7$
(B) $x = \log 10$
(C) $7 = \log x$
(D) $10 = \log x$

14. Evaluate the logarithmic expression $\log_{16} 4$.

(A) 0
(B) 0.5
(C) 1
(D) 2

15. Determine the concentration of hydrogen ions in bleach, with a pH of 12.8.
Recall that pH, $p(x)$, is defined by the equation
$p(x) = -\log x$, where the concentration of hydrogen ions, x, in a solution is measured in moles per litre.

(A) 1.3×10^{-13} mol/L
(B) 1.6×10^{-13} mol/L
(C) 1.3×10^{-12} mol/L
(D) 1.6×10^{-12} mol/L

16. Which expression is equivalent to ln 52 – ln 13?

(A) ln 39
(B) ln 39e
(C) ln 4
(D) ln 4e

17. Which logarithmic expression is **not** equivalent to the others?

(A) $\frac{\log 5}{\log 3}$
(B) $\log_9 25$
(C) $\frac{\log 25}{2\log 3}$
(D) $\log_{27} 100$

18. The equation of the logarithmic function that models a data set is $y = 8.2 + 0.7 \ln x$. Extrapolate the value of y when $x = 22$.

(A) $y = 10.4$
(B) $y = 10.8$
(C) $y = 11.1$
(D) $y = 11.3$

Selected Response Answer Sheet (UPPERCASE letters please!!!)

1. ___D	7. ___A	13. ___C
2. ___D	8. ___B	14. ___B
3. ___B	9. ___B	15. ___B
4. ___B	10. ___C	16. ___C
5. ___A	11. ___B	17. ___D
6. ___C	12. ___C	18. ___A

Constructed Response: Answer all the questions that follow showing all work.

1. Technetium-99, a radioactive isotope used in nuclear medicine, has a half-life of 6 hours. Set up an equation, and algebraically determine how long it would take for 500 micrograms of Technetium-99 to reduce to 100 micrograms. (6 marks)

$y = a \times b^{\frac{t}{h}}$

half-life so $b = \frac{1}{2}$; half-life is 6hrs so $h = 6$

initial amount is 500μg so $a = 500$

final amount is 100μg so $y = 100$

$\therefore 100 = 500\left(\frac{1}{2}\right)^{\frac{t}{6}}$

$\frac{100}{500} = \frac{500\left(\frac{1}{2}\right)^{\frac{t}{6}}}{500}$

$0.2 = 0.5^{\frac{t}{6}}$

$\log 0.2 = \log 0.5^{\frac{t}{6}}$

$\log 0.2 = \frac{t}{6}\log 0.5$

$6\left(\log 0.2\right) = 6\left(\frac{t}{6}\log 0.5\right)$

$6\log 0.2 = t\log 0.5$

$\frac{6\log 0.2}{\log 0.5} = \frac{t\cancel{\log 0.5}}{\cancel{\log 0.5}}$

$t \approx \boxed{13.93\text{yrs}}$

2. A laboratory assistant decided to observe the reproductive properties of a new strain of bacteria. The assistant started observing a population of 300 bacteria and noted that the bacteria population doubled every 5 minutes. Write a function to model this situation and use it to algebraically determine the time it will take for the population to reach 18 000 bacteria.(6 marks)

$y = a \times b^{\frac{t}{c}}$

population doubles $(b = 2)$ every 5 minutes $(c = 5)$
initial amount is 300 bacteria so $a = 300$
final amount is 18000 so $y = 18000$

$$\therefore 18000 = 300(2)^{\frac{t}{5}}$$

$$\frac{18000}{300} = \frac{300(2)^{\frac{t}{5}}}{300}$$

$$60 = (2)^{\frac{t}{5}}$$

$$\log 60 = \log(2)^{\frac{t}{5}}$$

$$\log 60 = \frac{t}{5}\log 2$$

$$5(\log 60) = 5\left(\frac{t}{5}\log 2\right)$$

$$5\log 60 = t\log 2$$

$$\frac{5\log 60}{\log 2} = \frac{t\cancel{\log 2}}{\cancel{\log 2}}$$

$$t \approx \boxed{29.53\,\text{min}}$$

3.$2500 is invested at 2.6% per year, compounded quarterly. Algebraically determine how long it will take for the investment to reach $3000. Use the compound interest formula: $A = P(1+i)^n$

(6 marks)

$$i = \frac{0.026}{4} = 0.0065$$

$$3000 = 2500(1.0065)^n$$

$$\frac{3000}{2500} = \frac{\cancel{2500}\,(1.0065)^n}{\cancel{2500}}$$

$$1.2 = 1.0065^n$$

$$\log 1.2 = \log 1.0065^n$$

$$\log 1.2 = n \log 1.0065$$

$$\frac{\log 1.2}{\log 1.0065} = \frac{n \cancel{\log 1.0065}}{\cancel{\log 1.0065}}$$

$$n = \frac{\log 1.2}{\log 1.0065} \approx 28.14053262 \text{ quarters} \approx \boxed{7.04\text{yrs}}$$

4. A laboratory that uses radioactive substances received a shipment of 500 g of thorium-227. Only 318.16 g of the thorium-227 remained 12.0 days later. Determine the half-life of thorium-227 algebraically using logarithms, to the nearest tenth of a day. The half-life equation is

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

(6 marks)

$$318.16 = 500\left(\frac{1}{2}\right)^{\frac{12}{h}}$$

$$\frac{318.16}{500} = \frac{\cancel{500}\left(\frac{1}{2}\right)^{\frac{12}{h}}}{\cancel{500}}$$

$$0.63632 = 0.5^{\frac{12}{h}}$$

$$\log 0.63632 = \log 0.5^{\frac{12}{h}}$$

$$\log 0.63632 = \frac{12}{h}\log 0.5$$

$$h\left(\log 0.63632\right) = h\left(\frac{12}{h}\log 0.5\right)$$

$$h\log 0.63632 = 12\log 0.5$$

$$\frac{h\log 0.63632}{\log 0.63632} = \frac{12\log 0.5}{\log 0.63632}$$

$$h \approx \boxed{18.4\text{days}}$$

5. Algebraically solve for x: $2^{3x+2} = 7^{x-3}$ (6 marks)

$\log 2^{3x+2} = \log 7^{x-3}$

$(3x+2)\log 2 = (x-3)\log 7$

$3x\log 2 + 2\log 2 = x\log 7 - 3\log 7$

$3x\log 2 + 2\log 2 - 2\log 2 = x\log 7 - 3\log 7 - 2\log 2$

$3x\log 2 = x\log 7 - 3\log 7 - 2\log 2$

$3x\log 2 - x\log 7 = x\log 7 - x\log 7 - 3\log 7 - 2\log 2$

$3x\log 2 - x\log 7 = -3\log 7 - 2\log 2$

$x(3\log 2 - \log 7) = -3\log 7 - 2\log 2$

$$\frac{x\cancel{(3\log 2 - \log 7)}}{\cancel{3\log 2 - \log 7}} = \frac{-3\log 7 - 2\log 2}{3\log 2 - \log 7}$$

$x \approx \boxed{-54.1}$

6. Algebraically solve for x: $e^{x+1} = 9$ (3 marks)

$\log e^{x+1} = \log 9$

$(x+1)\log e = \log 9$

$$\frac{(x+1)\cancel{\log e}}{\cancel{\log e}} = \frac{\log 9}{\log e}$$

$$x+1 = \frac{\log 9}{\log e}$$

$$x+1-1 = \frac{\log 9}{\log e} - 1$$

$$x = \frac{\log 9}{\log e} - 1$$

$x \approx 1.197224577$