

**Math 3201 Notes**  
**Chapter 4: Rational Expressions & Equations**

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Learning Goals: See p. 213 text.

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**§4.1 Equivalent Rational Expressions (2 classes)**

Read *Goal* p. 216 text.

1. Define and give an example of a **rational expression**. p. 216
  2. Define **non-permissible values** as they relate to rational expressions. p. 217
  3. Determine the **non-permissible values** for a rational expression. p. 217
  4. Define and give examples of **equivalent rational expressions**. p. 218
  5. Find a rational expression that is equivalent to a given rational expression using multiplication or division. p. 218
  6. Determine if two rational expressions are equivalent. p. 220
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Def<sup>n</sup> A **rational expression** is an expression of the form  $\frac{p}{q}$ , where  $p$  and  $q$  are polynomials and  $q \neq 0$ .

It is a fraction that has a polynomial in the numerator and a polynomial in the denominator and the denominator cannot be 0.

E.g.:  $\frac{3}{4}$ ;  $\frac{4}{r}, r \neq 0$ ;  $\frac{4x^2}{x-1}, x \neq 1$ ;  $\frac{1-t}{t-1}, t \neq 1$ ;  $\frac{q^2-1}{q^2-q-2}, q \neq -1, 2$ ;  $\frac{7t}{8y^2(x-1)}, y \neq 0, x \neq 1$

Non e.g.:  $\frac{3}{0}$ ,  $\frac{4}{\sqrt{w}}$ ,  $\frac{4\sqrt{x+1}}{x-1}$

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If you look back at the examples of rational expressions, you should notice that rational expressions with one or more variables in the denominator had **restrictions** on the variable(s). The restrictions on the denominator occur because division by zero is undefined, so we are not permitted to substitute certain values for the variable(s) in the denominator because they result in division by zero. These value(s) which cannot be substituted for the variable(s) are called **non-permissible values**. These non-permissible values lead to the restrictions on the rational expression.

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Def<sup>n</sup>: **Non-permissible values** are those values of the variable that makes the denominator of the rational expressions equal to zero.

E.g.: For  $\frac{x}{x^2-4} = \frac{x}{(x-2)(x+2)}$ , the non-permissible values are  $\pm 2$ , so the restrictions are  $x \neq \pm 2$ .

E.g.: For  $\frac{x+5}{7x^2+33x-10} = \frac{x+5}{(x+5)(7x-2)}$ , the non-permissible values are  $-5$  and  $\frac{2}{7}$ , so the restrictions are  $x \neq -5, x \neq \frac{2}{7}$ .

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**See *Need to Know*, bullets 1-3, p. 222**

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**Do # 1 b, 9 a, 10 b, c, d, 11a(i) & a(ii), 16, CYU/Practising pp. 222-224 text in your homework booklet.**

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### Equivalent Rational Expressions

Finding equivalent rational expressions is the same as finding equivalent numerical fractions. You must multiply (or divide) the numerator and denominator by the same number or expression. The original and resulting rational expressions are equivalent because multiplying both the numerator and the denominator by the same number or expression means you are multiplying by one. However, with rational expressions involving variables, you have to pay close attention to the restriction(s) on the variable(s).

E.g.:  $\frac{3}{4}$  and  $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$  are equivalent rational expressions.

E.g.:  $\frac{5}{10}$  and  $\frac{5}{10} \div \frac{5}{5} = \frac{1}{2}$  are equivalent rational expressions.

E.g.:  $\frac{4}{x}$  and  $\frac{4}{x} \times \frac{x}{x} = \frac{4x}{x^2}$  are equivalent rational expressions for  $x \neq 0$ .

E.g.:  $\frac{7x-14}{x-2}$  and  $\frac{7(x-2)}{x-2} \div \frac{x-2}{x-2} = \frac{7}{1} = 7$  are equivalent rational expressions for  $x \neq 2$ .

E.g.:  $\frac{5t}{7-t}$  and  $\frac{5t}{7-t} \times \frac{t+2}{t+2} = \frac{5t(t+2)}{(7-t)(t+2)} = \frac{5t^2+10t}{14+5t-t^2}$  are equivalent rational expressions for  $t \neq -2, 7$ .

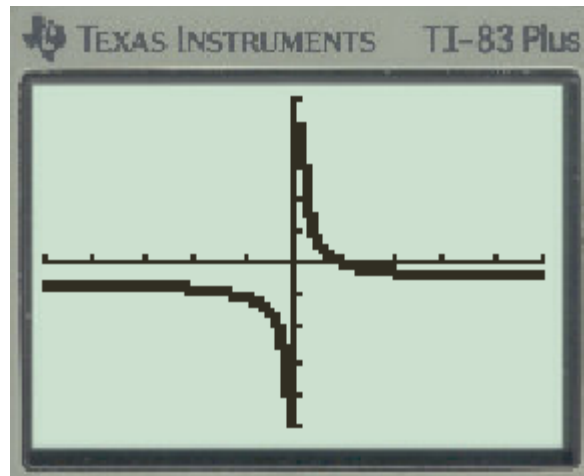
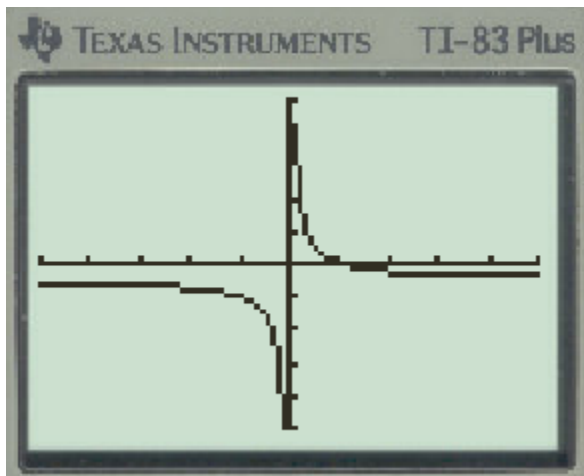
E.g.:  $\frac{1-t}{t-1}$  and  $\frac{-1(t-1)}{t-1} \div \frac{t-1}{t-1} = \frac{-1}{1} = -1$  are equivalent rational expressions for  $t \neq 1$ .

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Note: If the rational expressions contain at most one variable, you can determine if two rational expressions are equivalent by graphing both expressions. If one expression graphs on top of the other expression, they are equivalent.

E.g.: Determine if  $\frac{2-2x}{4x}$  is equivalent to  $\frac{1-x}{2x}$  for  $x \neq 0$ .

The graphs of  $y = \frac{2-2x}{4x}$  (left graph) and  $y = \frac{1-x}{2x}$  (right graph) graph on top of each other so  $\frac{2-2x}{4x}$  and  $\frac{1-x}{2x}$  are equivalent.



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**Sample Exam Question**

Identify the rational expression that is equivalent to  $\frac{6a^2}{a^2 + 2a}$ .

