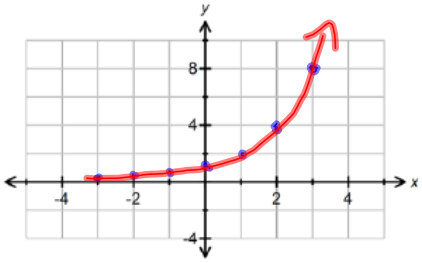
**Investigation:**

1. a. Complete the table of values and sketch the graph of: $y = 2^x$

| x | y |
|---|---|
| -3 | 0.125 |
| -2 | 0.25 |
| -1 | 0.5 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |




$2^{-3} = \left(\frac{1}{2}\right)^3 = 0.125$

$2^{-2} = \left(\frac{1}{2}\right)^2 = 0.25$

$2^{-1} = \left(\frac{1}{2}\right) = 0.5$

b. Identify the characteristics.

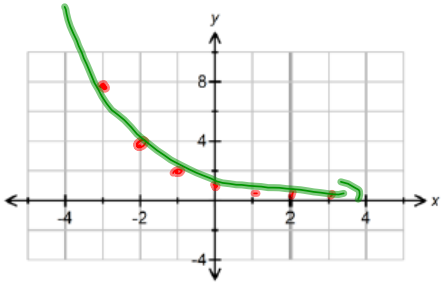
| | |
|---|---|
| x-intercept | none |
| y-intercept | 1 |
| End Behaviour | Q2 to Q1 |
| Increasing/Decreasing | increasing |
| Horizontal Asymptote | $y = 0$ |
| Domain | $x \in \mathbb{R}$ |
| Range | $\{y \mid y > 0, y \in \mathbb{R}\}$ |

c. Compare the pattern in the table of values with the b-value.

$b > 1$ increasing

2. a. Complete the table of values and sketch the graph of: $y = \left(\frac{1}{2}\right)^x$

| x | y |
|---|---|
| -3 | 8 |
| -2 | 4 |
| -1 | 2 |
| 0 | 1 |
| 1 | 0.5 |
| 2 | 0.25 |
| 3 | 0.125 |




$\left(\frac{1}{2}\right)^{-3} = \frac{2^3}{1} = 2^3 = 8$

$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$

b. Identify the characteristics.

| | |
|---|---|
| x-intercept | none |
| y-intercept | |
| End Behaviour | Q2 to Q1 |
| Increasing/Decreasing | decreasing |
| Horizontal Asymptote | $y = 0$ |
| Domain | $x \in \mathbb{R}$ |
| Range | $\{y \mid y > 0, y \in \mathbb{R}\}$ |

c. Compare the pattern in the table of values with the b-value.

$0 < b < 1$ so graph is decreasing

3. How are the graphs of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$ alike and how are they different?

Alike

Same x-int.

Same end behavior

horizontal asymptote

domain

range

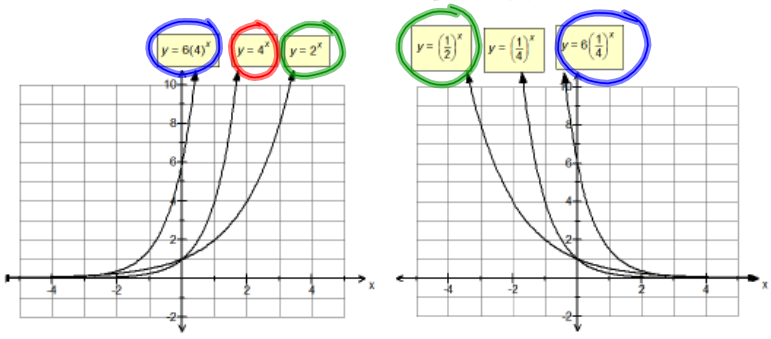
y-int

Different

increasing/decreasing

different b-value

4. Given the following graphs in the form: $y = a(b)^x$




Graph A Graph B

a. Complete the table.

| | a | b | y-intercept | increasing/ decreasing |
|---|---|---|---|---|
| $y = 2^x$ | 1 | 2 | 1 | increasing |
| $y = 4^x$ | 1 | 4 | 1 | in |
| $y = 6(4)^x$ | 6 | 4 | 6 | in |
| $y = \left(\frac{1}{2}\right)^x$ | 1 | 1/2 | 1 | d |
| $y = \left(\frac{1}{4}\right)^x$ | 1 | 1/4 | 1 | d |
| $y = 6\left(\frac{1}{4}\right)^x$ | 6 | 1/4 | 6 | d |

b. Compare the **a-value** with the **y-intercept**.

What conclusion can you make?

same thing

c. Compare the **b-value** with the **shape** of the graph.

What conclusion can you make?

