Math 3201 Notes

Chapter 1: Set Theory

Learning Goals: See p. 3 text.

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§1.1 **Types of Sets and Set Notation (2 classes)**

Outcomes:

1. Define and give examples and nonexamples (if possible) of the following. Also include the proper symbol or notation.
   a. **Set** p. 6
   b. **Set builder notation** p. 6 & notes
   c. **Element** of a set. p. 6
   d. **Universal set**. p. 6
   e. **Subset** of a set. p. 6
   f. **Venn Diagram** (see notes)
   g. **Complement** of a set. p. 6
   h. **Empty** set. p. 7
   i. **Disjoint** sets. p. 7
   j. **Number** elements in a set. p. 10
   k. **Finite** set. p. 10
   l. **Infinite** set. p. 10
2. Explain what is meant by each of the following:
   a. **Inclusive** p. 10
   b. **Exclusive** (see notes)
   c. **Mutually exclusive** p. 13
3. Write the following special sets on numbers in set notation. (see #17a, p. 18) and give the symbol for each set of numbers.
   a. **Natural** numbers
   b. **Whole** numbers
   c. **Integers**
   d. **Rational** numbers
   e. **Irrational** numbers
   f. **Real** number

Def^{n}: A **set** is a collection of objects that are distinguishable (can be identified as being different).

E.g.: The people doing Math 3201 at Heritage Collegiate make up a set.

E.g.: The people at Heritage Collegiate who are members of the cheerleading team make up a set.

E.g.: The letters of the alphabet make up a set.

E.g.: The part of Canadian currency that is in coins make up a set.

We can describe a set using words (see the last 4 examples) but we often place the elements of a set inside a two curly brackets (think left and right hand side of a violin) called **braces**. To name the set, we often use a symbol that helps us remember the elements of the subset.

E.g.: We can describe the letters of the alphabet as a set by writing $A = \{\text{a, b, c, d}\ldots\text{x, y, z}\}$.

E.g.: We can describe the portion of Canadian currency made up of coins as a set by writing
$C = \{\text{penny?, nickel, dime, quarter, loonie, toonie}\}$

Def^{n}: An **element** of a set is one of the objects in the set.

E.g.: For our set of Canadian currency made up of coins, one element would be a nickel, another element of this set would be a quarter, another would be a loonie and so on. Since mathematicians are lazy, we often use notation instead of writing out long sentences. So if $C = \{\text{penny, nickel, dime, quarter, loonie, toonie}\}$, instead of writing "a nickel is an element of the set containing Canadian currency made up of coins" we write "$\text{nickel} \in C$" where the symbol "$\in$" means "belongs to" or "is a member of". However, a 5-dollar bill would not belong to the set of Canadian currency made up of coins, so \$5 would NOT be an element of the set. In this case, we would write $\$5 \notin C$.

Def$^{\underline{n}}$: The **universal set** is the set containing ALL the elements **under consideration** in a particular context. It is often given the symbol U.

E.g.: For Canadian currency made up of coins, the universal set is
$U = \{\text{penny, nickel, dime, quarter, loonie, toonie}\}$.

Suppose we are considering the set of tablets with screens from 9 to 12 inches. One subset could be those tablets made by Apple and another could be those made by Samsung. So if $A = \{\text{iPad, iPad Air}\}$
and $S = \{\text{Galaxy Note, Galaxy Tab 2, Galaxy Tab}\}$ then
$U = \{\text{iPad, iPad Air, Galaxy Note, Galaxy Tab 2, Galaxy Tab}\}$.

The universal set would NOT be

$U = \{\text{iPad, iPad Air, Galaxy Note, Galaxy Tab 2, Galaxy Tab, VivoTab, Transformer, Latitude}\}$

because tablets such as the Asus VivoTab, the Asus Transformer, or the Dell Latitude are NOT under consideration in this context. We are only considering tablets from Apple and Samsung and NOT from Asus or Dell.

Def$^{\underline{n}}$: A **subset** of a given set consists of elements that are also in that given set.

E.g.: For our set of Canadian currency made up of coins, if $U = \{\text{penny, nickel, dime, quarter, loonie, toonie}\}$ is the given set, then $LD = \{\text{penny, nickel, dime, quarter}\}$ (the set of coins worth **L**ess than one **D**ollar) is a subset of *U*. Since mathematicians are lazy, instead of writing "*LD* is a subset of *U*", we write "$LD \subset U$", where $\subset$ means "is a subset of".

E.g.: For our set of Canadian currency made up of coins, if $U = \{\text{penny, nickel, dime, quarter, loonie, toonie}\}$ is the given set, then $MD = \{\text{toonie}\}$ (the set of coins worth **M**ore than one **D**ollar) is a subset of *U*. Using notation we write "$MD \subset U$".

Sample Exam Question

1. Which statement is true for sets *A*, *B*, and *C*?

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; $B = \{5, 10\}$; $C = \{3, 6, 9, 12\}$

a) *A* is a subset of *B*. $A \subset B$
b) *A* is a subset of *C*. $A \subset C$
c) *B* is a subset of *A*. $B \subset A$
d) *C* is a subset of *A*. $C \subset A$

Def$^{\underline{n}}$: The **complement** of a given set consists of the set of elements in the universal set that **do not** belong to the given set. We use the prime symbol (') to denote the complement. So if the given set is called T, then the complement is denoted T'.

E.g.: Suppose we look at the set of **all** Canadian currency.

Then $U = \{\text{penny, nickel, dime, quarter, loonie, toonie, \$5, \$10, \$20, \$50, \$100}\}$.

One subset of this universal set could be the collection of Canadian currency made up of coins. This given subset is $C = \{\text{penny, nickel, dime, quarter, loonie, toonie}\}$.

The complement of *C* is the collection of Canadian currency **not** made up of coins, In other words, the complement is that part of our currency made up of paper money.

So $C' = \{\$5, \$10, \$20, \$50, \$100\}$.

Note that if we combine *C* and C' we get *U*.

Def$^{\underline{n}}$: The **empty set** is the set with no elements in it. We use the symbols $\{\ \}$ or $\varnothing$ to denote the empty set, but the symbol $\varnothing$ is used most often.

E.g.: If $U = \{\text{penny, nickel, dime, quarter, loonie, toonie, \$5, \$10, \$20, \$50, \$100}\}$, then the set containing \$1,000,000 bills would be the empty set.

E.g.: The solution to the equation $x^2 = -2$ would be the empty set, since there are no real numbers we can square and get a negative number. So mathematically we write $x = \varnothing$.

Def$^{\underline{n}}$: Two or more sets with no elements in common are called **disjoint** sets.

E.g.: If $U = \{\text{penny, nickel, dime, quarter, loonie, toonie, \$5, \$10, \$20, \$50, \$100}\}$, then $C = \{\text{penny, nickel, dime, quarter, loonie, toonie}\}$ and $C' = \{\$5, \$10, \$20, \$50, \$100\}$ are disjoint sets.

E.g.: If $U = \{\text{penny, nickel, dime, quarter, loonie, toonie, \$5, \$10, \$20, \$50, \$100}\}$, then

$P = \{\text{penny}\}$, $PM = \{\$10, \$20, \$50\}$, and $D = \{\text{dime}\}$ are disjoint sets.

Often we are interested not in the actual elements in a set, but in the **number** of elements in a set.

Def $^{\underline{n}}$: The **number** of elements in a given set Q is denoted $n(Q)$.

E.g.: If $C = \{\text{penny, nickel, dime, quarter, loonie, toonie}\}$, then $n(C) = 6$.

E.g.: If $D = \varnothing$, then $n(D) = 0$.

Def $^{\underline{n}}$: A **finite** set is a set in which you can count the number of elements.

E.g.: $PM = \{\$10, \$20, \$50\}$ is a finite set with $n(PM) = 3$.

Def $^{\underline{n}}$: An **infinite** set is a set with an unlimited (or infinite) number of elements.

E.g.: The set of even numbers, $E = \{2, 4, 6, 8, \ldots\}$, is an infinite set and $n(E) = \infty$.

E.g.: The digits after the decimal in the constant pi (π) make up an infinite set.

Many infinite sets involve sets of numbers. Recall that there are six basic sets of numbers. You need to know these numbers and the symbol for each set. (See 17(a), p. 18 text)

1. The **natural**/counting numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

2. The **whole** numbers: $W = \{0, 1, 2, 3, 4, 5, \ldots\}$

3. The **integers**: I or $\mathbb{Z} = \{\ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

4. The **rational** numbers $(\mathbb{Q})$: Those numbers that can be written as a fraction of one integer divided by another. E.g.: $\dfrac{-15}{29}$. Why isn't $\dfrac{-15}{\pi}$ a rational number?

OR

Any decimal number that ends or repeats.

5. The **irrational** numbers $(\mathbb{Q}')$: Those numbers that cannot be written as a fraction of one integer divided by another.

OR

Any decimal number that does not end or does not repeat.

Irrational numbers are often radicals. E.g.: $\sqrt{15} = 3.872983346\ldots$. However, there are two special irrational numbers that do not involve radicals. One is pi (π) and the other is Euler's number (e).

E.g.: $\pi = 3.14159265358979323846264338327950 2\ldots$ OR $e = 2.71828182845904523536028747135 27\ldots$

Note that $\mathbb{Q}$ and $\mathbb{Q}'$ are disjoint sets. They have no elements in common. This means that if a number is rational it cannot be irrational and vice versa.

6. The **real** numbers $(\mathbb{R})$: The set of numbers you get when you combine the rational numbers and the irrational numbers. This includes every number that you know of at this point.