- A.  $\frac{2}{a}$ ;  $a \neq -2, 0$
- B.  $\frac{3}{a}$ ;  $a \neq -2, 0$
- C.  $\frac{3a}{a^2}$ ;  $a \neq -2, 0$
- D.  $\frac{6a}{a+2}$ ;  $a \neq -2, 0$

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**See *Key Ideas*, bullet 2, p. 222**

**Do # 1 a, 3 b, c, 4, 5, 14, 15, CYU/Practising pp. 222-224 text in your homework booklet.**

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## §4.2 Simplifying Rational Expressions (2 classes)

Read *Goal* p. 225 text.

1. Simplify rational expressions. pp. 226-228
2. Determine when a rational expression is simplified. p. 225
3. Determine when to identify the non-permissible values when simplifying a rational expression. p. 225
4. Identify errors in the “simplification” of a rational expression. p. 230

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### Simplifying Rational Expressions

Simplifying rational expressions is very similar to simplifying numerical fractions. The idea is to factor both the numerator and denominator and cancel like factors to get an equivalent expression. Don't forget to pay attention to the non-permissible values so you can state the restriction(s) on the variable(s). Remember that graphing can be used to check your work if there is no more than one variable. You have simplified a rational expression when the numerator and the denominator have no common factors (see TIP, p. 225).

$$\text{E.g.: } \frac{14}{35} = \frac{\cancel{7}(2)}{\cancel{7}(5)} = \frac{2}{5}$$

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$$\text{E.g.: } \frac{9r}{6r^3} = \left(\frac{9}{6}\right)\left(\frac{r}{r^3}\right) = \frac{3}{2}r^{1-3} = \frac{3}{2}r^{-2} = \left(\frac{3}{2}\right)\left(\frac{1}{r^2}\right) = \frac{3}{2r^2}; r \neq 0$$

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$$\text{E.g.: } \frac{8x+16}{x+2} = \frac{8(\cancel{x+2})}{\cancel{x+2}} = \frac{8}{1} = 8, \quad x \neq -2$$

Note that you determine the non-permissible values and the restrictions **after you factor but before you cancel** (see TIP p. 225).

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$$\text{E.g.: } \frac{3a^3 - 3a^2}{-12a + 12} = \frac{3a(\cancel{a-1})}{-12(\cancel{a-1})} = \frac{3a}{-12} = -\frac{\cancel{3}a}{\cancel{3}(4)} = -\frac{a}{4}; a \neq 1$$

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$$\text{E.g.: } \frac{1-x}{x-1} = \frac{-1(\cancel{x-1})}{\cancel{x-1}} = \frac{-1}{1} = -1, \quad x \neq 1$$

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$$\text{E.g.: } \frac{5-x}{x-5} = \frac{5-x}{x-5} \times \frac{-1}{-1} = \frac{-1(5-x)}{-1(x-5)} = \frac{(\cancel{x-5})}{-1(\cancel{x-5})} = \frac{1}{-1} = -1, \quad x \neq 1$$

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E.g.: Simplify  $\frac{36-x^2}{6-x}$

$$\frac{36-x^2}{6-x} = \frac{\cancel{(6-x)}(6+x)}{\cancel{6-x}} = 6+x; x \neq 6$$

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E.g.: Simplify  $\frac{10x^2-8x}{24x-30x^2}$

$$\frac{10x^2-8x}{24x-30x^2} = \frac{2x(5x-4)}{6x(4-5x)} = \frac{5x-4}{3(4-5x)} = \frac{5x-4}{12-15x} = \frac{\cancel{5x-4}}{-3(\cancel{5x-4})} = -\frac{1}{3}; x \neq 0, \frac{4}{5}$$

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E.g.: Simplify  $\frac{14a^2-21a^3}{35a^2}$

$$\frac{14a^2-21a^3}{35a^2} = \frac{\cancel{7a^2}(2-3a)}{\cancel{7a^2}(5)} = \frac{2-3a}{5}; a \neq 0$$

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E.g.: Simplify  $\frac{12x}{8x^2-20x^3}$

$$\frac{12x}{8x^2-20x} = \frac{\cancel{4x}(3)}{\cancel{4x}(2x-5)} = \frac{3}{2x-5}; x \neq 0, \frac{5}{2}$$

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### Sample Exam Question

Simplify  $\frac{3x^2-6x}{14-7x}$  [ANS:  $-\frac{3x}{7}; x \neq 2$ ]

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E.g.: Find the error in the following “simplification” of  $\frac{16+20x}{10x}$ .

$$\frac{16+20x}{10x} = \frac{16 + \cancel{20x}^2}{\cancel{10x}} = 16 + 2 = 18; x \neq 0$$

The error occurs in canceling  $20x$  and  $10x$  to get 2. The correct simplification is

$$\frac{16+20x}{10x} = \frac{\cancel{2}(8+10x)}{\cancel{2}(5x)} = \frac{8+10x}{5x}; x \neq 0$$

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E.g.: Find the errors in the following “simplification” of  $\frac{2a-2}{4a^2-4}$ .

$$\frac{2a-2}{4a^2-4} = \frac{2(a-1)}{4(a^2-1)} = \frac{\cancel{2}(a-1)}{\cancel{4}_2(a+1)(a-1)} = 2(a+1) = 2a+2; a \neq -1$$

There are 2 errors, one in the simplification  $\left(2a+1 \text{ instead of } \frac{1}{2a+1}\right)$  and one in the restrictions. The correct simplification is

$$\frac{2a-2}{4a^2-4} = \frac{2(a-1)}{4(a^2-1)} = \frac{\cancel{2}(a-1)}{\cancel{4}_2(a+1)(a-1)} = \frac{1}{2(a+1)} = \frac{1}{2a+2}; a \neq \pm 1$$

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**See Key Ideas & Need to Know, p. 229**

**Do #'s 1, 2 b, c, d, 3 a, c, 4 c, d, 5 b, c, 6-8, 13, CYU/Practising pp. 229-231 text in your homework booklet.**

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### §4.3 Multiplying and Dividing Rational Expressions (2 classes)

Read *Goal* on p. 232 text

Outcomes:

1. Compare and contrast multiplying and dividing rational expressions with multiplying and dividing numerical fractions pp. 232-236
2. Determine the restriction(s) on the variable(s) when multiplying or dividing rational expressions. pp. 232-236
3. Multiply and divide rational expressions. pp. 232-236

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#### Multiplying Rational Expressions

Multiplying rational expressions is similar to multiplying numerical fractions. However, you must watch for the restriction(s) on the variable(s).

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E.g.: Simplify  $\frac{5}{4} \times \frac{3}{7}$

We multiply across the top and across the bottom to get

$$\frac{5 \times 3}{4 \times 7} = \frac{15}{28}$$

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Sometimes, we can do some cancelling before we multiply.

E.g.: Simplify  $\frac{15}{14} \times \frac{7}{3}$

$$\frac{15}{14} \times \frac{7}{3} = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{14}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}} = \frac{5 \times 1}{2 \times 1} = \frac{5}{2}$$

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Multiplying rational expressions is similar except that we have to determine the non-permissible value(s) for the expression and use them to write the restrictions on the variable.