$b > 1$ → ↗ increasing

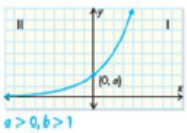
$0 < b < 1$ → ↘ decreasing

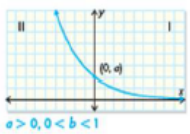
**Characteristics of Exponential Functions of the Form:**

$y = a(b)^x$ where $a > 0$ and $0 < b < 1$ or $b > 1$

- the number of x-intercepts: none
- y-intercept = a
- end behaviour: extends from Q2 to Q1
- equation of asymptote: $y = 0$
- domain: $x \in R$
- range: $y > 0$
- as x-values increase by 1, the y-values will increase/ decrease by a constant ratio equal to the b-value
- if b > 1 OR if 0 < b < 1

increasing from Q2 to Q1 decreasing from Q2 to Q1



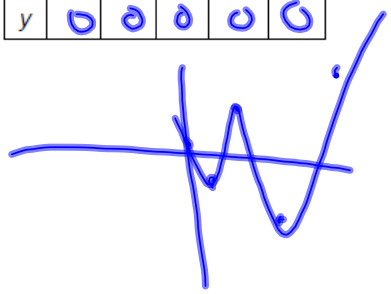



1. What will happen if $b=1$ ?

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 1 | 1 | 1 | 1 | 1 |

2. What will happen if $b=0$?

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 0 | 0 | 0 | 0 | 0 |



3. What will happen if $b<0$?

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 1 | -2 | 4 | -8 | 16 |

$1^x$

$b=1 \rightarrow 1=1$

$1^2=1$

$1^3=1$

$\vdots$

$b=0 \quad 0^x$

$b<0 \quad (-2)^x \quad (-2)^0=1$

$(-2)^1=-2$

$(-2)^2=4$

$(-2)^3=-8$

$(-2)^4=16$

***Journal Question:***

How are the functions $y=x^2$ and $y=2^x$ alike/different?

$y=x^2$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

$y=2^x$

| x | y |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

increasing → diff rates

exponential

diff y-intercepts

*Example 1:*

State the characteristics of each exponential function.

a) $y=9\left(\frac{2}{3}\right)^x$

b) $y=\frac{1}{2}(3)^x$

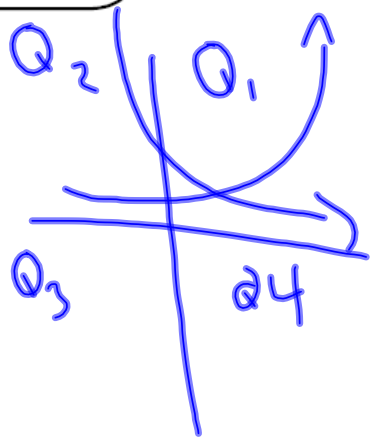
| | a) | b) |
|---|---|---|
| x-intercept | none | none |
| y-intercept | 9 | 1/2 |
| end behaviour | Q2 to Q1 | Q2 to Q1 |
| inc/dec | dec | inc |
| Equation of Asymptote | y=0 | y=0 |
| Domain | {x\|x∈R} | {x\|x∈R} |
| Range | {y\|y>0, y∈R} | {y\|y>0, y∈R} |

c) $y = e^x$

| | |
|---|---|
| x-intercept | none |
| y-intercept | 1 |
| end behaviour | Q2 & Q1 |
| inc/dec | inc |
| Equation of Asymptote | y=0 |
| Domain | $\{x \mid x \in \mathbb{R}\}$ |
| Range | $\{y \mid y > 0, y \in \mathbb{R}\}$ |

NOTE:
*e* is a constant known as Euler's number.
e = 2.718....

$y = (2.718...)^x$



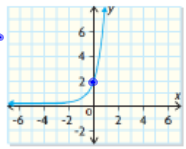

*Example 2:* (Ex. 3, p. 343)

Which exponential function matches each graph below? Explain why.

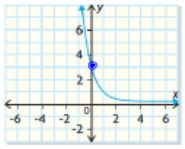
i) $y = 3(0.2)^x$ ii) $y = 4(3)^x$ iii) $y = 4(0.5)^x$ iv) $y = 2(4)^x$

C B D A

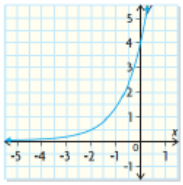
a) y-int=2



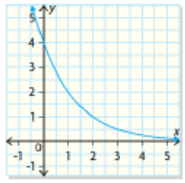
c)



b) inc



d) dec



*YOUR TURN:* p. 345

Which exponential function matches each graph below? Explain why.

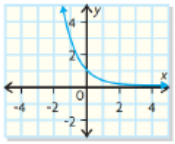
i) $y=(3)^x$ B

ii) $y=\frac{1}{3}(3)^x$ C

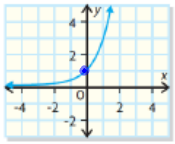
iii) $y=3\left(\frac{1}{3}\right)^x$ D

iv) $y=\left(\frac{1}{3}\right)^x$ A

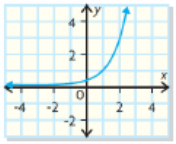
a)



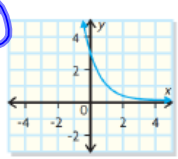
b)



c)



d)



Practice:

p. 347 - 350, #3, 4abcd, 5abcd, 9, 11, 12ace, 13, 15