Your Turn #1

Bombardier Recreational Product's (BRP) founder built the first tracked recreational vehicle in 1937. For the 2015 model year, BRP will produce the following snowmobile models: MXZ, Renegade, Summit, Freeride, GSX, Grand Touring, Expedition, Tundra, and Skandic.

1. What is the universal set in this context?

$U = \{ \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad \}$

2. Is U a finite set or an infinite set? ___________ How do you know? ________________________________
3. Identify one element of the universal set. ________________ $\in U$
4. Create a subset of U, named R, with 3 elements. $R = \{ \quad , \quad , \quad \}$
5. Create the complement of R, named R'.
   $R' = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$
6. Compare $n(R) + n(R')$ and $n(U)$. What do you notice?

7. Create three disjoints sets, one with 1 element, one with 2 elements, and one with 3 elements.

   $A = \{ \quad \}$ $B = \{ \quad , \quad \}$ $C = \{ \quad , \quad , \quad \}$

8. Would it be possible to create four disjoint sets, one with 1 element, another with 2 elements, a third with 3 elements, and a fourth with 4 elements? ________ Explain why or why not.

9. Create two subsets of U, P with 5 elements and Q with 3 elements, such that $Q \subset P$.

   $P = \{ \quad , \quad , \quad , \quad , \quad \}$ $Q = \{ \quad , \quad , \quad \}$

10. If $M = \{\text{Summit, Freeride}\}$ is the set of snowmobiles recommended for mountain riding, then $n(M) =$ ____
11. Describe any empty set for this context.

    The set of ____ $= \varnothing$

If a context has many sets and/or subsets, it is sometimes helpful to draw a diagram of the universal set and any subsets that are important in the given context. One such diagram is called a **Venn diagram** (see p. 6, text) which often consists of a rectangle (representing the universal set) surrounding one or more circles and/or ellipses (representing the subset(s)). On the top of page 12 of the text is a Venn diagram in which the entire grey rectangle represents the universal set of the animals under consideration in this context (*A*), the orange ellipse represents a subset of warm-blooded animals (*W*), the green circle represents a subset of mammals (*M*), the blue circle represents a subset of birds (*B*), and the purple circle represents a subset of cold-blooded animals (*C*).

Note that:

i. set *B* is entirely within set *W* which is entirely within set *A* so we can write $B \subset W \subset A$. Similarly, $M \subset W \subset A$.

ii. set *W* and set *C* do not overlap so they are disjoint sets. Similarly, set *B* & set *C* are disjoint sets, set *M* & set *C* are disjoint sets, and set *M* & set *B* are disjoint sets.

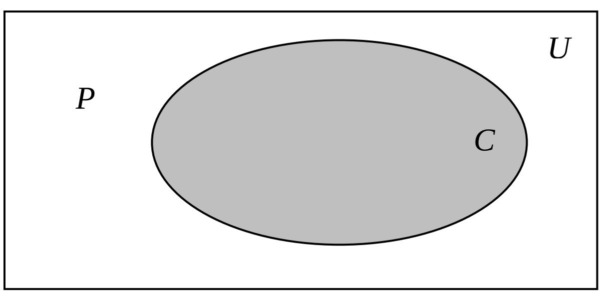
iii. an animal cannot be warm-blooded and also cold-blooded at the same time. This means that set W and set C must be disjoint (see note ii) but it also means that being warm-blooded and being cold-blooded are **mutually exclusive**. If an animal is one, it cannot be the other.

Def$^{\underline{n}}$: Two or more events are **mutually exclusive** if they cannot occur at the same time. Gender is mutually exclusive. When you flip a coin, you get either heads or tails. These events are mutually exclusive. Turning left and turning right are also mutually exclusive. In mathematics, being less than -2 and greater than +2 at the same time are mutually exclusive, so it is incorrect to write $-2 > x > 2$, as this is impossible.

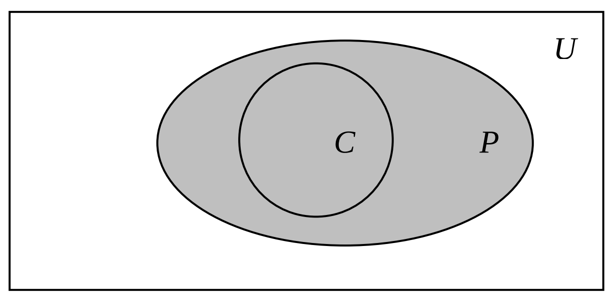
Your Turn #2

1. Let U be the set of Canadian currency, *C* be the set of Canadian currency consisting of coins, and *P* be the set of Canadian currency consisting of paper money. Which Venn diagram below would correctly represent this situation?

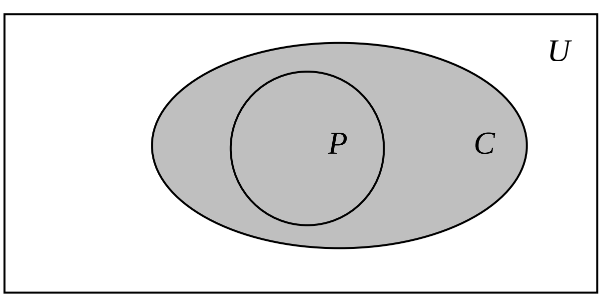
a)



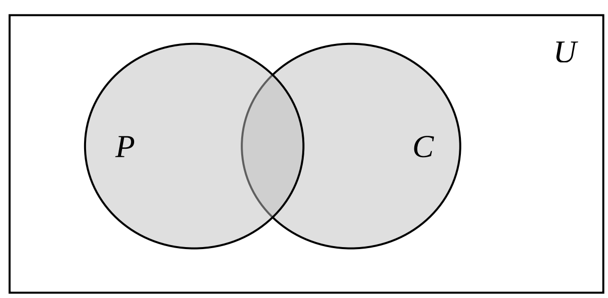
c)



b)



d)

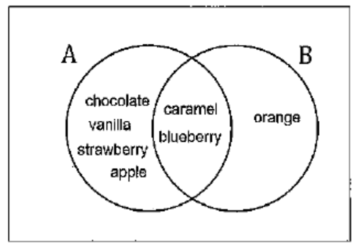


2. Are there any mutually exclusive events in #1? ______ Explain why or why not.

Sample Exam Question

2. Given the Venn diagram below, how many elements are in the complement of set *B*, $n(B')$?

   a) 1
   b) 2
   c) 4
   d) 6



E.g.: Suppose the set of integers $(\mathbb{Z})$ is the universal set. If *x* represents an element in the set of integers, find the number of elements in each of the following sets:

i. $\{-3 < x < 5\}$ ii. $\{-3 < x \le 5\}$ iii. $\{-3 \le x < 5\}$ iv. $\{-3 \le x \le 5\}$

i. $\{-3 < x < 5\}$ means the set of all the integers from -3 and 5 **exclusive**, meaning -3 and 5 are **not** included in the set. Therefore $\{-3 < x < 5\} = \{$-2, -1, 0, 1, 2, 3, 4$\}$ so the number of elements in $\{-3 < x < 5\}$ is 7. We could write $n(\{-3 < x < 5\}) = 7$. Note that, in this context, $\{-3 < x < 5\}$ means the same as "the set on integers **between** -3 and 5."

ii. $\{-3 < x \le 5\}$ means the set of all the integers from -3 and 5, **excluding** -3 but **including** 5. So -3 is **not** in the set but 5 **is** in the set. Therefore $\{-3 < x \le 5\} = \{$-2, -1, 0, 1, 2, 3, 4, 5$\}$ so the number of elements in $\{-3 < x \le 5\}$ is 8. We could write $n(\{-3 < x \le 5\}) = 8$.

iii. $\{-3 \le x < 5\}$ means the set of all the integers from -3 and 5, **including** -3 but **excluding** 5. So -3 **is** in the set but 5 is **not** in the set. Therefore $\{-3 \le x < 5\} = \{$-3, -2, -1, 0, 1, 2, 3, 4$\}$ so the number of elements in $\{-3 \le x < 5\}$ is 8. We could write $n(\{-3 \le x < 5\}) = 8$.

iv. $\{-3 \le x \le 5\}$ means the set of all the integers from -3 and 5 **inclusive**, meaning both -3 and 5 are included in the set. Therefore $\{-3 \le x \le 5\} = \{$-3, -2, -1, 0, 1, 2, 3, 4, 5$\}$ so the number of elements in $\{-3 \le x \le 5\}$ is 9. We could write $n(\{-3 \le x \le 5\}) = 9$. The "*Communication*" box on page 10 of the text notes that the phrase "from 1 to 5" means "from 1 to 5 **inclusive**." So in our example, if *x* is an integer, $\{-3 \le x \le 5\}$ means the same as "the integers from -3 to 5."

Your Turn #3

Suppose the set of natural numbers $(\mathbb{N})$ is the universal set. If *x* represents an element in the set of natural numbers, find the number of elements in each of the following finite sets:

i. $\{2 < x < 11\} = \{\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad\}$ and $n(\{2 < x < 11\}) =$ ______

ii. $\{2 \le x < 11\} = \{\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad\}$ and $n(\{2 \le x < 11\}) =$ ______

iii. $\{2 < x \le 11\} = \{\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad\}$ and $n(\{2 < x \le 11\}) =$ ______

iv. $\{2 \le x \le 11\} = \{\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad,\quad\}$ and $n(\{2 \le x \le 11\}) =$ ______

v. The set of natural numbers (P) between 2 and 11. $n(P) =$ ______ .

vi. The set of natural numbers (Q) from 2 to 11. $n(Q) =$ ______ .