E.g.: Simplify  $\frac{x^2 - 9}{x^2 - 4x} \times \frac{x - 4}{x - 3}$

We have to rewrite the expression so that everything is factored and cancel like factors. Don't forget to find the non-permissible values before you cancel. You could multiply across the top and across the bottom and then factor but this would greatly increase the difficulty of the factoring.

$$\frac{x^2 - 9}{x^2 - 4x} \times \frac{x - 4}{x - 3} = \frac{(x + 3)(\cancel{x - 3})}{(x)(\cancel{x - 4})} \times \frac{\cancel{x - 4}}{\cancel{x - 3}} = \frac{x + 3}{x}; x \neq 0, 3, 4$$

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E.g.: Simplify  $\frac{x^2}{x^2-4} \times \frac{x+2}{x}$

$$\frac{x^2}{x^2-4} \times \frac{x+2}{x} = \frac{\overset{x}{\cancel{x^2}}}{(\cancel{x+2})(x-2)} \times \frac{\cancel{x+2}}{\cancel{x}} = \frac{x}{x-2}; x \neq 0, \pm 2$$

E.g.: Simplify  $\frac{12x^3}{3x^2+6x} \cdot \frac{4x^3+8x^2}{5}$

$$\frac{12x^3}{3x^2+6x} \cdot \frac{4x^3+8x^2}{5} = \frac{12x^3}{3x(x+2)} \cdot \frac{4x^2(x+2)}{5} = \frac{\overset{16}{\cancel{48}} \overset{x^4}{\cancel{x^2}} (\cancel{x+2})}{\underset{5}{\cancel{15}} \cancel{x} (\cancel{x+2})} = \frac{16x^4}{5}; x \neq -2, 0$$

**See Key Ideas & Need to Know, p. 237**

**Do #'s 1 a, c, 2 a, b, 5 a, 6, CYU/Practising pp. 238-239 text in your homework booklet.**

### Dividing Rational Expressions

Dividing rational expressions is similar to dividing numerical fractions. However, you must watch for the restriction(s) on the variable(s).

E.g.: Simplify  $\frac{5}{4} \div \frac{3}{7}$

First we multiply the first fraction by the **reciprocal** of the second fraction and then we multiply across the top and across the bottom to get

$$\frac{5}{4} \div \frac{3}{7} = \frac{5}{4} \times \frac{7}{3} = \frac{5 \times 7}{4 \times 3} = \frac{35}{12}$$

Like multiplying numerical fractions, sometimes we can use cancellation after we multiply by the reciprocal.

E.g.: Simplify  $\frac{16}{25} \div \frac{32}{15}$

$$\frac{16}{25} \div \frac{32}{15} = \frac{16}{25} \times \frac{15}{32} = \frac{\overset{1}{\cancel{16}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{32}}} = \frac{1 \times 3}{5 \times 2} = \frac{3}{10}$$

Dividing rational expressions is similar except that we have to determine the non-permissible value(s) for the expression. This is a little bit trickier than with multiplying rational expressions.



E.g.: Simplify  $\frac{4t^2}{3s} \div \frac{2t}{s^2}$

$$\frac{4t^2}{3s} \div \frac{2t}{s^2} = \frac{4t^2}{3s} \times \frac{s^2}{2t} = \frac{4t^2 s^2}{6st} = \frac{2st}{3}, s \neq 0, t \neq 0$$

Do you see why  $t \neq 0$ ?

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E.g.: Simplify  $\frac{4(x-3)}{x+1} \div \frac{4}{x+1}$

$$\frac{4(x-3)}{x+1} \div \frac{4}{x+1} = \frac{\cancel{4}(x-3)}{\cancel{x+1}} \times \frac{\cancel{x+1}}{\cancel{4}} = x-3; x \neq -1$$

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E.g.: Simplify  $\frac{4x-12}{x+1} \div \frac{4}{x+1}$

$$\frac{4x-12}{x+1} \div \frac{4}{x+1} = \frac{\cancel{4}(x-3)}{\cancel{x+1}} \times \frac{\cancel{x+1}}{\cancel{4}} = x-3, x \neq -1$$

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E.g.: Simplify  $\frac{y-1}{y^2-y} \div \frac{y^2+y}{y^2-1}$

$$\begin{aligned} \frac{y-1}{y^2-y} \div \frac{y^2+y}{y^2-1} &= \frac{y-1}{y(y-1)} \div \frac{y(y+1)}{(y+1)(y-1)} = \frac{\cancel{y-1}}{y(\cancel{y-1})} \times \frac{(y+1)(y-1)}{y(\cancel{y+1})} \\ &= \frac{y-1}{y^2}, y \neq \pm 1, 0 \end{aligned}$$

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E.g.: Simplify  $\frac{a+2}{25-a^2} \div \frac{a+2}{a+5}$

$$\begin{aligned} \frac{a+2}{25-a^2} \div \frac{a+2}{a+5} &= \frac{a+2}{(5+a)(5-a)} \div \frac{a+2}{a+5} \\ &= \frac{a+2}{(5+a)(5-a)} \times \frac{a+5}{a+2} \\ &= \frac{\cancel{a+2}}{(\cancel{5+a})(5-a)} \times \frac{\cancel{a+5}}{\cancel{a+2}} \\ &= \frac{1}{5-a}; a \neq -2, \pm 5 \end{aligned}$$

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Note that division and multiplication of rational expressions can be combined as in the example below.

E.g.: Simplify  $\frac{3x+12}{x+4} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4}$

$$\begin{aligned} & \frac{3x+12}{x+4} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4} \\ &= \frac{3(x+4)}{x+4} \div \frac{12}{3x+4} \times \frac{2(x-3)}{x+4} \\ &= \frac{3(x+4)}{x+4} \times \frac{3x+4}{12} \times \frac{2(x-3)}{x+4} \\ &= \frac{\cancel{3}(x+4)}{\cancel{x+4}} \times \frac{3x+4}{\cancel{12}_2} \times \frac{\cancel{2}(x-3)}{x+4} \\ &= \frac{(3x+4)(x-3)}{2(x+4)}; x \neq -4, -\frac{4}{3} \end{aligned}$$

### Sample Exam Question

Simplify  $\frac{\frac{3(s-2)}{4(s+5)}}{\frac{9(s-2)}{s+5}}$

$$\frac{\frac{3(s-2)}{4(s+5)}}{\frac{9(s-2)}{s+5}} = \frac{3(s-2)}{4(s+5)} \times \frac{s+5}{9(s-2)} = \frac{\cancel{3}(s-2)}{4(s+5)} \times \frac{\cancel{s+5}}{9(s-2)} = \frac{3}{36} = \frac{1}{12}; s \neq -5, 2$$

See *Key Ideas & Need to Know*, p. 237

Do #'s 1 b, d, 2 c, d, 5 b, 7, CYU/Practising pp. 238-239 text in your homework booklet.

## §4.4 Adding and Subtracting Rational Expressions (2 classes)

Read *Goal* on p. 244 text

Outcomes:

1. Compare and contrast adding and subtracting rational expressions with adding and subtracting numerical fractions pp. 244-248
2. Determine the restriction(s) on the variable(s) when adding or subtracting rational expressions. pp. 244-248
3. Add and subtract rational expressions with like denominators. pp. 244-248
4. Add and subtract rational expressions with unlike denominators. pp. 244-248

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### Adding Rational Expressions

Adding rational expressions is similar to adding numerical fractions. It involves factoring the denominators and finding the LCM of the denominators.