Up to now we have used three different methods to denote the sets in these notes. The first method was a written description of the set such as "the set of natural numbers greater than 5." The second method was to list the elements of the set inside braces. For example, we could denote "the set of natural numbers greater than 5" by writing $\{6, 7, 8, 9, 10, \ldots\}$. The third method was to write an inequality inside a set of braces. For example, we could denote "the set of natural numbers greater than 5" by writing $\{x > 5, x \in \mathbb{N}\}$. We are now going to expand on this last method and denote sets using **set builder notation**.

Using set builder notation, we could denote "the set of natural numbers greater than 5" by writing $\{x \mid x > 5, x \in \mathbb{N}\}$, which is spoken "*x* such that *x* is greater than 5 where *x* belongs to the set of natural numbers."

"$x \mid$" means "*x* such that"

"$x > 5$" means "*x* is greater than 5"

"$x \in \mathbb{N}$" means "where *x* belongs to the set of natural numbers"

E.g.: Using set builder notation, the set of integers between -90 and 50 could be written $\{x \mid -90 < x < 50, x \in \mathbb{Z}\}$.

E.g.: Using set builder notation, the set of integers from -90 to 50 could be written $\{x \mid -90 \le x \le 50, x \in \mathbb{Z}\}$.

E.g.: Using set builder notation, the set of real numbers less than or equal to 3.15 could be written $\{x \mid x \le 3.15, x \in \mathbb{R}\}$. However, we can omit the "$x \in \mathbb{R}$" when dealing with the real numbers and just write $\{x \mid x \le 3.15\}$. It is assumed that the set under consideration is the real numbers unless we specify otherwise.

E.g.: Using set builder notation, the set of real numbers greater than $\frac{1}{2}$ could be written $\left\{x \mid x > \frac{1}{2}, x \in \mathbb{R}\right\}$ or just $\left\{x \mid x > \frac{1}{2}\right\}$.

Your Turn #4

Write each of the following sets using set builder notation.

i. "The set of natural numbers less than -7." ________________________________

ii. "The set of integers greater than 12." ________________________________

iii. "The set of integers between -50 and 80." ________________________________

iv. "The set of real numbers from $-\frac{1}{2}$ to 10.87." ________________________________

v. "The set of real numbers between -8 to -2." ________________________________

vi. "The set of real numbers greater than or equal to -8 and less than 10." ________________________

Read "In Summary" p. 14 text.
Do #'s 1, 2, 4, 6-9, 16, 17(a) CYU pp.14-18

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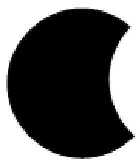
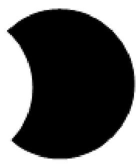
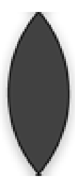
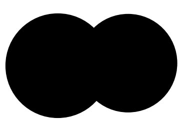
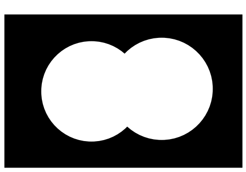
§1.2 **Exploring Relationships between Sets (1 class)**

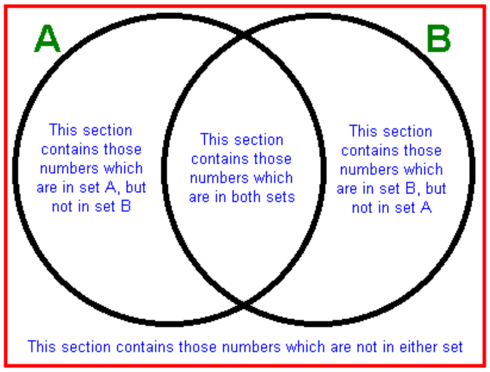
Outcomes:

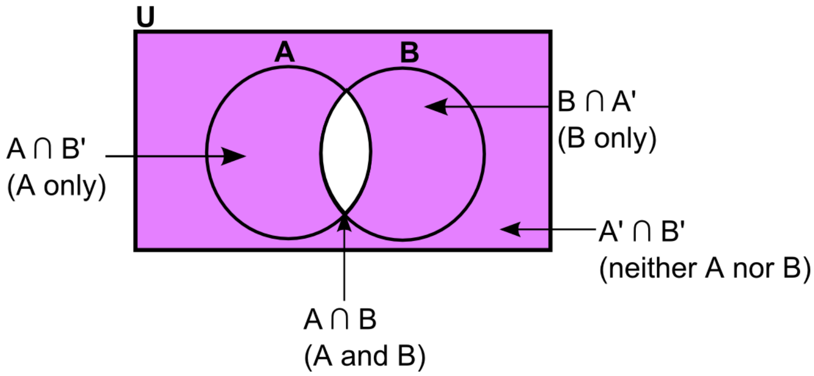
1. Explain what each region of a Venn diagram represents using connecting words (and, or, not) or set notation. pp. 19-21
2. Write the proper notation for the following sets:
   a) The set of elements in set *A* but not in set *B*.
   b) The set of elements in set *B* but not in set *A*.
   c) The set of elements in set *A* AND in set *B*.
   d) The set of elements in set *A* OR in set *B*.
3. Solve problems involving overlapping (non-disjoint) sets. pp. 19-21
4. Correct solution errors in problems involving Venn diagrams. E.g.: #5, p. 21

The Venn diagram on page 20 of the text contains two overlapping (non-disjoint) sets. Each region of the Venn diagram represents a different subset of the universal set.

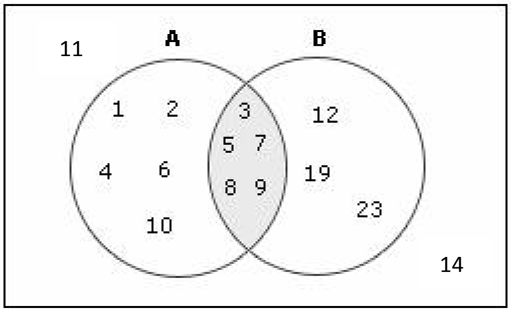
1. The orange region that has a  shape represents all the elements that are in set *A* but NOT in set *B*. $(A \setminus B)$
2. The blue region that has a  shape represents all the elements that are in set *B* but NOT in set *A*. $(B \setminus A)$
3. The blue/orange region that has a  shape where set *A* and set *B* overlap represents the set of elements in both set *A* AND set *B*. $(A \cap B)$
4. The region with the  shape represents the set of elements that are in set *A* OR in set *B*. $(A \cup B)$
5. The region with the  shape represents the set of elements in the universal set that are NOT in set *A* and also NOT in set *B*. $A' \cap B'$ or $(A \cup B)'$

You might also want to look at the Venn diagrams below. Note some different notation in the right diagram.





E.g.: For the Venn diagram to the right determine the elements:



a) in set U.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 19, 23\}$$

b) in set A.

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

c) in set B.

$$B = \{3, 5, 7, 8, 9, 12, 19, 23\}$$

d) in set A but not in set B. $(A \setminus B)$

$$A \setminus B = \{1, 2, 4, 6, 10\}$$

e) in set B but not in set A. $(B \setminus A)$

$$B \setminus A = \{12, 19, 23\}$$

f) in set A AND in set B. $(A \cap B)$

$$A \cap B = \{3, 5, 7, 8, 9\}$$

g) in set A OR in set B. $(A \cup B)$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 19, 23\}$

h) in the complement of set A. (A')

$A' = \{11, 12, 14, 19, 23\}$

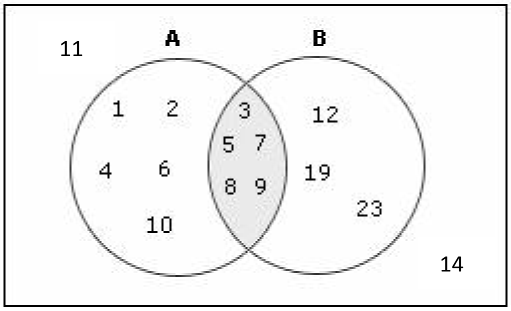
i) in the complement of set B. (B')

$B' = \{1, 2, 4, 6, 10, 11, 14\}$

j) that are NOT in set A and NOT in set B. $A' \cap B'$ or $(A \cup B)'$

$A' \cap B' = \{11, 14\}$

E.g.: For the Venn diagram to the right determine the number of elements:



1. in set U.

   $n(U) =$

2. in set A.

   $n(A) =$

3. in set B.

   $n(B) =$

4. in set A but not in set B.

   $n(A \setminus B) =$

5. in set B but not in set A.

   $n(B \setminus A) =$

6. in set A AND in set B.

   $n(A \cap B) =$

7. in set A OR in set B.

   $n(A \cup B) =$

8. in the complement of set *A*.

$n(A') =$

9. in the complement of set *B*.

$n(B') =$

10. that are NOT in set *A* and NOT in set *B*. $A' \cap B'$ or $(A \cup B)'$

$n(A' \cap B') =$

Read "In Summary" p. 20 text.
Do #'s 1-5 FYU pp. 20-21

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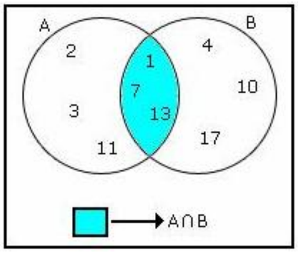
§1.3 **Intersection and Union of Two Sets (2 classes)**

Outcomes:

1. Define the **intersection** of two or more sets, identify the intersection in a Venn diagram, and denote it using the proper notation. pp. 22-23
2. Define the **union** of two or more sets, identify the union in a Venn diagram, and denote it using the proper notation. p. 23
3. Identify the set of elements that are in one set but not in another set and denote it using the proper notation. p. 23

Recall that the **intersection** $(\cap)$ of 2 sets is the set of elements that are in both sets.