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E.g.: Simplify  $\frac{5}{4} + \frac{3}{4}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is 4.
<b>Step 2:</b> Write both numerators over a single common denominator.	$\frac{5+3}{4}$
<b>Step 3:</b> Add the numerators.	$\frac{8}{4}$
<b>Step 4:</b> Reduce, if necessary	$\frac{8}{4} = 2$

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Sometimes, we have to find a common denominator before we add.

E.g.: Simplify  $\frac{15}{14} + \frac{3}{7}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is 14.
<b>Step 2:</b> Write equivalent fractions with the common denominator.	$\frac{15}{14}$ $\frac{3}{7} \times \frac{2}{2} = \frac{6}{14}$
<b>Step 3:</b> Write both numerators over a single common denominator.	$\frac{15+6}{14}$
<b>Step 4:</b> Add the numerators.	$\frac{21}{14}$
<b>Step 5:</b> Reduce, if necessary	$\frac{21}{14} \div \frac{7}{7} = \frac{3}{2}$

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Adding rational expressions is similar except that we have to determine the non-permissible value(s) for the expression so we can write the restrictions.

E.g.: Simplify  $\frac{m}{n} + \frac{m+1}{n}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is $n$ .
<b>Step 2:</b> Write both numerators over a single common denominator.	$\frac{m+m+1}{n}$
<b>Step 3:</b> Combine like terms.	$\frac{2m+1}{n}$
<b>Step 4:</b> Reduce, if necessary	$\frac{2m+1}{n}$ is already reduced.
<b>Step 5:</b> Find the non-permissible values.	$n = 0$
<b>Step 6:</b> Write the answer with any restrictions	$\frac{2m+1}{n}; n \neq 0$

E.g.: Simplify  $\frac{10m-1}{4m-3} + \frac{8-2m}{4m-3}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is $4m-3$ .
<b>Step 2:</b> Write both numerators over a single common denominator.	$\frac{10m-1+8-2m}{4m-3}$
<b>Step 3:</b> Combine like terms.	$\frac{8m+7}{4m-3}$
<b>Step 4:</b> Reduce, if necessary	$\frac{8m+7}{4m-3}$ is already reduced.
<b>Step 5:</b> Find the non-permissible values.	$m = \frac{3}{4}$
<b>Step 6:</b> Write the answer with any restrictions	$\frac{8m+7}{4m-3}; m \neq \frac{3}{4}$

E.g.: Simplify  $\frac{6}{n-3} + \frac{4}{n+2}$

<b>Step 1:</b> Factor both denominators.	Both denominators cannot be factored.
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $(n-3)(n+2)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{6}{n-3} \times \frac{(n+2)}{(n+2)} = \frac{6(n+2)}{(n-3)(n+2)}$ $\frac{4}{n+2} \times \frac{(n-3)}{(n-3)} = \frac{4(n-3)}{(n-3)(n+2)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{6(n+2)+4(n-3)}{(n-3)(n+2)}$

<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{6n+12+4n-12}{(n-3)(n+2)}$
<b>Step 6:</b> Combine like terms.	$\frac{10n}{(n-3)(n+2)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{10n}{(n-3)(n+2)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$n = -2, 3$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{10n}{(n-3)(n+2)}$ ; $n \neq -2, 3$

For many rational expressions, the denominators are NOT like, so we have to find a common denominator before we can add. We may have to factor the denominators before we can find the common denominator.

E.g.: Simplify  $\frac{4}{p^2-1} + \frac{3}{p-1}$

<b>Step 1:</b> Factor both denominators.	$p^2 - 1 = (p)^2 - (1)^2 = (p+1)(p-1)$ $p - 1 = p - 1$
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $(p+1)(p-1)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{4}{p^2-1} = \frac{4}{(p+1)(p-1)}$ $\frac{3}{p-1} = \frac{3}{p-1} \times \frac{p+1}{p+1} = \frac{3(p+1)}{(p+1)(p-1)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{4+3(p+1)}{(p+1)(p-1)}$
<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{4+3p+3}{(p+1)(p-1)}$
<b>Step 6:</b> Combine like terms.	$\frac{3p+7}{(p+1)(p-1)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{3p+7}{(p+1)(p-1)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$p = \pm 1$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{3p+7}{(p+1)(p-1)}$ ; $p \neq \pm 1$

E.g.: Simplify  $\frac{2}{x^2-x} + \frac{-2}{x^2-1}$

<b>Step 1:</b> Factor both denominators.	$x^2 - x = x(x-1)$ $x^2 - 1 = (x)^2 - (1)^2 = (x+1)(x-1)$
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $x(x+1)(x-1)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{2}{x^2-x} = \frac{2}{x(x-1)} \times \frac{(x+1)}{(x+1)} = \frac{2(x+1)}{x(x-1)(x+1)}$ $\frac{-2}{x^2-1} = \frac{-2}{(x+1)(x-1)} \times \frac{x}{x} = \frac{-2x}{x(x-1)(x+1)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{2(x+1) + -2x}{x(x-1)(x+1)}$
<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{2x+2 + -2x}{x(x-1)(x+1)}$
<b>Step 6:</b> Combine like terms.	$\frac{2}{x(x-1)(x+1)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{2}{x(x-1)(x+1)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$x = 0, \pm 1$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{2}{x(x-1)(x+1)}$ ; $x \neq 0, \pm 1$

E.g.: Simplify  $\frac{4x^2-20x}{x^2+2x-35} + \frac{3x-6}{x^2-12x+20}$

<b>Step 1:</b> Factor both the numerators and the denominators and cancel any like factors.	$\frac{4x^2-20x}{x^2+2x-35} = \frac{4x\cancel{(x-5)}}{(x+7)\cancel{(x-5)}} = \frac{4x}{x+7}$ $\frac{3x-6}{x^2-12x+20} = \frac{3\cancel{(x-2)}}{(x-10)\cancel{(x-2)}} = \frac{3}{x-10}$
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $(x+7)(x-10)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{4x}{x+7} = \frac{4x}{x+7} \times \frac{(x-10)}{(x-10)} = \frac{4x(x-10)}{(x+7)(x-10)}$ $\frac{3}{x-10} = \frac{3}{x-10} \times \frac{(x+7)}{(x+7)} = \frac{3(x+7)}{(x+7)(x-10)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{4x(x-10) + 3(x+7)}{(x+7)(x-10)}$

<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{4x^2 - 40x + 3x + 21}{(x+7)(x-10)}$
<b>Step 6:</b> Combine like terms.	$\frac{4x^2 - 37x + 21}{(x+7)(x-10)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{4x^2 - 37x + 21}{(x+7)(x-10)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$x = -7, 2, 5, 10$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{4x^2 - 37x + 21}{(x+7)(x-10)}$ ; $x \neq -7, 2, 5, 10$

E.g.: Find the error in the “simplification” below and write the correct simplification.

$$\frac{x}{5} + \frac{2}{3} = \frac{x+2}{5+3} = \frac{x+2}{8}$$

The error occurs when the denominators are added. The correct simplification is

$$\frac{x}{5} + \frac{2}{3} = \frac{x}{5} \times \frac{3}{3} + \frac{2}{3} \times \frac{5}{5} = \frac{3x}{15} + \frac{10}{15} = \frac{3x+10}{15}$$

E.g.: Chris purchased a motorboat and used it to travel 6 km upstream and then 6 km back downstream. If the river’s current is constant at 2 km/h, algebraically determine an expression which models the time taken to travel the 12km.

Let  $v$  be the speed of the boat in still water.

Let  $v+2$  be the speed of the boat going downstream.

Let  $v-2$  be the speed of the boat going upstream.

Upstream	Downstream
$d = 6\text{km}$	$d = 6\text{km}$
Speed = $v - 2$	Speed = $v + 2$
$t = \frac{6}{v-2}$	$t = \frac{6}{v+2}$

Since the total time is the sum of the time going upstream and then downstream, the expression for the total time taken to travel 12km is  $\frac{6}{v-2} + \frac{6}{v+2}$

See *In Summary*, p. 248

**Do #'s 1, 2, 3 b, d, 5 b, d, 6 a, c, 7 a, 10, CYU/Practising pp. 249-250 text in your homework booklet.**

## Subtracting Rational Expressions

Subtracting rational expressions is similar to subtracting numerical fractions. It involves factoring the denominators and finding the LCM of the denominators. However, you must watch for the restriction(s) on the variable(s).

E.g.: Simplify  $\frac{5}{4} - \frac{3}{4}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is 4.
<b>Step 2:</b> Write both numerators over a single common denominator.	$\frac{5-3}{4}$
<b>Step 3:</b> Subtract the numerators.	$\frac{2}{4}$
<b>Step 4:</b> Reduce, if necessary	$\frac{2}{4} = \frac{1}{2}$

Sometimes, we have to find a common denominator before we subtract.

E.g.: Simplify  $\frac{15}{14} - \frac{3}{7}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is 14.
<b>Step 2:</b> Write equivalent fractions with the common denominator.	$\frac{15}{14}$ $\frac{3}{7} \times \frac{2}{2} = \frac{6}{14}$
<b>Step 3:</b> Write both numerators over a single common denominator.	$\frac{15-6}{14}$
<b>Step 4:</b> Subtract the numerators.	$\frac{9}{14}$
<b>Step 5:</b> Reduce, if necessary	$\frac{9}{14}$ is already reduced.

Subtracting rational expressions is similar except that we have to determine the non-permissible value(s) for the expression so we can write the restrictions.

E.g.: Simplify  $\frac{m}{n} - \frac{m+1}{n}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is $n$ .
<b>Step 2:</b> Write both numerators over a single common denominator. Use parentheses to show that you are subtracting all of the second denominator.	$\frac{m-(m+1)}{n}$



<b>Step 3:</b> Change to addition and change the signs of all the terms inside the parentheses.	$\frac{m+(-m-1)}{n} = \frac{m-m-1}{n}$
<b>Step 4:</b> Add like terms.	$\frac{-1}{n}$
<b>Step 5:</b> Reduce, if necessary	$\frac{-1}{n}$ is already reduced.
<b>Step 6:</b> Find the non-permissible values.	$n = 0$
<b>Step 7:</b> Write the answer with any restrictions	$\frac{-1}{n}; n \neq 0$

E.g.: Simplify  $\frac{10m-1}{4m-3} - \frac{8-2m}{4m-3}$

<b>Step 1:</b> Find the lowest common denominator.	Lowest common denominator is $4m-3$ .
<b>Step 2:</b> Write both numerators over a single common denominator. Use parentheses to show that you are subtracting all of the second denominator.	$\frac{10m-1-(8-2m)}{4m-3} =$
<b>Step 3:</b> Change to addition and change the signs of all the terms inside the parentheses.	$\frac{10m-1+(-8+2m)}{4m-3} = \frac{10m-1-8+2m}{4m-3}$
<b>Step 4:</b> Combine like terms.	$\frac{12m-9}{4m-3}$
<b>Step 5:</b> Reduce, if necessary	$\frac{12m-9}{4m-3} = \frac{3(4m-3)}{4m-3} = 3$
<b>Step 6:</b> Find the non-permissible values.	$m = \frac{3}{4}$
<b>Step 7:</b> Write the answer with any restrictions	$3; m \neq \frac{3}{4}$

E.g.: Simplify  $\frac{6}{n-3} - \frac{4}{n+2}$

<b>Step 1:</b> Factor both denominators.	The denominators cannot be factored.
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $(n-3)(n+2)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{6}{n-3} \times \frac{(n+2)}{(n+2)} = \frac{6(n+2)}{(n-3)(n+2)}$ $\frac{4}{n+2} \times \frac{(n-3)}{(n-3)} = \frac{4(n-3)}{(n-3)(n+2)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{6(n+2)-4(n-3)}{(n-3)(n+2)}$

<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{6n+12-4n+12}{(n-3)(n+2)}$
<b>Step 6:</b> Combine like terms.	$\frac{2n+24}{(n-3)(n+2)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{2n+24}{(n-3)(n+2)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$n = -2, 3$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{2n+24}{(n-3)(n+2)}$ ; $n \neq -2, 3$

For many rational expressions, the denominators are NOT like, so we have to find a common denominator before we can subtract. We may have to factor the denominators before we can find the common denominator.

E.g.: Simplify  $\frac{4}{p^2-1} - \frac{3}{p-1}$

<b>Step 1:</b> Factor both denominators.	$p^2 - 1 = (p)^2 - (1)^2 = (p+1)(p-1)$ $p - 1 = p - 1$
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $(p+1)(p-1)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{4}{p^2-1} = \frac{4}{(p+1)(p-1)}$ $\frac{3}{p-1} = \frac{3}{p-1} \times \frac{p+1}{p+1} = \frac{3(p+1)}{(p+1)(p-1)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{4-3(p+1)}{(p+1)(p-1)}$
<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{4-3p-3}{(p+1)(p-1)}$
<b>Step 6:</b> Combine like terms.	$\frac{1-3p}{(p+1)(p-1)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{1-3p}{(p+1)(p-1)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$p = \pm 1$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{1-3p}{(p+1)(p-1)}$ ; $p \neq \pm 1$

E.g.: Simplify  $\frac{2}{x^2-x} - \frac{-2}{x^2-1}$

<b>Step 1:</b> Factor both denominators.	$x^2 - x = x(x-1)$ $x^2 - 1 = (x)^2 - (1)^2 = (x+1)(x-1)$
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $x(x+1)(x-1)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{2}{x^2-x} = \frac{2}{x(x-1)} \times \frac{(x+1)}{(x+1)} = \frac{2(x+1)}{x(x-1)(x+1)}$ $\frac{-2}{x^2-1} = \frac{-2}{(x+1)(x-1)} \times \frac{x}{x} = \frac{-2x}{x(x-1)(x+1)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{2(x+1) - (-2x)}{x(x-1)(x+1)}$
<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{2x+2+2x}{x(x-1)(x+1)}$
<b>Step 6:</b> Combine like terms.	$\frac{4x+2}{x(x-1)(x+1)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{4x+2}{x(x-1)(x+1)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$x = 0, \pm 1$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{4x+2}{x(x-1)(x+1)}$ ; $x \neq 0, \pm 1$