E.g.: In the Venn diagram to the right, the overlapping shaded area represents the intersection of set *A* and set *B*. In this example, the set of elements that are in set *A* AND also in set *B* is $A \cap B = \{1,\ 7,\ 13\}$ and $n(A \cap B) = 3$.

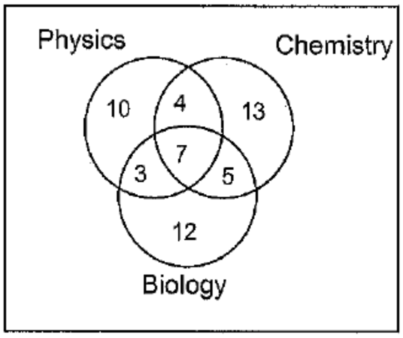


Your Turn #1: Complete the following statement using "OR" or "AND"

The set $A \cap B$ consists of the elements that are in set *A* _____ in set *B*.

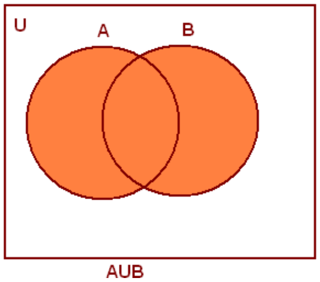
Given the Venn diagram to the right, how many students are in Physics AND Biology but NOT Chemistry?

a) 41
b) 25
c) 10
d) 3



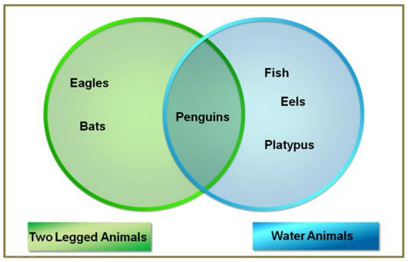
Def$^{\text{n}}$: The **union** $(\cup)$ of two sets is the set of all the elements that are in at least one of the two sets.

E.g.: In the Venn diagram to the right, the shaded area in the Venn diagram represents the union of set *A* and set *B*.

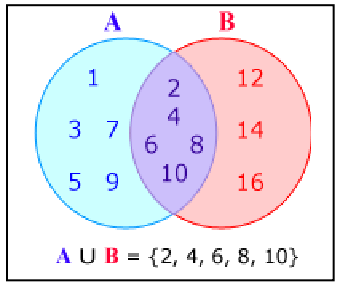


E.g.: In the Venn diagram to the right, the union of the set of two-legged animals and the set of water animals is $\text{TLA} \cup \text{WA} = \{\text{eagles, bats, penguins, fish, eels, platypus}\}$ and

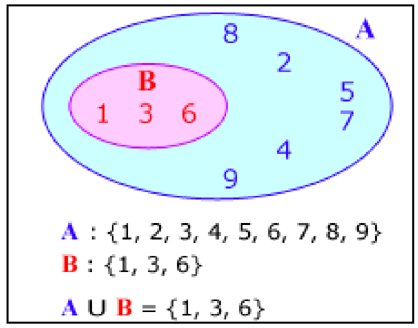
$n(\text{TLA} \cup \text{WA}) = 6$



E.g.: What is wrong with the Venn diagram to the right?



E.g.: What is wrong with the Venn diagram to the right?



Your Turn #2: Complete the following statement using "OR" or "AND"

The set $A \cup B$ consists of the elements that are in set A _____ in set B.

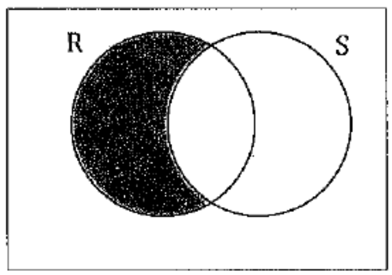
Do #'s 1-4, 6, pp. 32-33 text.

Sometimes we are interested set of elements that are in one set but not in another set.

Def^{n}: For two sets A and B, the set of elements that are in set A but NOT in set B is denoted "$A \setminus B$" which is pronounced "set A minus set B." Note that in each of the three diagrams at the bottom of the "Communication/Notation" box on page 23 of the text, the area in orange is $A \setminus B$.

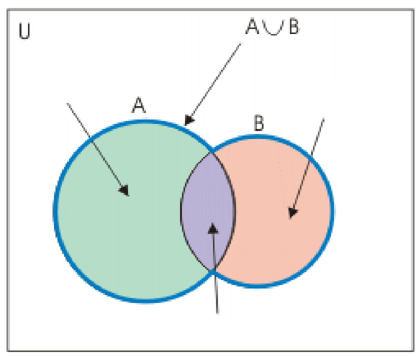
Sample Exam Question

In the diagram to the right, what does the shaded area represent?



a) $R \setminus S$
b) $R \cup S$
c) $S \setminus R$
d) $R \cap S$

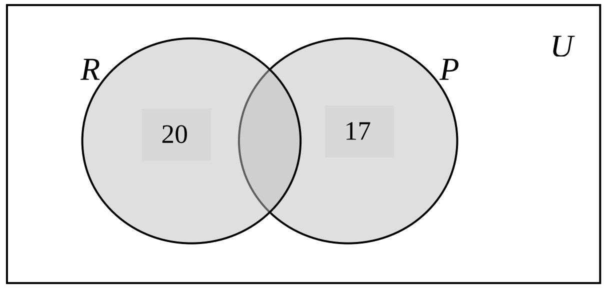
Your Turn #3: Finish labeling the arrows in the Venn diagram to the right.



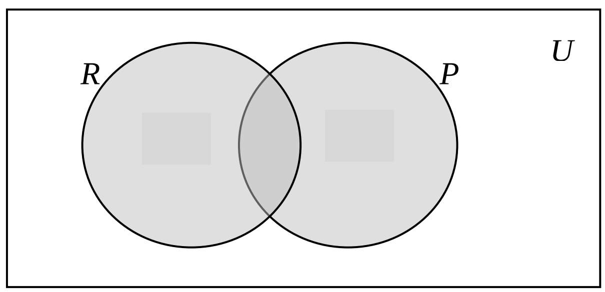
In problem solving using sets, we are often interested in the number of elements in one or more sets as opposed to the actual elements in the set. Consider the problem below.

In a mathematics class 20 students had forgotten their rulers and 17 had forgotten their pencils. “go and borrow them from someone” the teacher said. All 24 students left the room.

What is wrong with the Venn diagram below?

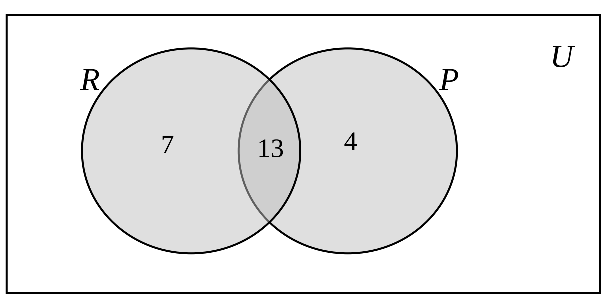


Hopefully, you should see that $20+17=37$, which is more than the total number of students in the class. This should tell you that there must be some students that forgot both their ruler and their pencil. So there must be a number in the intersection region in the Venn diagram. Can you determine how many students forgot both their ruler and their pencil? Fill in the diagram below with the correct numbers.



Determining the Number of Elements in a Set Using Reasoning

In the example above, since 37 is greater than 24, there must have been 37 – 24 = 13 students who forgot both their ruler and their pencil. Using notation we write $n(R\cap P)=13$. This means that 20 – 13 = 7 forgot only their ruler. Using notation, we write $n(R\setminus P)=7$. This also means that 17 – 13 = 4 forgot only their pencil. Using notation we write $n(P\setminus R)=4$. So the correct Venn diagram should look like the one below.

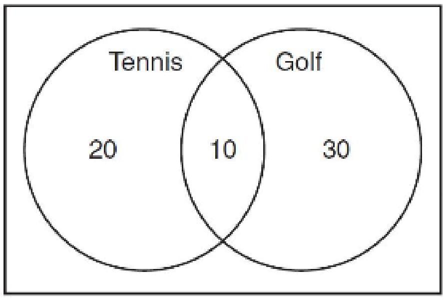


E.g.: A coach conducted a survey to determine how many students plan to try out for the tennis and golf teams. The results of the survey are shown below.

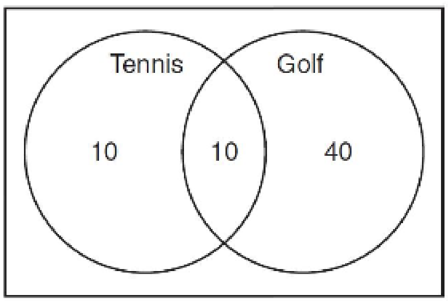
- A total of 20 students plan to try out for tennis.
- A total of 40 students plan to try out for golf.
- 10 students plan to try out for both tennis and golf.

Which Venn diagram best represents this information?