E.g.: Simplify  $\frac{4x^2-20x}{x^2+2x-35} - \frac{3x-6}{x^2-12x+20}$

<b>Step 1:</b> Factor both the numerators and the denominators and cancel any like factors.	$\frac{4x^2-20x}{x^2+2x-35} = \frac{4x\cancel{(x-5)}}{(x+7)\cancel{(x-5)}} = \frac{4x}{x+7}$ $\frac{3x-6}{x^2-12x+20} = \frac{3\cancel{(x-2)}}{(x-10)\cancel{(x-2)}} = \frac{3}{x-10}$
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $(x+7)(x-10)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{4x}{x+7} = \frac{4x}{x+7} \times \frac{(x-10)}{(x-10)} = \frac{4x(x-10)}{(x+7)(x-10)}$ $\frac{3}{x-10} = \frac{3}{x-10} \times \frac{(x+7)}{(x+7)} = \frac{3(x+7)}{(x+7)(x-10)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{4x(x-10) - 3(x+7)}{(x+7)(x-10)}$

<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{4x^2 - 40x - 3x - 21}{(x+7)(x-10)}$
<b>Step 6:</b> Combine like terms.	$\frac{4x^2 - 43x - 21}{(x+7)(x-10)}$
<b>Step 7:</b> Reduce, if necessary	$\frac{4x^2 - 43x - 21}{(x+7)(x-10)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$x = -7, 2, 5, 10$
<b>Step 9:</b> Write the answer with any restrictions	$\frac{4x^2 - 43x - 21}{(x+7)(x-10)}; \quad x \neq -7, 2, 5, 10$

E.g.: Simplify  $\frac{x-1}{x^2+x-6} - \frac{x-2}{x^2+4x+3}$

<b>Step 1:</b> Factor both the numerators and the denominators and cancel any like factors.	$\frac{x-1}{x^2+x-6} = \frac{x-1}{(x+3)(x-2)}$ $\frac{x-2}{(x+3)(x+1)} = \frac{x-2}{x^2+4x+3}$
<b>Step 2:</b> Find the lowest common denominator.	Lowest common denominator is $(x+3)(x-2)(x+1)$ .
<b>Step 3:</b> Write both expressions with the common denominator.	$\frac{x-1}{(x+3)(x-2)} \times \left(\frac{x+1}{x+1}\right) = \frac{(x-1)(x+1)}{(x+3)(x-2)(x+1)}$ $\frac{x-2}{(x+3)(x+1)} \times \left(\frac{x-2}{x-2}\right) = \frac{(x-2)(x-2)}{(x+3)(x+1)(x-2)}$
<b>Step 4:</b> Write both numerators over a single common denominator.	$\frac{(x-1)(x+1) - (x-2)(x-2)}{(x+3)(x-2)(x+1)}$
<b>Step 5:</b> Expand the numerator, if necessary.	$\frac{x^2 - 1 - (x^2 - 4x + 4)}{(x+3)(x-2)(x+1)} = \frac{x^2 - 1 + (-x^2 + 4x - 4)}{(x+3)(x-2)(x+1)}$
<b>Step 6:</b> Combine like terms.	$= \frac{4x - 5}{(x+3)(x-2)(x+1)}$
<b>Step 7:</b> Reduce, if necessary	$= \frac{4x - 5}{(x+3)(x-2)(x+1)}$ is already reduced.
<b>Step 8:</b> Find the non-permissible values.	$x = -3, -1, 2$
<b>Step 9:</b> Write the answer with any restrictions	$= \frac{4x - 5}{(x+3)(x-2)(x+1)}; \quad , x \neq -3, -1, 2$

E.g.: Find the error in the “simplification” below and write the correct simplification.

$$\frac{3x-2}{(x+2)(x-2)} - \frac{2x-4}{(x+2)(x-2)} = \frac{3x-2-2x-4}{(x+2)(x-2)} = \frac{x-6}{(x+2)(x-2)}; \quad x \neq \pm 2$$

The error occurs in the subtraction of  $2x-4$ . The correct simplification is

$$\begin{aligned} & \frac{3x-2}{(x+2)(x-2)} - \frac{2x-4}{(x+2)(x-2)} \\ &= \frac{3x-2-(2x-4)}{(x+2)(x-2)} \\ &= \frac{3x-2+(-2x+4)}{(x+2)(x-2)} \\ &= \frac{\cancel{x+2}}{(\cancel{x+2})(x-2)} \\ &= \frac{1}{x-2}; \quad x \neq \pm 2 \end{aligned}$$

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**Do #'s 3 a, c, 5 a, c, 6 b, d, 7 b, 8, 9, CYU/Practising pp. 249-250 text in your homework booklet.**

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## §4.5 Solving Rational Equations (2 classes)

Read *Goal* p. 253 text

Outcomes:

1. Define and give an example of a **rational equation**. p. 253
2. Solve rational equations algebraically. pp. 253-257
3. Identify the restriction(s) on the variable(s) in a rational equation. pp. 253-257
4. Identify **extraneous roots** when solving a rational equation. p. 254
5. Identify **inadmissible roots** when solving a rational equation. p. 254

Def<sup>n</sup>: A **rational equation** is an equation that contains one or more rational expressions.

E.g.:  $\frac{1}{x} = 3$ ,  $\frac{d}{v-3} - \frac{d}{v+3} = 8$ ,  $\frac{t}{30} + \frac{t}{40} = 1$

We can often solve rational equations by multiplying each term of the equation by the LCD of all the denominators in the equation. We have to determine the restrictions on the variables in the rational equations to identify any extraneous roots or inadmissible solutions.

Def<sup>n</sup>: An **extraneous root** is a solution to the equation that is not permissible in the original equation.

Def<sup>n</sup>: An **inadmissible solution** is a solution to the equation that is not valid in the context of the problem.

E.g.: Solve  $\frac{6}{t} + \frac{t}{2} = 4, t \neq 0$

<b>Step 1:</b> Find the lowest common denominator.	The LCD is $2t$ .
<b>Step 2:</b> Multiply each term of the equation by the LCD.	$2t\left(\frac{6}{t}\right) + 2t\left(\frac{t}{2}\right) = 2t(4)$
<b>Step 3:</b> Simplify each term. There should be no fractions remaining.	$12 + t^2 = 8t$
<b>Step 4:</b> Rearrange to get a quadratic equation.	$t^2 - 8t + 12 = 0$
<b>Step 5:</b> Factor or use the quadratic formula.	$(t-6)(t-2) = 0$
<b>Step 6:</b> Use ZPP to get the solutions.	ZPP $t-6 = 0$ or $t-2 = 0$ $t = 6$ or $t = 2$
<b>Step 7:</b> Check for extraneous roots or inadmissible solutions.	The restriction is $t \neq 0$ so both 2 and 6 are solutions.

Another way to solve rational equations is to combine expressions on each side of the equal side and cross-multiplying.