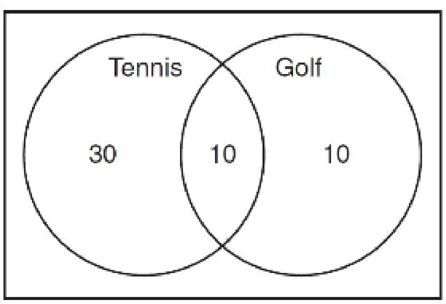
a)



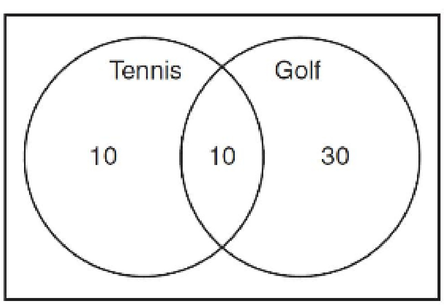
b)



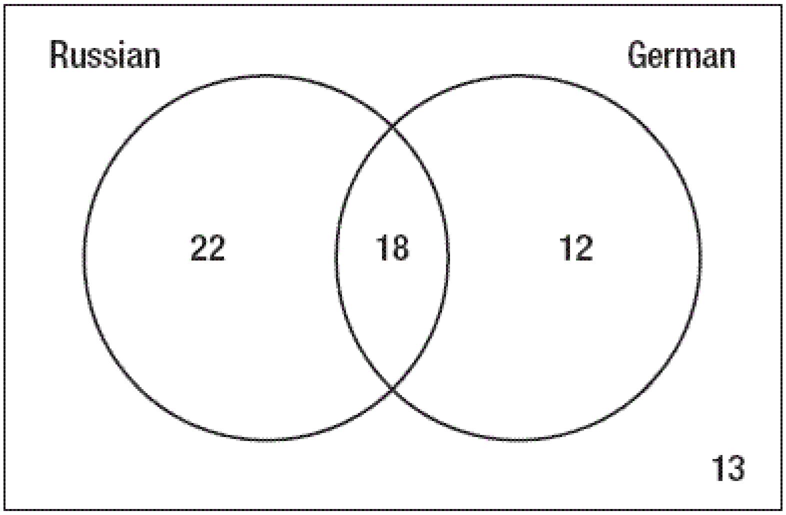
c)



d)



E.g.: Complete the description of the situation modeled by the Venn diagram below.



_____ people speak Russian. $n(R) =$ _____

_____ people speak German. $n(G) =$ _____

_____ people speak Russian but do not speak German. $n(R \setminus G) =$ _____

_____ people speak German but do not speak Russian. $n(G \setminus R) =$ _____

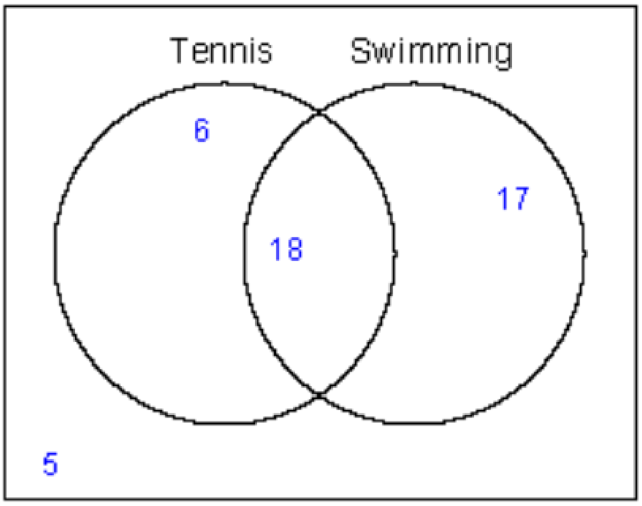
_____ people speak Russian and German. $n(R \cap G) =$ _____

_____ people speak Russian or German. $n(R \cup G) =$ _____

_____ people speak neither Russian nor German. $n(R \cup G)' =$ _____

_____ people were surveyed. $n(U) =$ _____

E.g.: Use the Venn diagram below to answer the questions that follow.



How many students like tennis OR swimming? ________

How many students do not like tennis? _______

How many students do not like either tennis OR swimming? ________

How many students like swimming? ________

How many students do not like swimming? _______

How many students like tennis? _______

How many students like both tennis AND swimming? ________

How many students only like tennis? _______

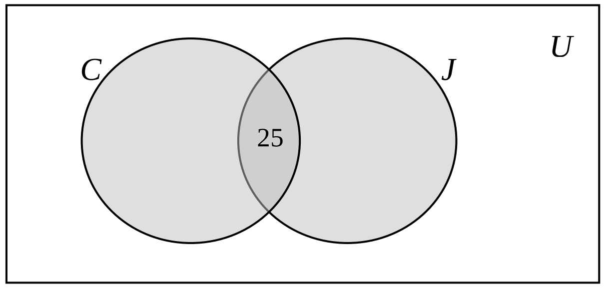
How many students only like swimming? _______

How many students do not like both tennis AND swimming? ________

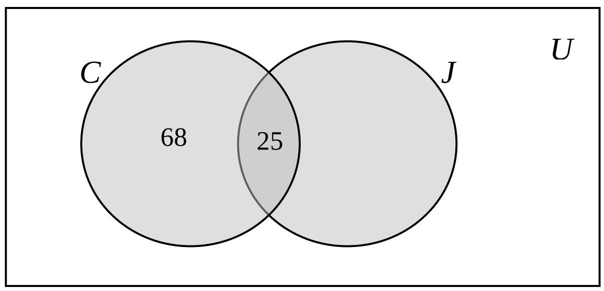
E.g.: At breakfast buffet, 93 people chose coffee for their beverage and 47 people chose juice. 25 people chose both coffee and juice. If each person chose at least one of the beverages, how many people visited the buffet?

Let C represent the set of people who chose coffee and J represent the set of people who chose juice.

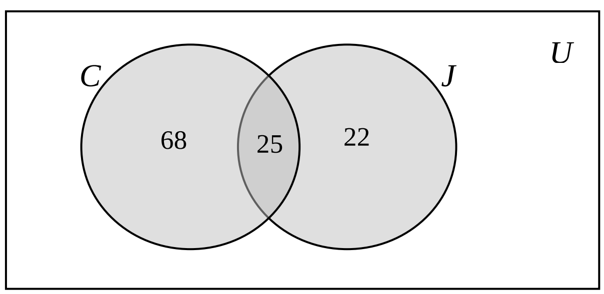
Since 25 people chose both coffee and juice, we know that the number of people in the intersection is of C and J is 25.



Since 93 students chose coffee then 93 – 25 = 68 people chose coffee only.



Since 47 students chose juice then 47 – 25 = 22 people chose juice only.



So 68 + 25 + 22 = 115 people visited the buffet.

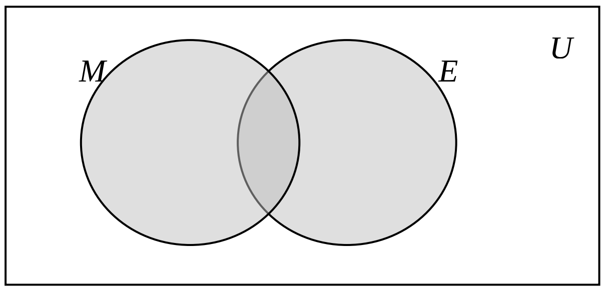
E.g.: 150 first year college students were interviewed to determine what courses they had registered in.

- 85 students registered for a math course.
- 70 students registered for an English course.
- 50 students registered for a math course and an English course.

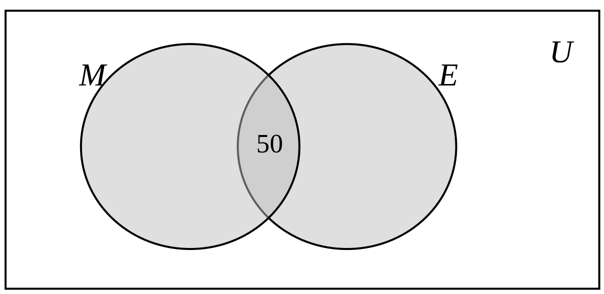
Determine:

a) how many students registered for a math course only.
b) how many students registered for an English course only.
c) how many students registered for a math course OR an English course.
d) how many students registered for neither a math course nor an English course.

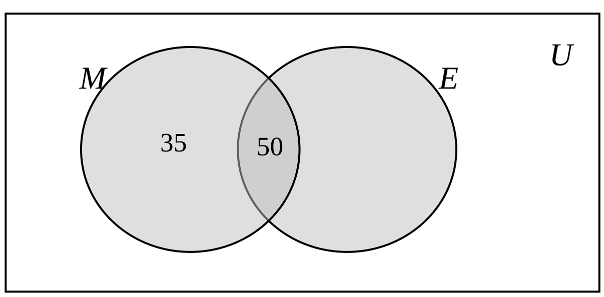
Let M represent the set of math students and E represent the set of English students. Let's draw a Venn diagram to represent this situation.



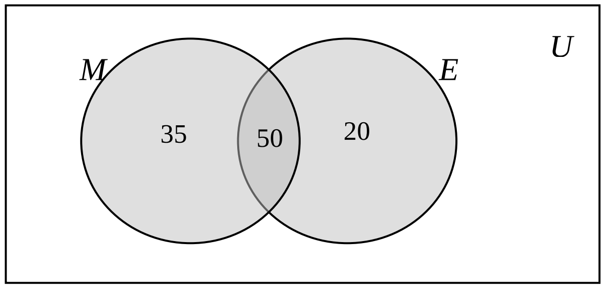
Since 50 students registered for a math and an English course, we know that the number of students in the intersection of M and E is 50



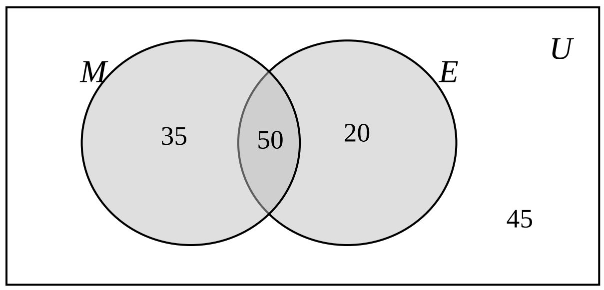
Since 85 students registered for a math course then 85 – 50 = 35 students registered for a math course only.