E.g.: Solve  $\frac{6}{t} + \frac{t}{2} = 4, t \neq 0$

<b>Step 1:</b> Find the lowest common denominator.	The LCD is $2t$ .
<b>Step 2:</b> Write each term on the left with a denominator of $2t$ .	$\frac{6}{t} \times \frac{2}{2} + \frac{t}{2} \times \frac{t}{t} = 4$ $\frac{12}{2t} + \frac{t^2}{2t} = 4$
<b>Step 3:</b> Write both expressions on the left over the common denominator.	$\frac{12+t^2}{2t} = 4$
<b>Step 4:</b> Cross multiply.	$12+t^2 = 8t$
<b>Step 5:</b> Rearrange to get a quadratic equation.	$t^2 - 8t + 12 = 0$
<b>Step 6:</b> Factor or use the quadratic formula.	$(t-6)(t-2) = 0$
<b>Step 7:</b> Use ZPP to get the solutions.	ZPP $t-6=0 \text{ or } t-2=0$ $t=6 \text{ or } t=2$
<b>Step 8:</b> Check for extraneous roots or inadmissible solutions.	The restriction is $t \neq 0$ so both 2 and 6 are solutions.

E.g.: Solve  $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$ ,  $x \neq -2, 0$

<b>Step 1:</b> Find the lowest common denominator.	The LCD is $5x(x+2)$ .
<b>Step 2:</b> Multiply each term of the equation by the LCD.	$5x(x+2)\left(\frac{3}{x+2}\right) - 5x(x+2)\left(\frac{1}{x}\right) = 5x(x+2)\left(\frac{1}{5x}\right)$
<b>Step 3:</b> Simplify each term. There should be no fractions remaining.	$5x(3) - 5(x+2) = 1(x+2)$
<b>Step 4:</b> Expand and combine like terms.	$15x - 5x - 10 = x + 2$ $10x - 10 = x + 2$ $9x = 12$
<b>Step 5:</b> Solve the equation.	$x = \frac{12}{9} = \frac{4}{3}$
<b>Step 6:</b> Check for extraneous roots or inadmissible solutions.	The restrictions are $x \neq -2, 0$ so $\frac{4}{3}$ is a solution.

E.g.: Solve  $\frac{10}{x(x-2)} + \frac{4}{x} = \frac{5}{x-2}$ ,  $x \neq 0, 2$

<b>Step 1:</b> Find the lowest common denominator.	The LCD is $x(x-2)$ .
<b>Step 2:</b> Multiply each term of the equation by the LCD.	$x(x-2)\left(\frac{10}{x(x-2)}\right) + x(x-2)\left(\frac{4}{x}\right) = x(x-2)\left(\frac{5}{x-2}\right)$

<b>Step 3:</b> Simplify each term. There should be no fractions remaining.	$10 + 4(x - 2) = 5x$
<b>Step 4:</b> Expand and combine like terms.	$10 + 4x - 8 = 5x$ $2 + 4x = 5x$
<b>Step 5:</b> Solve the equation.	$x = 2$
<b>Step 6:</b> Check for extraneous roots or inadmissible solutions.	The restrictions are $x \neq 0, 2$ so 2 is an extraneous root. Therefore we write $\emptyset$ .

E.g.: Solve  $\frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2 - 9y + 18}$ ,  $y \neq 3, 6$

<b>Step 1:</b> Factor the denominators.	$y - 3 = y - 3$ $y - 6 = y - 6$ $y^2 - 9y + 18 = (y - 3)(y - 6)$
<b>Step 2:</b> Find the lowest common denominator.	The LCD is $(y - 3)(y - 6)$ .
<b>Step 3:</b> Multiply each term of the equation by the LCD.	$(y - 3)(y - 6) \left( \frac{9}{y - 3} \right) - (y - 3)(y - 6) \left( \frac{4}{y - 6} \right) = (y - 3)(y - 6) \left( \frac{18}{(y - 3)(y - 6)} \right)$
<b>Step 4:</b> Simplify each term. There should be no fractions remaining.	$9(y - 6) - 4(y - 3) = 18$
<b>Step 5:</b> Expand and combine like terms.	$9y - 54 - 4y + 12 = 18$ $5y - 42 = 18$ $5y = 60$
<b>Step 6:</b> Solve the equation.	$y = 12$
<b>Step 7:</b> Check for extraneous roots or inadmissible solutions.	The restrictions are $x \neq 3, 6$ so 12 is a solution.

E.g.: Solve  $\frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2 - x - 6}$ ,  $x \neq -2, 3$

<b>Step 1:</b> Factor the denominators.	$x + 2 = x + 2$ $x - 3 = x - 3$ $x^2 - x - 6 = (x + 2)(x - 3)$
<b>Step 2:</b> Find the lowest common denominator.	The LCD is $(x + 2)(x - 3)$ .



<b>Step 3:</b> Multiply each term of the equation by the LCD.	$(x+2)(x-3)\left(\frac{3x}{x+2}\right) - (x+2)(x-3)\left(\frac{5}{x-3}\right) = (x+2)(x-3)\left(\frac{-25}{(x+2)(x-3)}\right)$
<b>Step 4:</b> Simplify each term. There should be no fractions remaining.	$3x(x-3) - 5(x+2) = -25$
<b>Step 5:</b> Expand and combine like terms.	$3x^2 - 9x - 5x - 10 = -25$ $3x^2 - 14x + 15 = 0$
<b>Step 6:</b> Factor or use the quadratic formula.	$3x^2 - 14x + 15 = 0$ $3x^2 - 9x - 5x + 15 = 0$ $3x(x-3) - 5(x-3) = 0$ $(x-3)(3x-5) = 0$
<b>Step 7:</b> Use ZPP to get the solutions.	ZPP $x-3=0$ or $3x-5=0$ $x=3$ or $x=\frac{5}{3}$
<b>Step 7:</b> Check for extraneous roots or inadmissible solutions.	The restrictions are $x \neq -2, 3$ , so we know that only $\frac{5}{3}$ is a solution.

See *In Summary*, p. 257

See <http://www.regentsprep.org/Regents/math/algtrig/ATE11/RationalEqPract.htm> for practice questions.

Do #'s 1, 4, 5 c, d, 6 b, d, CYU/Practising p. 258 text in your homework booklet.

### Solving Problems Using Rational Equations (2 classes)

It costs Mr. Williams \$500 to take some of his senior high English students to Trinity to see a play. On the day of the trip, 10 students don't show up which means that each student must pay an extra \$2.50 to cover the cost of the trip. How many students actually went on the trip?

Let  $n$  be the number of students who actually went to see the play.

Let  $n+10$  be the number of students who were originally going to see the play.

The smaller cost per student with the  $n+10$  students who were originally going is  $\frac{500}{n+10}$ .

The larger cost per student with only the  $n$  students who actually went is  $\frac{500}{n}$ .