Since 70 students registered for an English course then 70 – 50 = 20 students registered for an English course only.



35 + 50 + 20 = 105 students registered for a math or an English course so 150 – 105 = 45 students registered for neither a math course nor an English course.



So,

35 students registered for a math course only.
20 students registered for an English course only.
105 students registered for a math course OR an English course.
45 students registered for neither a math course nor an English course.

Do #'s 8-10, 11 b,c, 15, 16, pp. 33-34 text.

Determining the Number of Elements in a Set Using a Formula

We can also determine the number of elements in a set using a formula. If the sets overlap (are non-disjoint), then the number of elements in the union of the sets is the sum of the number of elements in each set minus the number of elements in the overlap (intersection). This is known as the **Principle of Inclusion and Exclusion**. As a formula, the Principle of Inclusion and Exclusion is

**** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ****

Note that this formula can be rearranged to find any one of the other expressions in the formula.

E.g.: Cody asked some people at Heritage Collegiate what type of sports quad they liked.

- 16 students liked the Suzuki QuadRacer.
- 11 students liked the Yamaha Raptor.
- 21 students liked the QuadRacer or the Raptor.
- All student liked either the QuadRacer or the Raptor.

How many students liked the QuadRacer and the Raptor?

Let Q represent the set of people who like the QuadRacer and R represent the set of people who like the Raptor. Then

$n(Q)=16; \quad n(R)=11; \quad n(Q\cup R)=21; \quad n(Q\cap R)=?$

Using the Principle of Inclusion and Exclusion gives

$$n(Q\cup R)=n(Q)+n(R)-n(Q\cap R)$$
$$21=16+11-n(Q\cap R)$$
$$21=27-n(Q\cap R)$$
$$21-27=27-27-n(Q\cap R)$$
$$-6=-n(Q\cap R)$$
$$\frac{-6}{-1}=\frac{-n(Q\cap R)}{-1}$$
$$6=n(Q\cap R)$$

So 6 people liked the QuadRacer and the Raptor.

Sometimes we are interested in the number of elements that are in one set but not in another set. We can derive a formula for this situation. If the sets overlap, the number of elements in one set but not in the other set is the number of elements in that set minus the intersection of the two sets. As a formula, this can be written as

*** $n(A\setminus B)=n(A)-n(A\cap B)$ ***

To find the number of elements that are in one set but not in another set we could also find the number of elements in the union of the two sets and subtract the number of elements in the second set. As a formula, this can be written as

*** $n(A\setminus B)=n(A\cup B)-n(B)$ ***

Note that both of these formulas can also be rearranged.

E.g.: 33 people were asked if they liked table tennis or archery.

- 19 liked archery.
- 11 like only archery.
- All students liked at least one of archery or tennis.

How many liked table tennis and archery?

Let A represent the set of students who like archery and T represent the set of students who like table tennis.

$n(A)=19; \quad n(A\setminus T)=11; \quad n(A\cap T)=?$

Substituting into the formula $n(A\setminus T)=n(A)-n(A\cap T)$ gives

$$11=19-n(A\cap T)$$
$$11-19=19-19-n(A\cap T)$$
$$-8=-n(A\cap T)$$
$$\frac{-8}{-1}=\frac{-n(A\cap T)}{-1}$$
$$8=n(A\cap T)$$

So 8 people liked table tennis and archery.

We can also use a formula to find the number of elements in the union of two sets if we are given the number in the universal set and the number in the complement of the union of the two sets. As a formula this is written as

**** $n(A\cup B)=n(U)-n(A\cup B)'$ ****

Summary

$n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$n(A\setminus B)=n(A)-n(A\cap B)$ OR $n(A\setminus B)=n(A\cup B)-n(B)$

$n(A\cup B)=n(U)-n(A\cup B)'$

E.g.: 8, p. 33 using formulas.

Let V represent the set of people who like vanilla and C represent the set of people who like chocolate.

$n(U)=80; \quad n(V\setminus C)=20; \quad n(V\cap C)=11; \quad n(V\cup C)'=9; \quad n(C\setminus V)=?$

Substituting into $n(V\cup C)=n(U)-n(V\cup C)'$ gives

$$n(V\cup C)=n(U)-n(V\cup C)'$$
$$n(V\cup C)=80-9$$
$$n(V\cup C)=71$$

So 71 people like vanilla or chocolate.

Substituting into $n(V\setminus C)=n(V)-n(V\cap C)$ gives

$$n(V\setminus C)=n(V)-n(V\cap C)$$
$$20=n(V)-11$$
$$20+11=n(V)-11+11$$
$$31=n(V)$$

So 31 people like vanilla ice cream.

Substituting into $n(C\setminus V)=n(V\cup C)-n(V)$ gives

$$n(C\setminus V)=71-31$$
$$n(C\setminus V)=40$$

So 40 people like only chocolate ice cream.

Do # 9, p. 33 text using the formulas.
Do #’s 1, 2, 4-7, p. 38 text.

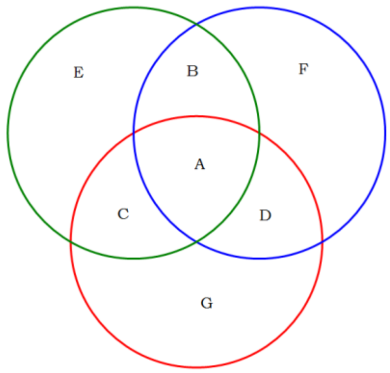
§1.4 **Applications of Set Theory (2 classes)**

Outcomes:

1. Derive the formula for the **Principle of Inclusion and Exclusion** for 3 non-disjoint (overlapping) sets. p. 40
2. Solve problems involving sets using reasoning and formulas. pp. 39-54

Now we will go from problems involving two overlapping sets to problems with 3 overlapping sets. You should be able to solve these problems using reasoning and using formulas. In each case, you should draw a Venn diagram to help you.

Principle of Inclusion and Exclusion for 3 non-disjoint (Overlapping) Sets



To derive the formula for the number of elements in the union of three overlapping sets we will use the diagram to the right.

The union of the 3 sets is everything in the green, blue, and red circles combined. But in combining everything in the 3 circles, we count some areas twice (B, C, D) and one area 3 times (A). So we need to subtract the areas we counted twice. In doing so, we subtract area A 3 times, so we need to add this area to complete the formula.

So the number of elements in the 3 sets is:

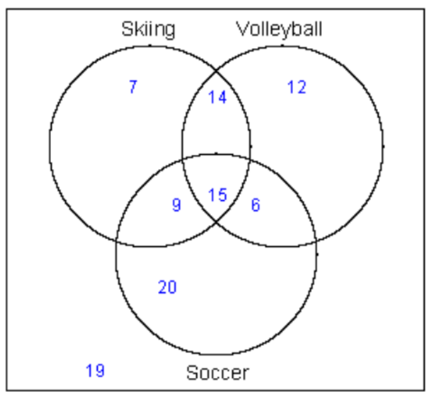
(G) + # (R) + # (B) – # intersection (GB) – # intersection (BR) – # intersection (GR) + #intersection (RGB)

So for 3 overlapping sets A, B, and C

**** $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$****

This is the **Principle of Inclusion and Exclusion** for 3 non-disjoint (overlapping) sets. Note that this formula can be rearranged.

E.g.: Use the Venn diagram below to answer the questions that follow.



How many students do not like skiing or volleyball? ______
How many students like volleyball or soccer? ______
How many students like both skiing and soccer but not volleyball? ______
How many students only like skiing? ______
How many students do not like either volleyball or soccer? ______
How many students like skiing or soccer but not volleyball? ______
How many students like volleyball or soccer but no skiing? ______
How many students like skiing or soccer? ______
How many students like both skiing and soccer? ______
How many students do not like both skiing and volleyball? ______

E.g.: If D represents the set of people who like dance music, R represents the set of people who like rock music and W represents the set of people who like Wrap (Rap) music, determine each of the following.