Since the difference in cost is 2.50 we can write

$$\frac{500}{n} - \frac{500}{n+10} = 2.50; \quad n \neq -10, 0$$

We multiply each term by  $n(n+10)$  to get rid of the fractions. This gives

$$n(n+10)\left(\frac{500}{n}\right) - n(n+10)\left(\frac{500}{n+10}\right) = n(n+10)(2.50)$$

$$500(n+10) - 500n = 2.5n(n+10)$$

$$500n + 5000 - 500n = 2.5n^2 + 25n$$

$$5000 = 2.5n^2 + 25n$$

$$2.5n^2 + 25n - 5000 = 0$$

$$\frac{2.5n^2}{2.5} + \frac{25n}{2.5} - \frac{5000}{2.5} = \frac{0}{2.5}$$

$$n^2 + 10n - 2000 = 0$$

$$(n+50)(n-40) = 0$$

ZPP

$$n = -50 \quad \text{or} \quad n = 40$$

So 40 students actually went to see the play.

Do you see that  $n = -50$  is an **inadmissible solution** because it is a root of

$\frac{500}{n} - \frac{500}{n+10} = 2.50$ ;  $n \neq -10, 0$  but does not make sense in the context of this problem because the number of students going on the trip cannot be negative?

**Do # 17 Practising p. 260 text in your homework booklet.**

E.g.: Chris purchased a motorboat and used it to travel 6 km upstream and then 6 km back downstream in a total of 4 hours. If the river's current is constant at 2 km/h, algebraically determine an equation which models this situation and use it to find the speed of the boat in still water.

Let  $v$  be the speed of the boat in still water.

Let  $v+2$  be the speed of the boat going downstream.

Let  $v-2$  be the speed of the boat going upstream.

Upstream	Downstream
$d = 6\text{km}$	$d = 6\text{km}$
Speed = $v - 2$	Speed = $v + 2$
$t = \frac{6}{v-2}$	$t = \frac{6}{v+2}$

Since the total time is 4hrs, we can write

$$\frac{6}{v-2} + \frac{6}{v+2} = 4$$

We multiply each term by  $(v-2)(v+2)$  to get rid of the fractions. This gives

$$(v-2)(v+2)\left(\frac{6}{v-2}\right) + (v-2)(v+2)\left(\frac{6}{v+2}\right) = (v-2)(v+2)(4)$$

$$6(v+2) + 6(v-2) = 4(v-2)(v+2)$$

$$6v + 12 + 6v - 12 = 4(v^2 - 4)$$

$$12v = 4v^2 - 16$$

$$\frac{0}{4} = \frac{4v^2}{4} - \frac{12v}{4} - \frac{16}{4}$$

$$0 = v^2 - 3v - 4$$

$$0 = (v-4)(v+1)$$

*ZPP*

$$v = 4 \quad \text{or} \quad v = -1$$

Since speed is positive, we know that  $v \neq -1$  (inadmissible root), so the speed of the boat in still water is 4km/h.

**Do # 14, Practising p. 259 text in your homework booklet.**

E.g.: Pria, working alone, can paint a room in 2 hours. If Paul works alone, he can paint the same room in 3 hours. How long would it take to paint the room if they work together?

	Time to Paint Room (hrs)	Fraction of Room Painted in 1 Hour	Fraction of Room Painted in $x$ Hours
Pria	2	$\frac{1}{2}$	$\frac{1}{2}x$
Paul	3	$\frac{1}{3}$	$\frac{1}{3}x$
Both	$x$	$\frac{1}{x}$	$\frac{1}{x} \bullet x = 1$

The equation that models this situation is  $\frac{1}{2}x + \frac{1}{3}x = \frac{1}{x} \bullet x$  or  $\frac{x}{2} + \frac{x}{3} = 1$

Multiplying all the terms by 6 and solving gives

$$6\left(\frac{x}{2}\right) + 6\left(\frac{x}{3}\right) = 6(1)$$

$$3x + 2x = 6$$

$$5x = 6$$

$$x = \frac{6}{5} = 1.2 \text{ hrs or } 1 \text{ hr, } 12 \text{ minutes}$$

It will take 1.2 hrs to paint the room if they both work together.

**Do # 10 Practising p. 259 text in your homework booklet.**

E.g.: Pria, working alone, can paint a room in 4 hours. If Pria and Paul work together, they can paint the same room in 2.4 hours. How long would it take Paul to paint the room if he works alone?

	Time to Paint Room (hrs)	Fraction of Room Painted in 1 Hour	Fraction of Room Painted in 2.4 Hours
Pria	4	$\frac{1}{4}$	$\frac{1}{4} \times 2.4 = \frac{2.4}{4}$
Paul	$x$	$\frac{1}{x}$	$\frac{1}{x} \times 2.4 = \frac{2.4}{x}$
Both	2.4	$\frac{1}{2.4}$	$\frac{1}{2.4} \times 2.4 = 1$

The equation that models this situation is  $\frac{2.4}{4} + \frac{2.4}{x} = \frac{1}{2.4} \times 2.4$  or  $\frac{2.4}{4} + \frac{2.4}{x} = 1$

Multiplying all the terms by  $4x$  and solving gives

$$4x\left(\frac{2.4}{4}\right) + 4x\left(\frac{2.4}{x}\right) = 4x(1)$$

$$2.4x + 9.6 = 4x$$

$$2.4x + 9.6 - 2.4x = 4x - 2.4x$$

$$9.6 = 1.6x$$

$$\frac{9.6}{1.6} = \frac{\cancel{1.6}x}{\cancel{1.6}}$$

$$x = 6$$

It will take 6 hrs for Paul to paint the room alone.

**Do #'s 11, 8, Practising p. 259 text in your homework booklet.**

E.g.: Mrs Harnum bought a box of cheer-leader T-shirts for \$450. She kept one for herself and one for her daughter and sold the rest for \$560, making a profit of \$5 on each T-shirt. How many T-shirts were in the case?

Let  $x$  represent the number of T-shirts in the box.

Profit/shirt = Revenue/shirt – Cost/shirt so we can write the equation

$$5 = \frac{560}{x-2} - \frac{450}{x}, \quad x \neq 2, 0$$

Multiplying both sides by  $x(x-2)$  gives

$$5x(x-2) = \frac{560}{x-2}x(x-2) - \frac{450}{x}x(x-2)$$

$$5x(x-2) = 560x - 450(x-2)$$

$$5x^2 - 10x = 560x - 450x + 900$$

$$5x^2 - 10x - 560x + 450x - 900 = 0$$

$$5x^2 - 120x - 900 = 0$$

$$5x^2 - 120x - 900 = 0$$

$$x = \frac{-(-120) \pm \sqrt{(-120)^2 - 4(5)(-900)}}{2(5)}$$

$$x = \frac{120 \pm \sqrt{14400 + 18000}}{10}$$

$$x = \frac{120 \pm \sqrt{32400}}{10}$$

$$x = \frac{120 + 180}{10} \quad \text{or} \quad x = \frac{120 - 180}{10}$$

$$x = 30 \quad \text{or} \quad x = -6$$

Since  $x = -6$  is an inadmissible root, there were 30 shirts in the box.

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**Do #'s 15, 18, Practising p. 260 text in your homework booklet.**

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**Do #'s 2, 3 d, 4 b, d, 5 c, d, 6 a, d, 7, 9 a, c, 10 b, d, 11, Practising p. 265-266 text in your homework booklet.**

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