$n(D) = \_\_\_\_$

$n(R) = \_\_\_\_$

$n(W) = \_\_\_\_$

$n(D \cap R) = \_\_\_\_$

$n(D \cap W) = \_\_\_\_$

$n(R \cap W) = \_\_\_\_$

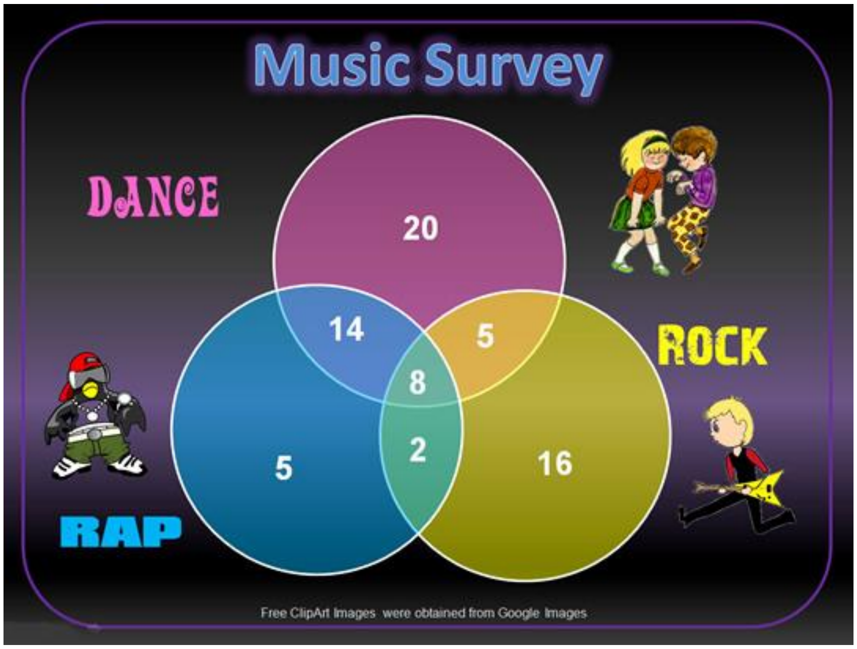
$n(D \cap R \cap W) = \_\_\_\_$

$n(D \cup R \cup W) = \_\_\_\_$

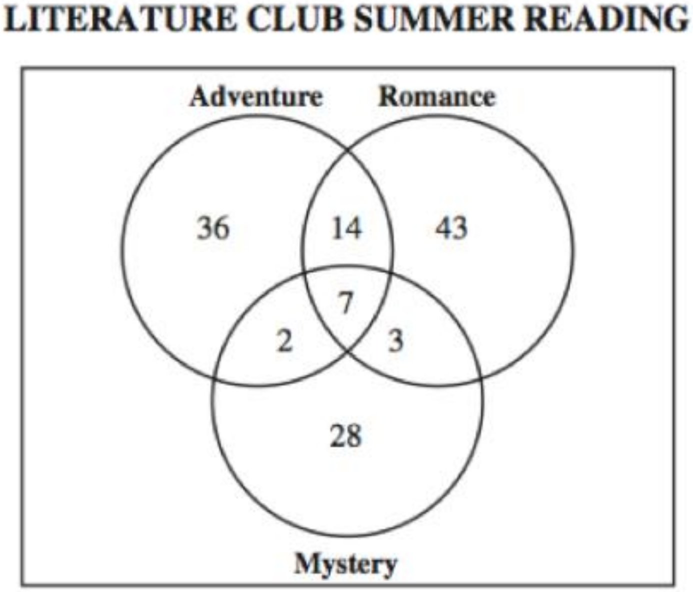
$n(D \setminus R \setminus W) = \_\_\_\_$

$n(R \setminus D \setminus W) = \_\_\_\_$

$n(W \setminus D \setminus R) = \_\_\_\_$



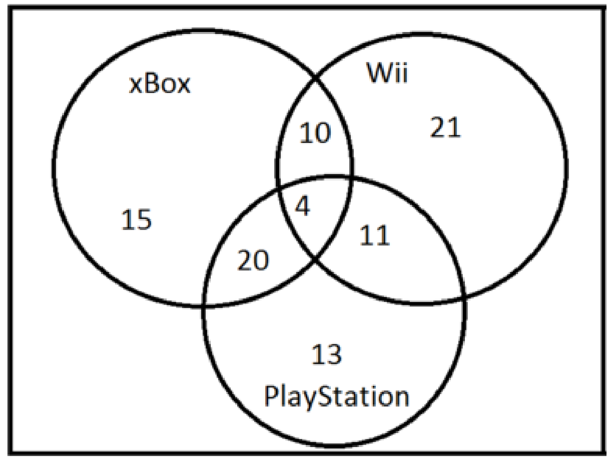
The Venn diagram below shows the types of novels that members of the literature club read during their summer break.



Which of the following statements is NOT supported by the information in the Venn diagram?

a) 21 members read both an adventure novel and a romance novel.
b) 64 members read only an adventure or a mystery novel.
c) 26 members read all three types of novels.
d) 67 members read a romance novel.

E.g.: The Venn diagram below shows the gaming preferences for 100 students.



How many students like xBox? ______

How many students like Wii? ______

How many students like PlayStation? ______

How many students like all 3 gaming systems? ______

How many students like xBox and Wii? ______

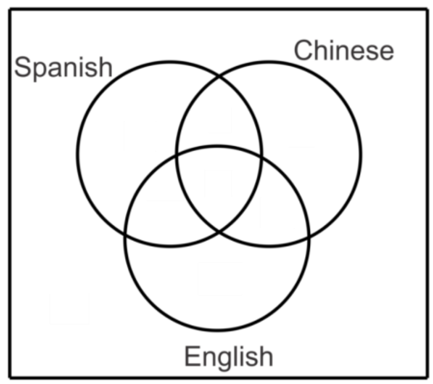
How many students like Wii and PlayStation? ______

How many students like PlayStation or xBox? ______

How many students do not like any gaming systems? ______

Problem Solving with 3 Overlapping Venn Diagrams

E.g.: 105 students were surveyed to see if they spoke Spanish, Chinese, or English. The results of the survey are given below.



- 4 students did not speak either of the languages.
- 51 students spoke Spanish.
- 26 students spoke Chinese.
- 89 students spoke English.
- 11 students spoke Spanish and Chinese.
- 41 students spoke Spanish and English.
- 21 students spoke Chinese and English.

How many students spoke all three languages?

Let *x* be the number of students who spoke all three languages. So $n(S \cap C \cap E) = x$

Since 4 students do not speak any of the three languages, then 105 – 4 = 101 do speak at least one of the three languages. So $n(S \cup C \cup E) = 101$. From the survey results above we know that:

$n(S) = 51$

$n(C) = 26$

$n(E) = 89$

$n(S \cap C) = 11$

$n(S \cap E) = 41$

$n(C \cap E) = 21$

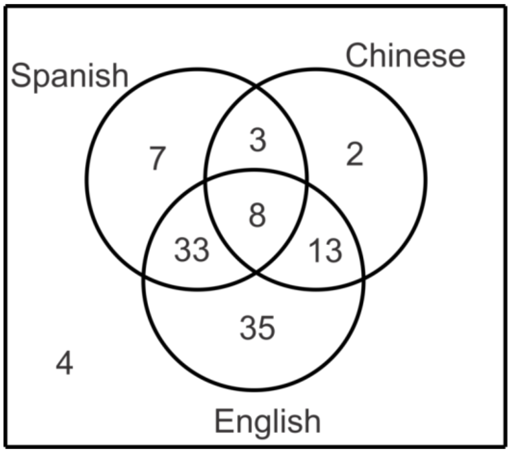
Substituting into the formula (**Principle of Inclusion and Exclusion)**

$n(S \cup B \cup C) = n(S) + n(C) + n(E) - n(S \cap C) - n(B \cap E) - n(S \cap E) + n(S \cap C \cap E)$ gives

$$101 = 51 + 26 + 89 - 11 - 41 - 21 + x$$
$$101 = 93 + x$$
$$8 = x$$

So 8 people speak all three languages.

Complete your Venn diagram and see if it matches the diagram below.



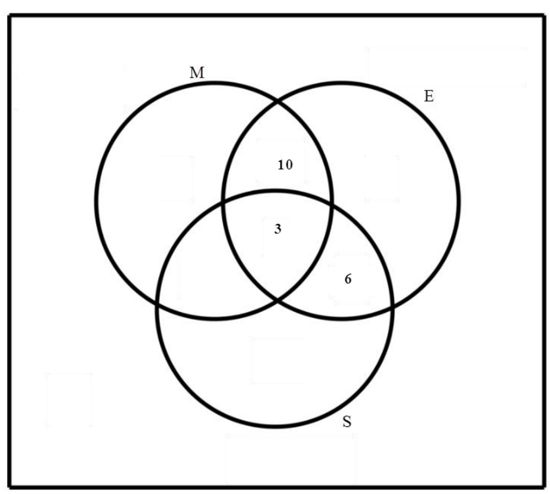
Sample Exam Question

37 Level II students selected courses in Math, Science, and English for their graduating year. Each student selected a course in at least one of the three subjects.

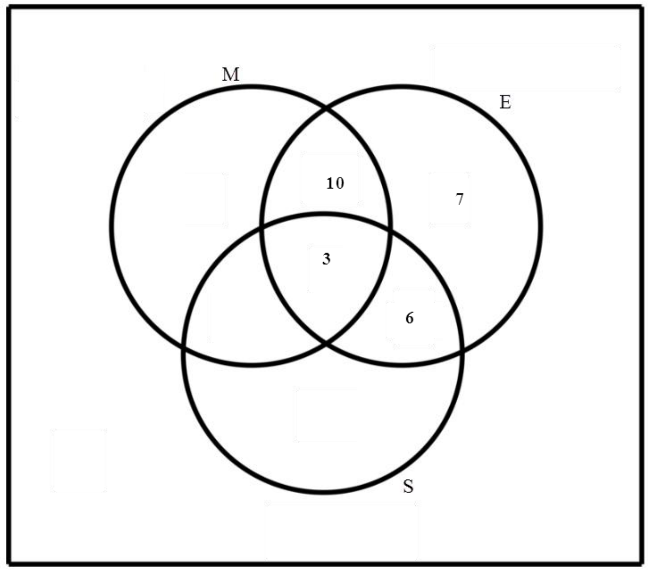
- 22 students selected a math course.
- 16 students selected a science course.
- 26 students selected an English course.
- 6 students selected a science and an English course but not a math course.
- 10 students selected a math and an English course but not a science course.
- 3 students selected a science, a math, and an English course.

How many students selected only a science course?

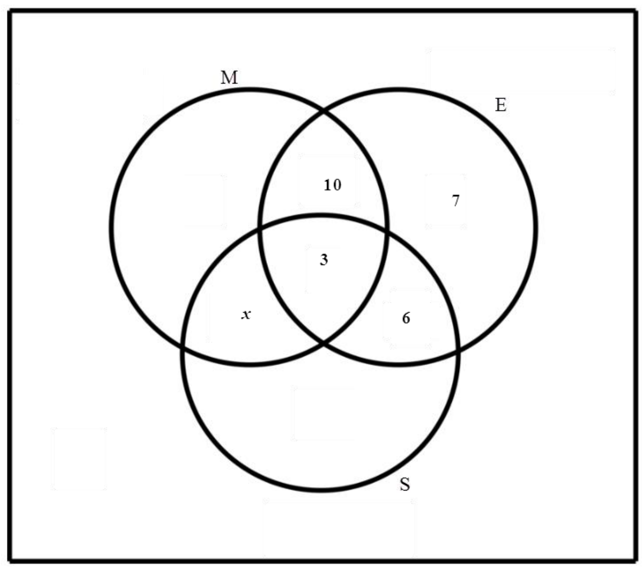
Let's fill in what we know so far on a Venn diagram.



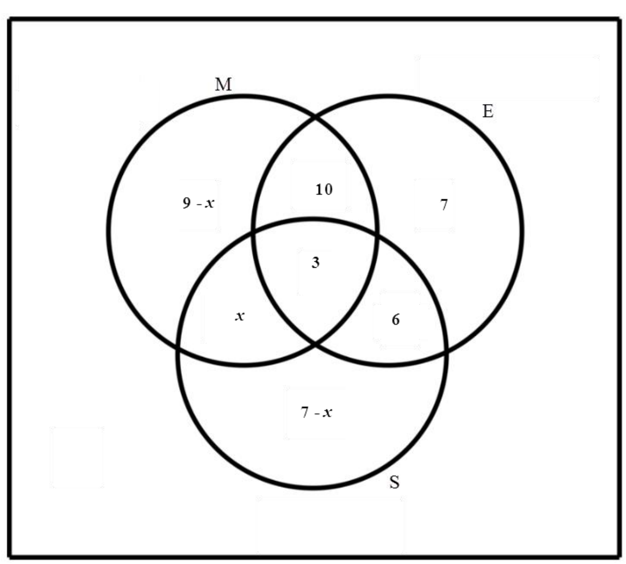
Since 26 students selected an English course, the number of students in the remaining region for the English circle must be 26 – 10 – 3 – 6 = 7.



Let *x* be the number of students who selected a math and a science course but not an English course.



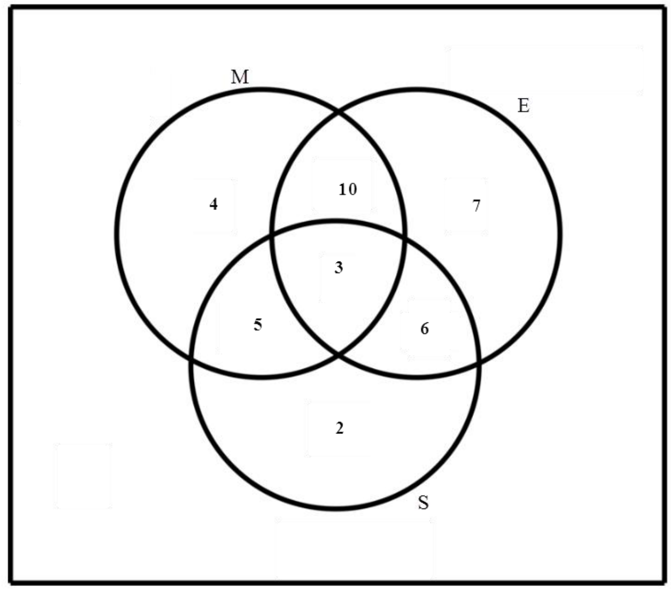
Then the number of students in the remaining region for the math circle must be $22-10-3-x=9-x$ and the number of students in the remaining region for the science circle must be $16-6-3-x=7-x$.



Since the total number of students is 37, then
$$10+3+6+7+7-x+9-x+x=37$$
$$42-x=37$$
$$x=5$$

So 2 students selected only a science course.

As a check you should make sure these numbers add to 37.

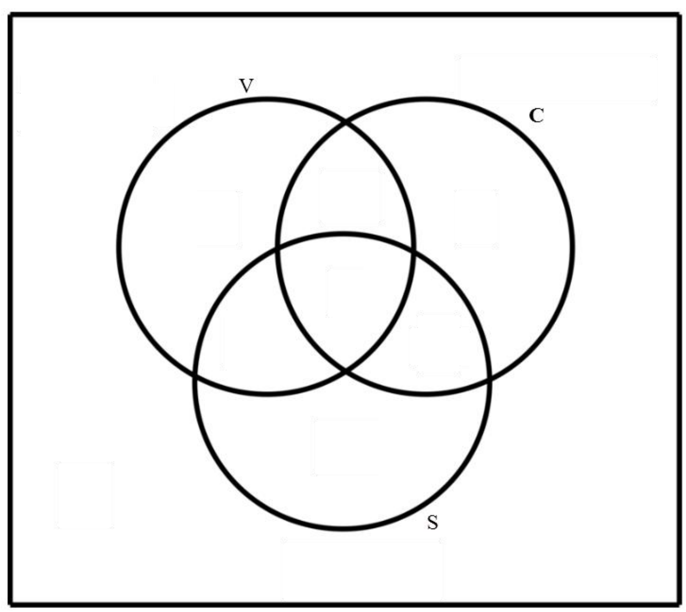


Your Turn

A survey of 38 children gave the following results.

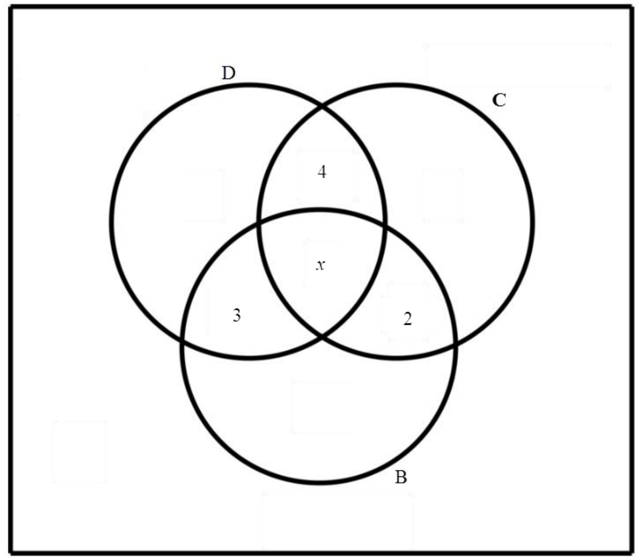
- All children liked at least one flavour of vanilla, chocolate, and strawberry ice cream.
- 25 children liked vanilla ice cream.
- 20 children liked chocolate ice cream.
- 16 children liked strawberry ice cream.
- 3 children liked chocolate and strawberry but not vanilla ice cream.
- 8 children liked chocolate and vanilla but not strawberry ice cream.
- 5 children liked vanilla, chocolate, and strawberry ice cream.

How many students like only strawberry ice cream?
[Ans: 6]

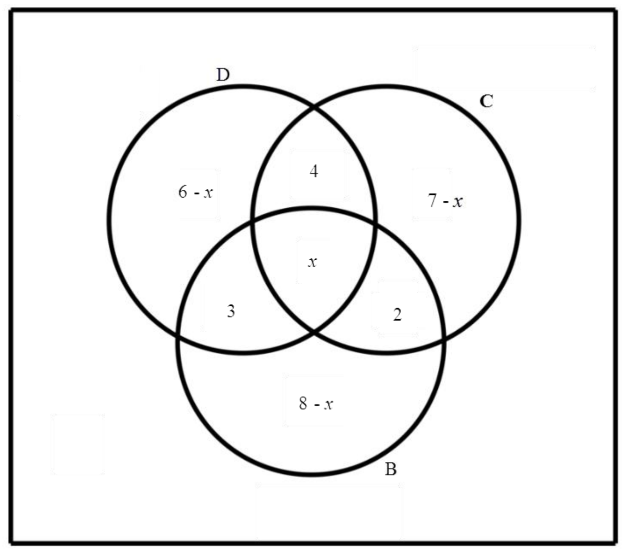


E.g.: Example 1 a), p. 40 text.

Let x be the number of children with a dog, a cat, and a bird.



The number of students in the remaining region for the dog circle must be $13-4-3-x=6-x$, the number of students in the remaining region for the bird circle must be $13-2-3-x=8-x$, and the number of students in the remaining region for the cat circle must be $13-4-2-x=7-x$.



The numbers in these regions must add up to 28, so

$$\begin{aligned}&4+3+2+x+6-x+7-x+8-x=28\\&30-2x=28\\&-2x=-2\\&x=1\end{aligned}$$

So 1 child has a cat, a dog, and a bird.

Do # 4, p. 56 text.
Do #'s 1, 2, 4, 6, 9, pp. 51-52 text.
Do #'s 2-7, p. 58 text.